Bank Capital Structure and Productivity Growth

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First version: May 2018

This version: February 2023

Abstract

We study the long-run impact of macroprudential policies in a dynamic general equilibrium model with endogenous productivity growth and an endogenous balance sheet structure. Banks choose liabilities depending on their risk perceptions, and pecuniary externalities create a feedback mechanism between banking and entrepreneurial activity that amplifies the financial accelerator. Macroprudential tools can dampen this mechanism, and we uncover a non-monotonic relationship between long-run growth and the intensity of macroprudential rules. In particular, by performing a welfare analysis, we show that a benevolent regulator faces a trade-off between growth and risk.

Keywords: Macroprudential Policy, Capital Requirements, Economic Growth, Risk, DSGE

JEL classification: O16, E44, E58

1 Introduction

There exists ample empirical evidence that a financial system with fewer constraints promotes entrepreneurship and innovation, thus fostering economic growth (Brown et al. (2009), Aghion et al. (2010), Brown et al. (2012), Aghion et al. (2014), Beck et al. (2014), Hsu et al. (2014), Bassetto et al. (2015), Acharya and Xu (2017)). In the wake of the Great Recession of 2008, abundant theoretical and empirical literature had developed, exploring the role of macro-prudential policies in mitigating the adverse effects of financial constraints. A particular focus is given to the role of these policies in reducing the risk of systemic failure (e.g., Blanchard et al. (2010), Galati and Moessner (2013)), as well as their role in smoothing business cycle fluctuations coming from the so-called financial accelerator mechanism (see, e.g., Hanson et al. (2011), Galati and Moessner (2013) and Borio (2011))

However, a general understanding of the long-run impacts of macro-prudential tools is still lacking from both an empirical and a theoretical perspective. In this paper, we aim to contribute to this understanding by first developing a theoretical model that incorporates a link between credit frictions and long-term endogenous economic growth. Precisely, entrepreneurs in our economy have the skills to produce new innovations but need to rely on external funds, which are provided by financial intermediaries (bankers) who endogenously choose the composition of their balance sheet. We show that if intermediaries take excessive risk, the financial sector is more exposed to fire sales externalities. The more important the effects of the externality, the less banks will be able to finance innovation. Having laid out the link between credit frictions and long-term growth, we then use our model to address the long-term consequences, if any, of macro-prudential policy rules.

In our setup, the agency problem between bankers and their creditors in the capital market is based on Gertler et al. (2012). Bankers lend to entrepreneurs by raising funds in the capital market and can do so by issuing contingent and non-contingent debt. The latter is relatively favored by bankers because it allows them to raise more funds, thus increasing leverage. However, non-contingent debt, by preventing bankers from hedging against adverse shocks, increases the risk of their balance sheet. Since banks' assets correspond to claims on innovation profits, negative shocks to innovation profits adversely impact the balance sheet of all bankers, creating a pecuniary externality. If bankers could jointly choose the composition of their balance sheet, they would take into account this externality and would choose a less risky funding scheme, favoring contingent debt. However, in the decentralized equilibrium, bankers fail to internalize this effect and choose a riskier balance sheet.

It is the market failure prompted by the pecuniary externality that leads to the need for macroprudential interventions. Broadly speaking, these interventions are meant to induce bankers to be more heavily funded with loss-absorbing liabilities. In more detail, motivated by the guidance provided by the Basel Committee on Banking Supervision - commonly referred to as the Basel III reform package - we restrict attention to macroprudential policies implemented through capital requirements (BCBS (2010)). Moreover, we illustrate the quantitative properties of our model by applying three distinct rule designs. We obtain that macroprudential policies have two main implications. First, they dampen cycle volatility in the banking sector and reduce the amplification mechanism of financial frictions on the business cycles. Second, more interestingly, there exists a non-monotonic relationship between the magnitude of the policy intervention and long-term economic growth. Given this non-monotonicity, and to evaluate the optimal intervention, we derive a welfare function and optimize each design we consider within a reasonable parameter interval. This allows the regulator to calibrate the intensity of the intervention to achieve the highest welfare within a given policy design. We show that the optimal intervention is not necessarily associated with the highest economic growth.

Our work is closely related to Queralto (2019), which also examines the link between financial frictions and long-term endogenous economic growth. Still, it is particularly interested in the long-term recovery from a financial crisis. Queralto borrows the amplification mechanism of financial shocks from Bernanke and Gertler (1989) and Kiyotaki and Moore (1997) and, similarly to our model, shows that the endogenous nature of innovations amplifies this mechanism. However, in contrast to our setup, bankers in his model can only raise funds by issuing non-contingent debt in such a way that the steady state level of economic growth is determined independently from the risk structure of assets. Our analysis also differs due to our focus on policy and the link between risk and growth.[1]

Anzoategui et al. (2016) also link financial frictions and economic growth, with a focus on business cycle persistence. In their model, the financial sector is not modeled directly, and financial frictions appear as an exogenous liquidity shock. This shock reduces resources available to both technology adoption and development, inducing a fall in TFP growth and triggering a highly persistent downturn. As in Queralto (2019), the growth rate of the economy is independent of the risk structure of assets. Our paper also relates to Moran and Queralto (2017), who developed a New Keynesian model featuring endogenous TFP change and studied the impact of the zero lower bound constraints on monetary policy in the medium run. They obtain that the zero lower bound constraints impose sizable and permanent output and TFP losses.

[1]Consistently with our approach, Queralto (2019) presents evidence supporting the persistent, long-term effects of the financial crisis. In turn, Ridder (2016) presents evidence from firm-level data in support of the link between the financial crisis and reduced long-term growth.

The interaction between the macroprudential and monetary policy is studied by de Groot (2014) and Angeloni and Faia (2013), focusing on how macroprudential policy design must take into account the interconnections between macroprudential and monetary policy. Our approach differs from theirs because we assume flexible and also endogenous TFP changes. However, the risk-taking channel of monetary policy emphasized in these papers is related to the risk channel emphasized here.

Optimal bank capital is discussed quantitatively in Miles et al. (2013) and Angelini et al. (2015). These papers report long-run social costs and benefits of a financial system funded more heavily with loss-absorbing liabilities. They also study how capital requirements modify the required rate of return in banking activities and how capital requirements induce a decrease in bank failure rate. But there is no endogenous growth in their models, and there is no consideration of how risk affects the balanced growth path state itself.

The remainder of the paper is organized as follows. Section 2 explains the model. The subsequent section details our calibration strategy and discusses the computational challenges in solving the model. Section 4 motivates the macroprudential policy and discusses the details of the rules we considered in the study. In this section, we also compare the results under different rules. The final section concludes the paper. We added the derivation of some equations from the model, the computational strategy, and details on the steady state to the online appendix.

2 The Model

Our benchmark environment follows Gertler et al. (2012), but there are several modifications. We endogenize productivity growth in line with the work of Queralto (2019). The economy is closed, and prices are flexible. Banks have a single activity: financing innovative activity. These financial institutions obtain funds in the credit markets to fund innovations created by an entrepreneurial sector. Banks can raise funds by two means: a short-term debt paying a noncontingent rate of return and/or a long-term debt contract paying a contingent rate of return. An agency problem between the banker and its creditors appears as financial friction. A banker faces a meaningful trade-off between risk exposure and returns by changing the balance sheet composition of the financial institution it manages. A banker optimally addresses the trade-off between hedging against balance sheet risk and tightening its incentive constraint: raising funds through equity tightens the incentive constraints. Still, it allows banks to hedge their balance sheet restriction partially.

