Computational Models Using the Equatorial Beta Plane

M. Bancroft*
Department of Physics & Astronomy, Bates College, Lewiston, ME 04340, USA
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The goal of this research was to study the problems that arise due to the mathematical formula used/required when studying geophysical behavior along and near the equator. Error can arise because the formula for the Coriolis force has a sine term contained within it, which returns a zero when applied to behavior running parallel to the equator. In order to account for this we use the equatorial beta plane, which is an approximation for the Coriolis force. This research is attempting to use python framework Dedalus and Bates super computer Leavitt to create models of geophysical behavior. For these models we are examining Rossby waves in the Tachocline region of the sun. First we will model these waves at a latitude above the equator of the sun then we will look at the model as it is adjusted to model waves near the equator.

I. INTRODUCTION

The goal of this thesis is to explore if we can effectively use the periodic equatorial beta plane to create simulations of naturally occurring geostrophic phenomena. In order to talk about the equatorial beta plane we must first examine the Coriolis force. This is a fictitious force that involves a curved path an object follows when traveling latitudinally across a rotating body. This force is given in the equation below where f is the Coriolis force, Ω is the angular velocity and ψ is the angle from the equator.

$$f = 2\Omega sin(\psi) \tag{1}$$

An issue arises with this equation when considering waves that propagate parallel to the equator and stay at or near zero latitude. This gives us a value for ψ of zero which outputs a value of zero for the Coriolis force. This issue can be navigated around by using the small angle approximation. This allows us to rewrite $sin(\psi)$ as roughly equal to y. From here we are able to use an alternate equation known as the equatorial beta plane.

The equatorial beta plane allows us to maintain a Coriolis force greater than zero when considering waves parallel to the equator. This is achieved by using the latitudinal distance from the equator as the only variable. The equation for the equatorial beta plane can be found below where again; f is the Coriolis force, y is the latitudinal distance from the equator, and β is the beta plane constant $\frac{2\Omega}{a}$. Ω is the reference frame's angular rotation rate and a is its radius.

$$f = \beta y \tag{2}$$

The test case we are building this simulation for is based off the paper β-Plane Magnetohydrodynamic Turbulence in the Solar Tachocline. In this paper they examine naturally occurring equatorial Rossby waves in the Tachocline region of the sun. This region (shown below) is important in driving solar magnetic activity and may also play a role in the mixing processes of the solar interior.

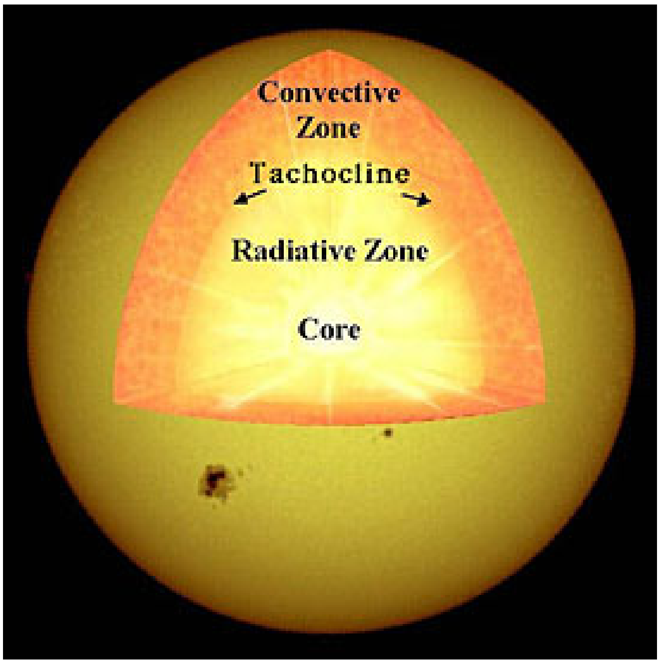

FIG. 1: Model of the Sun

II. METHODOLOGY

Our goal is to regenerate the results of this paper by running computational simulations on Bates College's high-performing computing cluster(HPCC). The HPCC is named after the twentieth century astronomer Henrietta Swan Leavitt. Leavitt includes eleven compute nodes, each with twentyeight cores. Each core runs one programming command instruction, and the multicore setup allows for several tasks to be run simultaneously. Depending on the magnitude of our simulation, we chose how many nodes and cores to run it on. We are able

*Electronic address: mbancrof@bates.edu

to communicate with Leavitt remotely from my windows desktop which is running a virtual Ubuntu machine.

The program used to write our scripts is the text editor Emacs. These simulations have two files included with them one Python script thats solves the system of differential equations and one run script which instructs Leavitt on how to run the Python script.

Dedalus is a spectral method differential equation package for python built by my advisor Jeffrey Oishi and his colleagues. It uses bases in order to generate sparse matrices that lead to solutions. Dedalus has a few bases that the user can pick from when writing the code. The two we will use are Chebyshev polynomials and Fourier series. Both of these bases have there advantages when implemented in solving differential equations. Fourier series are much faster and cheaper in terms of processing power. However, it is not as stable as Chebyshev polynomials which are reliable for these problems.

One disadvantage the Fourier series also has lies within the nature of the bounds. This basis requires the beta plane to be periodic. However, graphing the linear beta plane periodically elicits a discontinuous function. Therefore, we must adapt the equations for the y component in order to create a stable, periodic, continuous form of the Coriolis force. To do this we use sponge layers which will give us inaccurate results at the edges of the simulation however this does not effect the accuracy of the simulation near the equator. To do this we introduced new equations for y where Y(y) is the control equation and Z(y) is the damping equation.

$$Y(y) = \frac{L_y(1+\zeta)}{\pi\zeta} arctan\left(\frac{\zeta sin(\theta)}{1+\zeta cos(\theta)}\right) \tag{3}$$

$$Z(y) = \frac{(1-\zeta)^2}{2}\frac{(1-cos(\theta))}{(1+\zeta^2+2\zeta cos(\theta))} \tag{4}$$

In these equations ζ is the control parameter for the sponge layer, L_y is the length in longitude, and $\theta = \frac{\pi}{L_y}$ These adjustments allow us to run the scripts using a Fourier series basis with the equatorial beta plane being both periodic and continuous.

III. RESULTS

IV. DISCUSSION

V. APPENDICES

VI. BIBLIOGRAPHY

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