The final good is homogeneous. The entrepreneurial sector uses funds borrowed from banks to finance the creation of new firms. These new firms are interpreted as varieties that compete in a monopolistically competitive market serving as input to a perfectly competitive final good sector. Total factor productivity (TFP) is growing by means of expanding varieties, as in Romer (1990). Bankers can perfectly monitor entrepreneurial activities, capturing all profits from monopolistically competitive intermediary goods producers. We abstract from both fiscal and monetary policy. This is done for simplicity. It should be clear that it is straightforward to extend our model to consider both fiscal and monetary policy. Still, we use the simple case to focus on our point.[2]

2.1 Households

The household sector follows the extension proposed by Gertler et al. (2012) to the framework developed by Gertler et al. (2010) and Gertler and Karadi (2011). There is a continuum of households with unity mass. A mass f of households is composed of workers, and the remaining $(1-f)$ mass is formed by bankers. A fixed fraction of workers is endowed with special skills that allow them to work in entrepreneurial activities. The remaining fraction of workers elastically supply work to the intermediate sector.

Bankers can be interpreted as specialists in finding investment opportunities and monitoring businesses that receive their funds. In this sense, bankers manage financial institutions that transfer funds from lenders to borrowers. While the relationship between banks and the entrepreneur sector is assumed to be frictionless, financial friction limits how much funding bankers can raise. Because the flow of funds is forcibly bounded, the financial friction puts a wedge between how much a bank pays for its debt and how much return it earns for its assets. This spread between the cost and revenue rate of return implies that banks accumulate net worth over time. To avoid the fact that bankers can potentially overcome financial friction, job turnover between workers and bankers is introduced.

In every period, a banker becomes a worker with probability $(1-\sigma)$, *i.i.d.* drawn across time and bankers. At the end of the career, the banker transfers his net worth to his family. To retain the proportion of bankers and workers, a fraction $(1-\sigma)\frac{f}{1-f}$ of workers start their career as bankers. Each entering banker receives a start-up net worth transfer so that they can operate in the financial markets.

The household maximizes utility by choosing an optimal plan for consumption C_t, labor L_t, one period of noncontingent debt D_t, contingent debt $\bar{E}_t$ and capital holdings K_{t+1}.

The utility function is a GHH-type function. Even though our main concern is the long run, short-run dynamics have a key role within the model because the risk is incorporated into the model solution, and

[2] In what follows, we present the model with two innovations or shocks. The qualitative results hold within a reasonable calibration with a single shock on the productivity of intermediary firms.

this format of the utility function abstracts from the wealth effect on labor supply. A counterfactual labor supply would be unrealistic and lead to an incorrect interpretation of the results.

$$U(C_t, L_t) = \frac{(C_t - \Psi H_t \frac{1}{1+\epsilon} L_t^{1+\epsilon})^{1-\rho} - 1}{1-\rho} \tag{1}$$

where H_t is a term governing labor disutility. For simplicity, this term is assumed to be equal to the technological level of the economy, namely,

$$H_t = A_t \tag{2}$$

This assumption is necessary to establish a balanced growth path with constant hours but also greatly reduces computational difficulties. Defining a welfare function is straightforward, and it reduces computational effort. Because of the GHH format and because the economy features long-run growth, labor disutility is required to grow at the level pace of output over the long term. We could then forcibly assume an *ad hoc* format for the evolution of the labor disutility term H_t, but the goal would be the same. We preferred a simpler approach.

The household problem is then

$$\max_{(C_t, L_t, \bar{E}_t, K_{t+1}, D_t)} \mathbb{E}_t \sum_{i=0}^{\infty} \beta^i U(C_{t+i}, L_{t+i}) \tag{3}$$

subject to the budget constraint

$$C_t + K_{t+1} + D_t + q_t \bar{E}_t \leqslant R_t^k K_t + (1-\delta) K_t + W_t L_t + R_t D_{t-1} + \bar{E}_{t-1} \phi \left[q_t + \pi_t \right] + \xi_t \tag{4}$$

where R_t^k is the rental rate of capital in period t, R_t is the risk-less bond return, q_t is the price of bank equity, $\phi \pi_t$ is the dividend paid by the bank[3], W_t is the wage rate in the intermediate sector, and ξ_t is the net transfers from bankers and labor income from skilled household members. The capital depreciation rate is denoted by δ.

The marginal utility of consumption $U_{c,t}$ is

$$U_{c,t} = \left(C_t - \Psi A_t \frac{1}{1+\epsilon} L_t^{1+\epsilon} \right)^{-\rho} \tag{5}$$

[3] We assumed that bankers are repacking their assets and selling them to households.

We define the *household stochastic discount factor* between periods t and $t+i$, $\Lambda_{t,t+i}$

$$\Lambda_{t,t+i} = \frac{\beta^i U_{c,t+i}}{U_{c,t}}$$

Decisions on bond and equity holding must satisfy the following optimality conditions:

$$1 = \mathbb{E}_t(\Lambda_{t,t+1})R_{t+1} \tag{6}$$

$$1 = \mathbb{E}_t(\Lambda_{t,t+1}R^e_{t+1}) \tag{7}$$

where

$$R^e_{t+1} = \phi\frac{q_{t+1} + \pi_{t+1}}{q_t} \tag{8}$$

Decisions on capital holdings must satisfy

$$1 = \mathbb{E}_t\left(\Lambda_{t,t+1}(1 - \delta + R^k_{t+1})\right) \tag{9}$$

Labor supply is simply given by

$$\Psi A_t L_t^{\epsilon} = W_t \tag{10}$$

We next turn to the description of the entrepreneurial sector, which is a force of growth within the economy.

2.2 Entrepreneurs

Our entrepreneurial sector follows closely that in Queralto (2019). There is an unbounded mass of prospective entrepreneurs with the ability to introduce new varieties of intermediate goods. Each entrepreneur combines resources (final output) and skilled labor hired in a competitive market to create a new firm variety. One alternative interpretation is that there exists a competitive sector that produces varieties, and we call this sector "entrepreneurs."

Existing and newly created firms face the risk of an exogenous exit shock that occurs with probability $(1-\phi)$. The probability of survival is thus ϕ between adjacent periods. Let A_t be the number of existing varieties in operation in period t, and Z_t be the mass of new firms created on date t. The law of movement of the number of operating firms is thus given by

$$A_{t+1} = \phi(A_t + Z_t) \tag{11}$$

Entrepreneurs need to obtain funding to finance entry and production. We assume that entrepreneurs have no internal funds and must rely on external funds provided by banks: entrepreneurs have to pay the entire entry cost by borrowing from financial intermediates. In a sense, bankers are specialists in monitoring, aiming to capture the whole financing market.

Contracts between banks and entrepreneurs are assumed to be *frictionless*. Essentially, to raise funds, an entrepreneur issues a security that is perfectly contingent on the success of his or her project. In that sense, the bank that lends to the entrepreneur effectively owns the future financial rights of the financed variety. Banks hoard risk in their balance sheet. [4]

The production of new varieties is given by

$$Z_t = \Upsilon_t^z \mathcal{R}_t^{\eta} (A_t L_{S,t})^{1-\eta} \tag{12}$$

where $\mathcal{R}_t$ is the amount of materials used in production in units of the final output, A_t is the aggregate technological level, and $L_{S,t}$ is the number of skilled workers hired.

Endogenous Growth. Assuming that the production of Z_t depends on A_t through augmenting $L_{S,t}$ (which is exogenous and constant) is key to generating endogenous growth. As Z_t increases with A_t, the following period's technological level A_{t+1} grows, and so does the production of new varieties in the following period Z_{t+1}.

The specification in (12) also allows for exogenous variation in aggregate entrepreneurial productivity, captured by the random variable Υ_t^z, which follows an AR(1) process as

$$\log(\Upsilon_t^z) = \rho_z \log(\Upsilon_{t-1}^z) + \epsilon_t^z$$

where ϵ_t^z is an *i.i.d.* innovation.

Denoting the skilled labor wage by $W_{S,t}$, the expression for the marginal cost of producing a variety MC_t is given by

$$MC_t = \frac{1}{\Upsilon_t^z} \left(\frac{1}{\eta}\right)^{\eta} \left(\frac{W_{S,t}/A_t}{1-\eta}\right)^{1-\eta} \tag{13}$$

The entrepreneur has no funding to finance the sunk cost MC_t. To obtain funds, he or she issues a perfect state-contingent claim on the future profits of the produced variety to the bank that has funded the entrepreneur. Let J_t be the price of a unit of entrepreneur equity. Given the free entry of entrepreneurs, the

[4] Some may prefer to see banks as entrepreneurs.

price of the variety issued must equal its production sunk cost, i.e.,

$$J_t = MC_t \tag{14}$$

Once the entrepreneur has paid the sunk cost in period t, the variety is ready to operate in period $t+1$. A proportion $(1-\phi)$ of new entrants will never produce, as the exit shock happens after the production and entry of varieties. Financing the production of these varieties is risky: the profits from funded varieties may never come due to the exogenous exit shock, or profits might be temporarily low due to exogenous variation in the real sector.

The aggregate supply of skilled labor is assumed to be inelastic and fixed at $\bar{L}_S$. We can then derive a relation between the price of the entrepreneur equity J_t and the number of new entrants Z_t.

$$J_t = \left[\frac{1}{\Upsilon_t^z}\right]^{\frac{1}{\eta}} \left[\frac{1}{\eta}\right] \left[\frac{1}{\bar{L}_S}\right]^{\frac{1-\eta}{\eta}} \left[\frac{Z_t}{A_t}\right]^{\frac{1-\eta}{\eta}} \tag{15}$$

This equation can be interpreted as an aggregate supply curve of new entrant firms as a function of the price of the unit of entrepreneur equity J_t. To define an equilibrium between the supply and demand of varieties, we now turn to describe the banking sector.

2.3 Financial Intermediaries

Our financial sector closely follows Gertler et al. (2012), but banks finance entrepreneurial activities instead of physical capital, as in the original paper. A banker is a household member endowed with the skills to manage a financial institution. In each period, any household member receives an i.i.d. signal. With a probability σ, the banker continues as a banker in the next period, and with probability $(1-\sigma)$, the banker becomes a worker in the following period. If the banker receives a signal to leave the profession, he transfers his net worth to his family. As explained above, job turnover is necessary to prevent bankers from superseding financial friction by hoarding accumulated profits.

Banks lend funds obtained in capital markets to entrepreneurs and capture the entire financial intermediation sector. For simplicity, we assume that there is no interbank market. Banks purchase securities issued by entrepreneurs that are perfectly state-contingent, providing them with funds used to create new varieties. These securities represent claims on the future stream of profits generated by firm variety, given by $\{\phi^i \pi_{t+i}\}_{i=1}^{\infty}$. Observe that ϕ appears here to clarify that survival in each future period is insecure.

Maturity Transformation. Banks engage in maturity transformation as they hold long-term assets (claims on entrepreneurial projects) and fund these assets with short-term liabilities (beyond their own capital and rights on their assets). Banks also finance these long-term assets with long-term liabilities, i.e., equity and own net worth.

Limited Enforcement. At the end of the period, after borrowing funds, a banker can default on his or her debt and divert a fraction $\Theta(x_t)$ of resources, with creditors only being able to recover the remaining part. Because of the limited enforcement problem, households recognize in advance that excessive debt will lead to default and, therefore, optimally choose not to lend too much from bankers. In equilibrium, this will lead to an endogenous leverage constraint within the banking sector.

For any individual bank, the flow of funds constraint implies that the value of loans funded within a given period, $J_t s_t$, must equal the sum of bank net worth n_t and funds raised from the household, either outside equity $q_t e_t$ or debt d_t. The balance sheet identity is

$$J_t s_t = n_t + q_t e_t + d_t \tag{16}$$

Banks can fund themselves with two debt instruments: a risk-free one-period bond that pays a noncontingent return and a contingent claim on its assets, which we call simply *outside equity*. This contingent liability has an important feature for bankers: while it may be costly during good times, the instrument provides hedging value against unfavorable shocks. During a downturn, this contingent liability partially absorbs the loss from adverse shocks to the bank's assets and transfers it to the bank's lenders. Because of household risk aversion, there is a steady-state premium paid on this contingent liability over the risk-free rate.

While banks may issue new outside equity, they raise inside equity only through retained earnings. In particular, the bank's net worth n_t at t is the gross payoff from assets funded at $t-1$, net of returns to outside equity shareholders and depositors. Let R_t^z be the gross rate of return on a unit of the bank's assets from $t-1$ to t:

$$R_t^z = \frac{\phi(J_t + \pi_t)}{J_{t-1}} \tag{17}$$

The parameter ϕ makes explicit the probability that a variety potentially becomes obsolete between periods. It turns out that because the obsolescence shock is i.i.d., the rate of return is affected by ϕ directly, and bankers need not care about specific assets in their books. A bank's net worth is the residual value of the bank after repaying its creditors, which means that

$$n_t = R_t^z J_{t-1} s_{t-1} - R_t^e q_{t-1} e_{t-1} - R_t d_{t-1} \tag{18}$$

We are now in place to describe the problem of the individual banker. The banker's objective is to maximize net worth at the end of his or her career. By being a household member, he discounts future earnings using the household stochastic discount factor.[5]

$$V_t = \mathbb{E}_t\left[\sum_{i=1}^{\infty}(1-\sigma)\sigma^{i-1}\Lambda_{t,t+i}n_{t+i}\right] \tag{19}$$

Financial friction takes the form of a simple agency problem: after the bank obtains funds, the banker managing the bank may divert a fraction of the funds to his or her family. By recognizing this possibility, households limit the funds they want to lend to banks.

We assume that the fraction of funds the banker may divert depends on the bank's composition of liabilities. In particular, we assume that noncontingent debt is a more efficient disciplinary device than outside equity, based on the work of Calomiris and Kahn (1991). We assume that bankers find it easier to divert funds raised in the form of outside equity. Because the return on outside equity is contingent, it may be harder for the shareholder to understand if less return is due to a diverting of funds or a phase of low returns on risky assets due to the business cycle.

Let us define the fraction of the bank's assets that are financed with outside equity:

$$x_t = \frac{q_t e_t}{J_t s_t} \tag{20}$$

After the bank has obtained funds, it can choose to divert the fraction $\Theta(x_t)$ of funds or not. We follow Gertler et al. (2012) and model the divertable fraction as the following convex function of x_t:

$$\Theta(x_t) = \theta\left(1 + \varepsilon x_t + \frac{\kappa}{2}x_t^2\right), \quad \Theta'(x_t) > 0 \tag{21}$$

As the authors, we allow for the possibility that efficiency is possible for some values of x_t (in the sense that ϵ is negative), but we focus on calibrations that ensure that $\Theta'(x)$ is positive.

Let $V(s_t, n_t, x_t)$ be the maximized value of the bank objective V_t at the end of period t. Observe first that n_t is a state variable. The following incentive constraint must hold:

$$V_t \geqslant \Theta(x_t)J_t s_t \tag{22}$$

[5] Observe that because a banker might leave the profession, the discount of the future is different from the discount of the regular household.

Putting (16) and (18) together, we derive the evolution of a bank's net worth as a function of s_{t-1}, x_{t-1} and n_{t-1}:

$$n_t = [R_t^z - x_{t-1}R_t^e - (1-x_{t-1})R_t]J_{t-1}s_{t-1} + R_t n_{t-1} \tag{23}$$

Let Φ_t be the maximum leverage ratio (asset-to-net-worth ratio) for the bank on date t. Then, we have

$$\frac{n_t}{n_{t-1}} = [R_t^z - x_{t-1}R_{t-1}^e - (1-x_{t-1})R_t]\Phi_{t-1} + R_t \tag{24}$$

In equilibrium, the term in brackets is positive, so the net worth growth rate is increasing in the leverage ratio. One could use recursion to show that the banker's value function is given by [6]

$$V_t(n_t, x_t, s_t) = (\mu_{z,t} + x_t\mu_{e,t})J_t s_t + v_t n_t \tag{25}$$

with

$$\mu_{z,t} = \mathbb{E}_t[\Lambda_{t,t+1}\Omega_{t+1}(R_{t+1}^z - R_{t+1})] \tag{26}$$

$$\mu_{e,t} = \mathbb{E}_t[\Lambda_{t,t+1}\Omega_{t+1}(R_{t+1} - R_{t+1}^e)] \tag{27}$$

$$v_t = \mathbb{E}_t[\Lambda_{t,t+1}\Omega_{t+1}R_{t+1}] \tag{28}$$

The shadow value of an extra unit of net worth is given by

$$\Omega_{t+1} = 1 - \sigma + \sigma[\Phi_{t+1}(\mu_{z,t+1} + x_{t+1}\mu_{e,t+1}) + v_{t+1}] \tag{29}$$

Here, $\mu_{z,t}$ is the discounted excess value of assets over deposits, $\mu_{e,t}$ is the discounted excess value from substituting outside equity for deposits, and ν_t is savings in deposit costs. We refer to $\mu_{z,t} + x_t\mu_{e,t}$ as the discounted total bank profit with structure x_t.

Provided that the excess return on lending activity is positive, that is, $\mu_{z,t} + x_t\mu_{e,t} > 0$, the banker finds it profitable to raise funds through equity and debt to finance varieties. Then, it follows that the incentive constraint (22) holds with equality.

From (22) and (25), we can find the endogenous leverage ratio

$$\Phi_t = \frac{v_t}{\theta\left(1 + \varepsilon x_t + \frac{\kappa}{2}x_t^2\right) - (\mu_{z,t} + x_t\mu_{e,t})} \tag{30}$$

[6] We provide a formal derivation in online Appendix B.

Observe that this leverage ratio depends only on aggregate states; that is, it does not depend on banking-specific factors. Given that, we can aggregate the banking sector's net worth and consider the system as if there were a representative bank.

We need now to characterize the choice of x_t. Given the format of the value $V_t(\cdot)$, it can be shown that x_t is increasing in the ratio of the excess value from substituting outside equity for deposit finance $(\mu_{e,t})$ to the excess value on assets over the deposit $(\mu_{s,t})$ as follows [7]

$$(\mu_{z,t} + x_t \mu_{e,t})\theta(\epsilon + \kappa x_t) = \theta(1 + \epsilon x_t + \frac{\kappa}{2} x_t^2)(\mu_{e,t}) \tag{31}$$

By definition, the economy must satisfy

$$J_t s_t = \Phi_t n_t \tag{32}$$

In addition, Φ depends only on aggregate states, and one can aggregate across banks, so that

$$J_t S_t = \Phi_t N_t \tag{33}$$

In equilibrium, the total supply of assets $A_t + Z_t$ must be clear with the total demand of S_t, so that

$$J_t(A_t + Z_t) = \Phi_t N_t \tag{34}$$

This relation clarifies how the constraint on banks' ability to raise funds (Φ_t) may limit TFP growth. From (11) and (33)

$$J_t A_{t+1} = \phi \Phi_t N_t \tag{35}$$

The next period's technological level A_{t+1} depends directly on how much net worth the banking system has N_t, the asset prices J_t, the survival rate ϕ and the endogenous Φ_t. The production of new varieties relies on an effective labor input. For that reason, there is a strong negative feedback between the banks' ability to lend and the corresponding technology growth. A normal-sized shock may lead to a long-lasting and severe downturn in economic growth and postpone recovery to the balanced growth path.

[7] The relation is valid for the no-policy case. If any macroprudential policy described below is active, this relation changes.

2.4 Evolution of Aggregate Net Worth

Aggregate net worth for the banking system N_t is the sum of the net worth of existing banks $N_{o,t}$ and entering banks $N_{y,t}$, where “o” stands for “old” and “y” stands for “young”.

$$N_t = N_{o,t} + N_{y,t} \tag{36}$$

The net worth of existing banks is equal to earnings on assets held in the previous periods net of its funding costs through debt and outside equity times the probability of surviving until the current period:

$$N_{o,t} = \sigma \{R_t^z J_{t-1} S_{t-1} - R_t^e q_{t-1} E_{t-1} - R_t D_{t-1}\} \tag{37}$$

As (33) clarifies, the absence of net worth of entering banks results in no assets financed by these institutions. Therefore, each entering bank must receive a “start-up” net worth; otherwise, it cannot lend. We assume that each entering bank receives a fraction ξ of the total assets value of existing banks. We can then aggregate it as

$$N_{y,t} = \xi(1-\sigma)[J_{t-1} S_{t-1}] \tag{38}$$

Thus, aggregate net worth is then

$$N_t = \sigma[R_t^z J_{t-1} S_{t-1}] - \sigma[R_t^e q_{t-1} E_{t-1} + R_t D_{t-1}] + \xi(1-\sigma) J_{t-1} S_{t-1} \tag{39}$$

We next finish the model description with the real sector production, which includes both intermediary and final sectors.

2.5 Final Output and Intermediates Producers

The real sector we modeled is quite standard. Prices are flexible. The final good is produced in a competitive sector that aggregates a continuum of measure A_t of intermediates

$$Y_t = \left[\int_0^{A_t} Y_t(s)^{\frac{\upsilon-1}{\upsilon}} ds\right]^{\frac{\upsilon}{\upsilon-1}} \tag{40}$$

Given the aggregator above, the demand for each intermediate s by the final good sector is

$$Y_t(s) = \left[\frac{P_t(s)}{P_t}\right]^{-\upsilon} Y_t \tag{41}$$

where the price level P_t is defined as

$$P_t = \left[\int_0^{A_t} P_t(s)^{1-\upsilon} ds\right]^{\frac{1}{1-\upsilon}} \tag{42}$$

Equation (41) gives the demand facing each intermediate good producer s. Its production is given by

$$Y_t(s) = \Upsilon_t^y [K_t(s)]^\alpha L_t(s)^{1-\alpha} \tag{43}$$

where $K_t(s)$ is capital holding and $L_t(s)$ is the labor employed. Υ_t^y is an exogenous stochastic process common to all intermediate firms, evolving according to

$$\log(\Upsilon_t^y) = \rho_y \log(\Upsilon_{t-1}^y) + \epsilon_t^y$$

where ϵ^y is an i.i.d. innovation. The objective of intermediate producers is to maximize profits. Their objective is to choose a vector $\boldsymbol{x}_s := (P_t(s), Y_t(s), K_t(s), L_t(s))$ to solve

$$\pi_t = \max_{\boldsymbol{x}_s} P_t(s) Y_t(s) - W_t L_t(s) - R_t^k K_t(s)$$

subject to the demand (41) and production function (43). The solution yields the following optimality conditions:

$$W_t = \frac{\upsilon - 1}{\upsilon}(1-\alpha)\frac{Y_t}{L_t} \tag{44}$$

$$R_t^k = \frac{\upsilon - 1}{\upsilon}\alpha\frac{Y_t}{K_t} \tag{45}$$

Each intermediate producer sets its price to a constant markup over marginal cost – the ratio of price to marginal cost equals $\frac{\upsilon}{\upsilon-1}$. The per-period profits of intermediate producers, π_t, are then

$$\pi_t = \frac{1}{\upsilon}\frac{Y_t}{A_t} \tag{46}$$

Combining (40) with the first-order conditions for intermediate producers and with equilibrium in factor markets, we obtain an expression for the final output

$$Y_t = \Upsilon_t^y A_t^{\frac{1}{\upsilon-1}} (K_t)^\alpha L_t^{1-\alpha} \tag{47}$$

To have a labor-augmenting production function in conformity with well-known macroeconomic evidence, we need to choose υ such that $\frac{1}{\upsilon-1} = 1-\alpha$, so that

$$Y_t = \Upsilon_t^y K_t^\alpha (A_t L_t)^{1-\alpha}$$

2.6 Equilibrium

Market clearing in every market is required. For securities, this implies that

$$J_t(A_t + Z_t) = \frac{v_t}{\theta(1 + \varepsilon x_t + \frac{\kappa}{2} x_t^2) - (\mu_{z,t} + x_t \mu_{e,t})} N_t \tag{48}$$

where x_t is given by the solution to equation (31). Observe that we used $S_t = A_t + Z_t$.

The market-clearing condition for outside equity requires that the demand for outside equity $\bar{e}_t$ equal the supply from banks

$$q_t \bar{e}_t = x_t \cdot J_t(A_t + Z_t) \tag{49}$$

The flow of funds requires that total deposits must equal aggregate bank assets net outside equity and net worth plus net flows from the interaction with international financial markets.

$$D_t = (1 - x_t) J_t S_t - N_t \tag{50}$$

In the labor market, the equilibrium conditions are that labor demand meets labor supply, that is,

$$W_t = \frac{\upsilon - 1}{\upsilon}(1-\alpha)\frac{Y_t}{L_t} = \Psi A_t L_t^\varepsilon \tag{51}$$

Finally, the economy uses the output to finance consumption, investment in physical capital, and investment in new varieties. The economy-wide resource constraint is then

$$Y_t = C_t + I_t + \mathcal{R}_t \tag{52}$$

where $I_t = K_{t+1} - (1-\delta)K_t$.

3 Calibration and Solution of the Model

Altogether, there are 15 parameters in our model, not including parameters describing the exogenous forcing process. Because we incorporate risk in a balanced growth path concept and such an approach is somewhat new in the related literature (meaning that there are few tools available), we rely on calibrated parameters. Our paper is closely related to Comin and Gertler (2006), Anzoategui et al. (2016), and Moran and Queralto (2017), so much of our environmental structure and parameters can be validated within the existing literature. We calibrated the model to match figures from annual data, as those papers did.

We used annual data from 1951 to 2017 taken from Fernald (2014) for data on productivity. Given that the model does not take into account factor utilization, we used utilization-adjusted TFP data, first logged and then hp-filtered. The technological parameters were chosen such that the annual growth rate of the economy is approximately 1.42%. To meet this target, we performed a two-step procedure. First, we fixed $\bar{L}$ to 12.5%, which is reasonably close to estimates of the proportion of the US labor force holding an advanced degree or Ph.D. Next, we fixed the firm death rate ϕ at 10% annually, based on firm mortality evidence from Daepp et al. (2015) and aligned with the analogous parameter value chosen by Moran and Queralto (2017). Finally, we then adjusted the value of η so that TFP growth meets the target. The resulting value for η is 0.0476.

The discount factor β was set to 0.9943, aimed at achieving a 2% annual risk-free real interest rate to be internally consistent with our target of 1.42% for the annual TFP growth rate. The risk aversion coefficient ρ was set to a standard value of 1, i.e., log utility. We set the value of 1/2 for the inverse Frisch elasticity of labor supply ε, within the range estimate from Peterman (2016). The labor disutility coefficient Ψ was set to 0.1816 so that the working time is reasonable nearly 8 hours. We set the annual depreciation rate of physical capital δ to 10% and the capital share of production α to 0.33. As explained earlier, the value for υ is tied by the value for α, implying a value of 2.4925. The same approach was taken by Queralto (2019) and Moran and Queralto (2017). For such a value of υ, the resulting markup is 1.67, close to the value used by Comin and Gertler (2006), which was 1.6.

At the core of the financial friction, there are five parameters related to the financial sector in our model, namely $(\sigma, \xi, \theta, \varepsilon, \kappa)$. In the calibration of this set of parameters, we follow closely Gertler et al. (2012). We considered the banker survival rate σ so that the average life of a banker is nearly 8 years, adjusted upward to reach a banking spread of $R^z - R$ of 1.25% annually. We adjusted θ so that the risk-free rate R is nearly 2%. Concerning capital

Table 1: Calibrated Parameters

Parameter	Value	Description
β	0.9943	Discount Factor
ρ	1.0000	Risk aversion
Ψ	0.1816	Utility weight of labor
ε	0.5000	Inverse Frisch elasticity of labor supply
ϕ	0.9000	Firm survival rate
$\bar{L}$	0.1250	Skilled labor supply
η	0.0476	Materials share in firm creation
α	0.3300	Capital share of production
δ	0.1000	Depreciation rate
υ	2.4925	Intermediates producers' elasticity of substitution
σ	0.8947	Survival rate of bankers
ξ	0.0750	Transfer to entering bankers
θ	0.4279	Asset diversion constant term
ϵ	-0.2823	Asset diversion linear term
κ	2.7224	Asset diversion quadratic term
σ_z	0.0032	Entrepreneurial productivity shock standard error
σ_y	0.0035	Intermediary productivity shock standard error
ρ_z	0.6667	Persistence of entrepreneurial productivity shock
ρ_y	0.8800	Persistence of intermediary productivity shock

structure, we choose parameters κ and ξ to reach an aggregate leverage ratio (assets to the sum of inside and outside equity) of 4 and a ratio of outside to inside equity of 2/3. Finally, we adjusted ε to achieve an equity finance ratio x reasonably near 10%. The calibrated value of ε is negative. All those targets are taken from Gertler et al. (2012).

In our model, banks' balance sheet structure depends on macroeconomic risk. Bankers choose their balance sheet structure to optimally address the trade-off between short-term debt and equity finance from their individual perspectives. To properly incorporate risk in our solution, we borrow the concept of the risk-adjusted steady state from Coeurdacier et al. (2011). The risk-adjusted steady state is defined as the point in the state space in which agents choose to stay if all of the exogenous processes is at the respective mean (that is, all shocks have dissipated) on the current date and if they recognize that the future is risky. In our case, we work with a risk-adjusted balanced growth path.

In a perfect environment of foresight, if there are multiple assets, they must pay the same return in the steady state. In our model, this would leave the household indifferent between short-term debt and outside equity. In the absence of risk, there would not be any hedging value for bankers to issue outside equity, that is, $\mu_e = 0$. To solve these issues, we incorporate future risk into agents' decision problem and work with a risk-adjusted balanced growth path. Effectively, we took second-order Taylor expansions around the risk-adjusted balanced growth path of equations where risk matters (i.e., where the conditional expectation operator appears).

To fully characterize the risk-adjusted balanced growth path, we adopt the algorithm described by Coeurdacier et al. (2011) and implemented by Gertler et al. (2012). We first computed the deterministic balanced growth path and simulated the log-linearized dynamics of the model around such a balanced growth path. Then, we computed relevant second moments and used them to derive a tentative risk-adjusted balanced growth path. We then simulated the log-linearized model once again, but this time around, the tentative risk-adjusted balanced growth path. Finally, we checked whether the moments used to construct the risk-adjusted balanced growth path were consistent with moments from a subsequent simulation. If not, we continue iterating until convergence is achieved.

As risk matters to the concept of a balanced growth path that we considered in solving the model, modifying the characterization of the exogenous forcing process definitely changes the balanced growth path. In more regular quantitative macroeconomic frameworks, the concept of a steady state is typically deterministic. This means that the characterization of the exogenous forcing process does not affect the steady state itself but only affects the dynamics of the model up to a first-order approximation. Because we consider risk in our solution concept, the characterization of the exogenous forcing process does affect the balanced growth path; therefore, this characterization must be done in a manner similar to the calibration of *every* parameter of the model.

In order to calibrate the exogenous forcing processes, we chose parameters first for the persistence of shocks and then for the variance of shocks. We found that the theoretical persistence of the model counterparts was broadly unchanged when we changed the parameters of the model but fixed ρ_z and ρ_y. Given that, we first choose the parameters. Then, we calibrated the variance of innovations to match the variability of the output data. We first set ρ_y to 0.88, as did Comin and Gertler (2006), and then we adjusted ρ_z to match the autocorrelation of the annual output series, which is 0.51. This procedure resulted in a ρ_z value equal to 0.6667. To match output variability in the data, we took as given the autocorrelation parameters and then set σ_z at 0.0032. Finally, we adjusted σ_y to match the output variability in annual frequency, which is 2.60%, resulting in σ_y equal to 0.0035.

We next turn to the discussion of the model and the role of macroprudential policy. After a brief discussion, we present the results for three different policy implementations aiming to illustrate the numerical properties of the model.

4 Discussion of the Model and Macroprudential Policy

4.1 Reasoning for Macroprudential Policy

Within our model, there is a motivation for macroprudential policy. At any given date t, each asset of each bank has the same price J_t, implying the existence of a *pecuniary externality*. When assets pay an unexpectedly low rate of return, each bank finds itself forced to partially liquidate its assets to meet its short-term noncontingent obligations. By doing so, banks aim to alleviate pressure from their balance sheet. By selling assets in the market (or alternatively, funding less innovation), individual banks push down the price of assets held by *every* bank within the financial system. The decreasing asset price enters into the second round of pressure on banks' balance sheets in a strongly adverse feedback scheme. That is, there is a *contamination* effect of other banks' balance sheets. As the fire sales externality takes place and the banks' asset marketed-to-market value collapses, negative feedback emerges, deepening the crisis. The endogenous nature of growth enhances the crisis, further amplifying the financial accelerator effect.

There is an alternative view. Individual banks' value function depends on, among other things, asset and liability rates of return. Because each bank is atomistic in its own perspective within the system, it simply ignores the impact of its own actions on these rates of return. However, if the banking system as a whole took actions in a coordinated fashion, then the actions taken by the whole system would be different from the result of the aggregation of the decentralized decisions taken by each bank individually. In short, the banking system would make a decision in a manner that incorporates the spillover effects caused by the pecuniary externality. Because individual banks fail to internalize the hedging value of issuing outside equity from a social point of view, there is a role for macroprudential policy.

A macroprudential policy can operate to alleviate the negative feedback force described above, essentially ameliorating the financial accelerator mechanism within the financial system. If individual banks are hedged against adverse shocks, the whole system can be better off, as the financial accelerator mechanism would be partially dampened. A simple implementation plan is to urge banks to issue more state-contingent debt, which we call *outside equity*. With a liability structure that is heavier based on outside equity than on noncontingent debt, each bank would be better hedged against headwinds.

Such a schedule comes at some cost though. By inducing banks to rely more heavily on equity finance, moral hazard issues increase. A tighter incentive constraint for bankers reduces their ability to raise funds in capital markets such that there might be an optimal policy intensity for each policy design considered. A welfare-maximizing regulator must therefore consider not only the second-order effect (i.e., the amelioration of risk) but also the first-order effect (i.e., tightening bankers' incentive constraint). Designing cyclically changing prudential rules affects the second-order effects as well as the dynamics of balance sheet recomposition. As we discuss below, inducing banks to rely more on equity allows them to adjust their balance sheet structure (i.e., x_t) more frequently than their balance sheet size.

4.2 Operating Macroprudential Policy

We restrict our attention to policies that fit into the following description. Each policy considered below is intended to increase the incentives of *individual banks* to issue outside equity such that these institutions properly internalize the fire sale externality when making decentralized decisions. We follow Gertler et al. (2012) and let the incentive distortion be in the form of a subsidy to the issuance of outside equity. To finance policy expenditures, a tax rate on banks' assets is charged. The tax rate is set so that there is fiscal neutrality; i.e., the collection of taxes is exactly enough to finance the policy in every state and every period. In such cases, the balance sheet identity becomes

$$(1 + \tau_t^z)s_t J_t = N_t + D_t + (1 + \tau_t^e)q_t e_t \tag{53}$$

while fiscal neutrality implies that revenues equal expenditures

$$\tau_t^z s_t J_t = \tau_t^e q_t e_t \tag{54}$$

which can be simplified using the notation of x_t

$$\tau_t^z = \tau_t^e x_t \tag{55}$$

The first observation is that both the tax τ_t^z and subsidy τ_t^e are time-varying; i.e., they can change along the business cycles. Although our main concern is the long run, the response of the policy to short-term fluctuations is key to determining the resulting covariance matrix of endogenous variables. As explained earlier, such a matrix is used to compute the risk-adjusted balanced growth path. In that sense, each policy considered modifies this matrix of the variance in different manners and thus alters the trajectory of the economy in a very particular manner. These restrictions on the policy design are used for two reasons. First, the banker's value function has a closed-form solution, which is computationally appealing. Second, this approach allows us to compare with other papers in the literature on macroprudential policy, i.e., Gertler et al. (2012), de Groot (2014) and Liu (2016). With these restrictions, the value function of banks is:

$$V_t(s_t, x_t, n_t) = [(\mu_{z,t} - \tau_t^z \nu_t) + (\mu_{e,t} + \tau_t^e \nu_t)x_t]J_t s_t + \nu_t n_t \tag{56}$$

We then need to choose a rule for τ_t^e. That is, we need to describe the decision regarding the subsidy rate at every period t. Once the subsidy rate rule is defined, so is the tax rate, given the fiscal neutrality constraint. We reserve the description for the rules of τ_t^e for three different subsections to be explored later. Every policy sets a different rule for τ_t^e.

We now turn to briefly discuss how the policies operate. The following description fits all policies considered, so we reserve the description of policy rules for subsequent subsections. Whenever a regulator increases incentives to individual banks to issue outside equity, two main forces play a role within the banking sector. The key difference between policies is exactly which force in play is stronger, as we show next. First, the encouraged – subsidized – issuance of outside equity induces banks to optimally decide to increase the share of their assets that are funded with this instrument; i.e., the share of x increases. This increase in x tightens the incentive constraint of bankers, reducing their abilities to raise funds, all things held equal. Consequently, banks can finance fewer varieties on each date, and the growth potential of the economy is lower. Second, the increase in x makes the whole banking system better off in terms of hedging against adverse shocks to the return of bank assets. In case a negative shock affects banks' assets, the increased proportion of liabilities in contingent obligations helps banks partially transfer the loss to their creditors. [8]

Therefore, we can distinguish two quite different effects associated with inducing banks to rely more on equity finance. First, there is a moral hazard effect, which is associated with a tighter incentive constraint for bankers, given the higher value of x. Second, there is a general equilibrium effect by which the reduced risk increases the banker's continuation payoff. This second effect increases the banker's value function multipliers in net terms in the risk-adjusted balanced growth path; i.e., the discounted excess value of banks $\mu_z + x\mu_e$ increases with the policy. Intuitively, the reduced bank risk gives bankers less room to extract hedging value when choosing their balance sheet structure μ_e. The better-hedged system leads to higher μ_z and lower μ_e. The main reason is the relative scarcity of risk-free assets, which induces a reduction in R relative to other returns. From equations (26)-(28), the increase in x causes an increase in μ_z and a decrease in μ_e. Since $\mu_z \gg \mu_e$ and $x \in (0, 1)$, the general equilibrium effect is positive, so $\mu_z + x\mu_e$ increases. The shadow price of a unit of net worth tomorrow Ω increases in response to that, as shown in equation (29). The bank's saving in terms of deposit cost ν also increases when the policy is active because now net worth tomorrow is more valuable. Additionally, the increase in Ω implies a further increase in μ_z and partially cancels out the decrease in μ_e. These two increases in the multipliers imply higher leverage, all things held equal. The tightening of the incentive constraint, however, decreases the leverage ratio. From equation (30), these multipliers, together with the divertable fraction play a role in modifying the leverage ratio Φ.

To make the comparison of rules straightforward, we use rules with a single parameter each, considering only linear cases for simplicity. We reinforce that the analysis conducted here is meant to be illustrative and not the results of statistical testing; our main goal is normative rather than positive. We present 2 below the risk-adjusted balanced growth path for the baseline model together with each optimization rule.

4.3 GKQ Policy Case

We begin with the policy case developed by Gertler et al. (2012), which we simply call the GKQ rule henceforth. The rule for the subsidy is the following.

$$\tau_t^e = \frac{\tau}{\nu_t} \tag{57}$$

where τ is the policy parameter.

[8] Obviously, a bank's creditor then requires a premium to carry such risk. We explain in section 4.6 the details of such risk premiums under each policy.

Table 2: Risk-Adjusted Balanced Growth Path

		No Policy	GKQ	Capital Ratio	Credit-to-GDP
Growth rate	$g(\%)$	1.4248	1.4250	1.4250	1.4250
Output	Y	10.2601	10.2602	10.2602	10.2602
Consumption	C	8.1493	8.1492	8.1492	8.1492
Labor	L	8.0094	8.0094	8.0094	8.0094
Capital	K	16.9639	16.9639	16.9640	16.9639
Net worth	N	5.3444	5.3322	5.3303	5.3328
Risk-free return	$R(\%)$	1.9509	1.9509	1.9509	1.9509
Risky return	$R^z(\%)$	3.1298	3.1237	3.1227	3.1240
Credit spread	$R^z - R(\%)$	1.1789	1.1728	1.1718	1.1731
Outside equity ratio	x	0.1108	0.1645	0.1716	0.1622
Leverage ratio	Φ	6.0274	6.0436	6.0411	6.0428
Deposit cost	ν	2.3797	2.3973	2.4011	2.3961
Excess equity cost	μ_e	0.0004	0.0004	0.0003	0.0004
Bank asset profit	μ_z	0.0268	0.0271	0.0271	0.0271
Subsidy	$\tau^e(\%)$	NA	0.1710	0.1939	0.1633
Tax	$\tau^z(\%)$	NA	0.0281	0.0333	0.0265
Aggregate leverage	$\Phi/(1+x\Phi)$	3.6133	3.0303	2.9677	3.0521
Equity ratio	$1/x\Phi$	1.4967	1.0056	0.9640	1.0205
Optimal Policy Intensity	τ	NA	0.0041	0.0113	0.0052
Welfare	$\mathcal{W}$	728.9176	728.9833	728.9944	728.9794

The first reason is clarity, at least at the level of modeling.[9] The authors argue that with these features, the policy operates as a countercyclical capital requirement. First, the subsidy increases the steady-state value of x_t, serving as a capital requirement instrument. Second, x_t will vary countercyclically. The release of the requirement is important to the extent that it helps increase banking leverage after an adverse shock to banks' assets, thus making recovery quicker than it would be otherwise.

Operating as a countercyclical capital buffer is a desired feature of all policies. Initially, suppose that the economy is in a boom phase. The capital requirement is countercyclical, so it induces banks to build up a stock of outside equity during the boom phase. By doing so, banks tend to be at least partially better off in terms of hedging against adverse shocks. When a downturn arises, the cutback in the subsidy rate urges banks to rely more heavily on noncontingent debt than on contingent debt, thus lowering the value for x during the downturn. This change in the balance sheet structure of banks increases the leverage within the financial sector by increasing bank value function multipliers on the net ($\mu_z + x\mu_e$ and ν) and alleviating the moral hazard incentive constraint ($\Theta(x)$). Altogether, the sustained increase in leverage after an adverse shock due to the countercyclical capital requirement mechanism ultimately accelerates the recovery of the bank's balance sheet size. The recovery of the financial system balance sheet translates into greater availability of funds for new varieties, and the economy recovers to its balanced growth

[9] This format allows the calculations to become simpler than they would be otherwise.

path. The negative feedback mechanism between outside banking finance and growth is somewhat dampened. This countercyclicality is quite important because it helps banks adjust their balance sheet structure x_t rather than their balance sheet size. At the end of the day, such cyclicality increases aggregate demand for new varieties and decreases demand variability; i.e., demand for varieties becomes more stable.

As shown in 1, the general equilibrium effect dominates the moral hazard effect when the policy parameter τ is relatively low. This happens because of the convexity of Θ on x_t. The increase in the discounted excess rate value of banks $(\mu_z + x\mu_e)$ is dominated by the moral hazard effect $\Theta(x)$ for every value of τ, as shown in the right-upper chart. The increase in the private value of an extra unit of deposit (or the savings in terms of deposit costs) ν allows an increase in leverage. The higher leverage translates into a higher growth rate of the economy, from equation (35). The optimal policy is achieved at the point $\tau = 0.0041$.

Although the difference in growth appears to be only marginal between policies, this difference has key implications for welfare computations. As we show in Appendix B, welfare depends on the growth rate of the economy and the first and second moments of utility components, consumption C, and labor L. Of course, the policy affects all three channels.

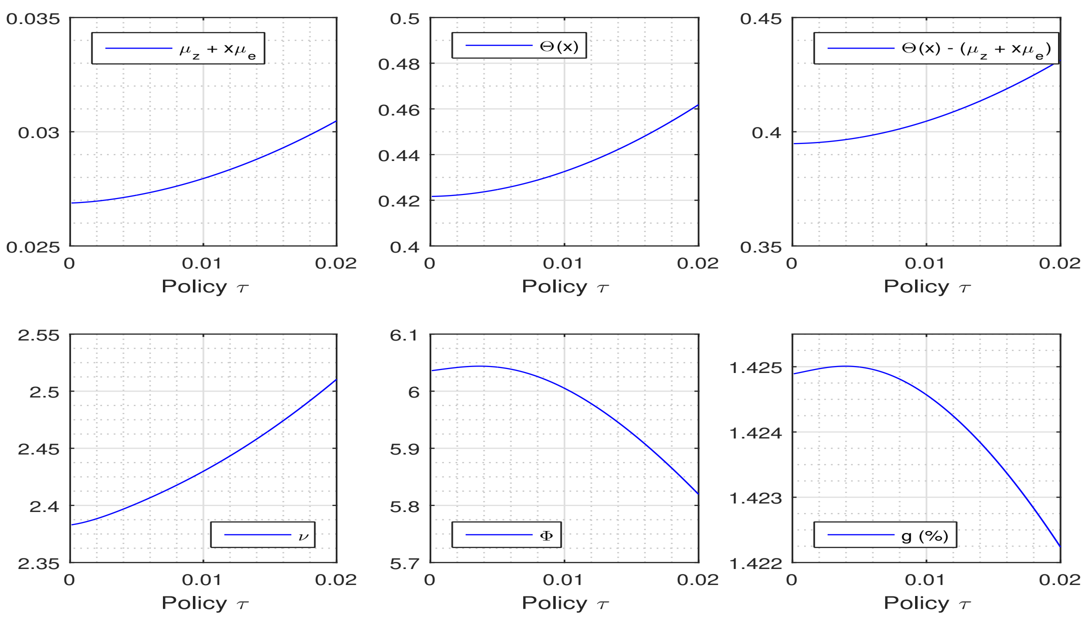

Figure 1: Response of Selected Variables to GKQ Policy

4.4 Capital Ratio Rule

We next discuss the results under the format of the policy proposed by Liu (2016). The author equipped the model proposed by Gertler et al. (2012) with a different rule for the macroprudential subsidy rate, based on ideas developed by Hanson et al. (2011). The goal of the policy is the same as the original rule: mitigate future risk. The same intuition for the subsidy operating as a countercyclical capital buffer continues to apply under such a rule. Essentially, intuition is desired for any rule. Moreover, intuition applies to any rule we considered. The key difference then is how it applies.

The subsidy rule we considered has the following format, with τ being the policy parameter.

$$\tau_t^e = \tau x_t \tag{58}$$

Liu (2016) argues that in his rule, the subsidy acts as a stronger capital buffer. The author numerically shows that in his model, under this rule, the subsidy reacts such that the financial sector recovers its balance sheet size quicker than it would under the rule proposed by Gertler et al. (2012). Under such a rule, compared with the GKQ rule, the better response of banking leverage to adverse shocks leads to higher finance capacity and finally higher economic growth.

There is a reasonably simple explanation for why the capital ratio rule is so effective in mitigating bank risk. The design of the rule allows the subsidy to increase together with the fraction of assets funded by outside equity, x. Therefore, there is a positive feedback mechanism in action. First, higher τ^e increases x, as do the other policies. However, higher x translates into higher τ^e, which in turn raises the banker's incentive to increase x again, as in a virtuous cycle. This positive feedback has a robust impact on modifying the risk structure of assets. Importantly, this rule allows the outside equity finance rate x to increase only marginally for a large subset of the interval considered for τ. This happens at a cost; if the policy is very intense, the distortion of a bank decision is markedly strong such that the moral hazard effect is largely dominant at the margin.

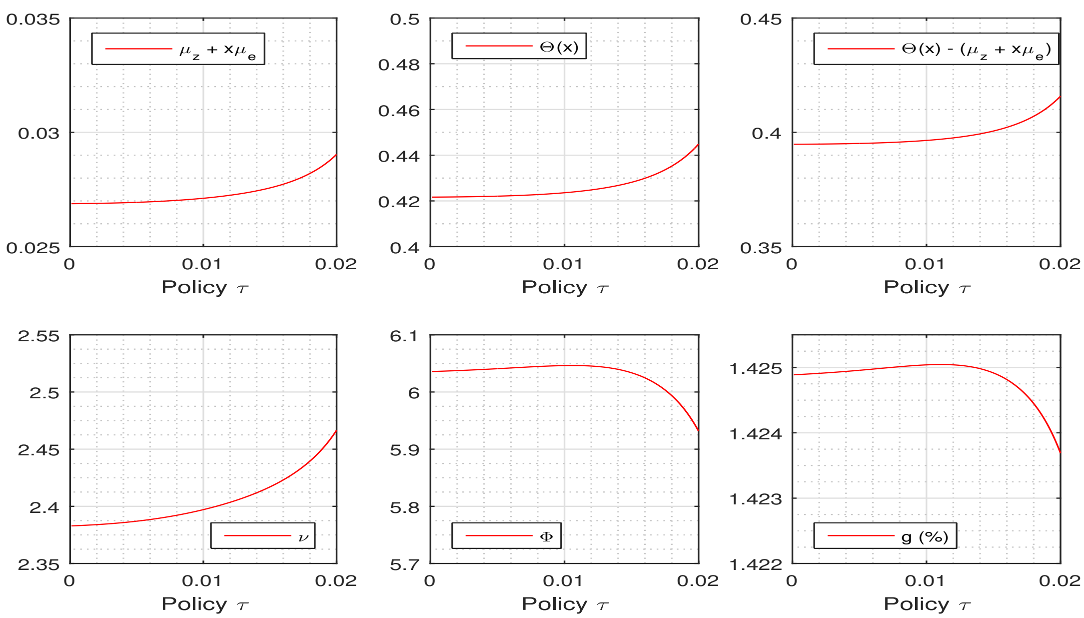

Figure 2: Response of Selected Variables to Capital Ratio Policy

4.5 Credit-to-GDP Ratio Rule

To complete the illustration of the model, we propose a simple rule that differs from the two rules previously specified. We motivate the rule with insights from the literature on macroprudential policy. A general conclusion of the study of Angelini et al. (2015) is that a prudential rule that increases the capital requirement based on a credit-to-GDP gap tends to reduce output volatility. Additionally, Borio (2011) argues that indicators based on credit-to-GDP gap measures are useful to detect or predict financial distress in advance.

Along these lines, we test a macroprudential subsidy rule based on a credit-to-GDP ratio measure. The policy parameter is again τ.[10] The policy rule is designed as

$$\tau_t^e = \frac{\tau}{10}\frac{J_t(A_t + Z_t)}{Y_t} \tag{59}$$

Here, J_t is the price of one unit of assets held by banks, and $A_t + Z_t$ is the total asset amount held by banks. $J_t(A_t + Z_t)$ is therefore the aggregate balance sheet size, while Y_t is total output.

The rule operates as the others do. The procyclicality of the credit-to-GDP ratio gives the subsidy rate to the same property. In line with Borio (2011), the rule might be especially useful during the build-up of the vulnerability phase, i.e., the boom phase of the economy. During this boom phase, banks' balance sheets expand faster than output due to the financial accelerator mechanism. This in turn increases the credit-to-GDP measure, raising incentives for banks to issue outside equity. Nevertheless, in line with the suggestion of Borio (2011), the release of the capital requirement might be inefficiently slow: this release under such a rule happens in a manner in which the maximum welfare value is poor relative to the others presented, as shown in the bottom of table 2.

5 Policy Comparison

We next turn to a brief discussion of the results of the three policy cases considered. To access the optimal policy parameter, we looped the model for each policy case within the range [0.0001-0.0200] for the policy parameter, linearly spaced with 200 nodes. Therefore, we looped each rule 200 times, changing only the policy parameter along the nodes. For each policy parameter, we computed the risk-adjusted steady state. Equipped with both steady-state values of the endogenous variables and the variance matrix of these variables, it is straightforward to compute welfare. The optimal policy intensity is defined in each case as the policy parameter value that maximizes the welfare function.[11]

The risk-adjusted balanced growth path for the baseline model and for each policy design we considered is presented in table 2. As expected, each policy has an optimal policy that is better off (in terms of welfare) than the

[10] The algorithm we used to solve the model crashes if the policy parameter is too far from zero under this rule for the subsidy. Normalizing the parameter by 10 allows us to overcome this issue.

[11] Welfare function details are provided in online Appendix B.

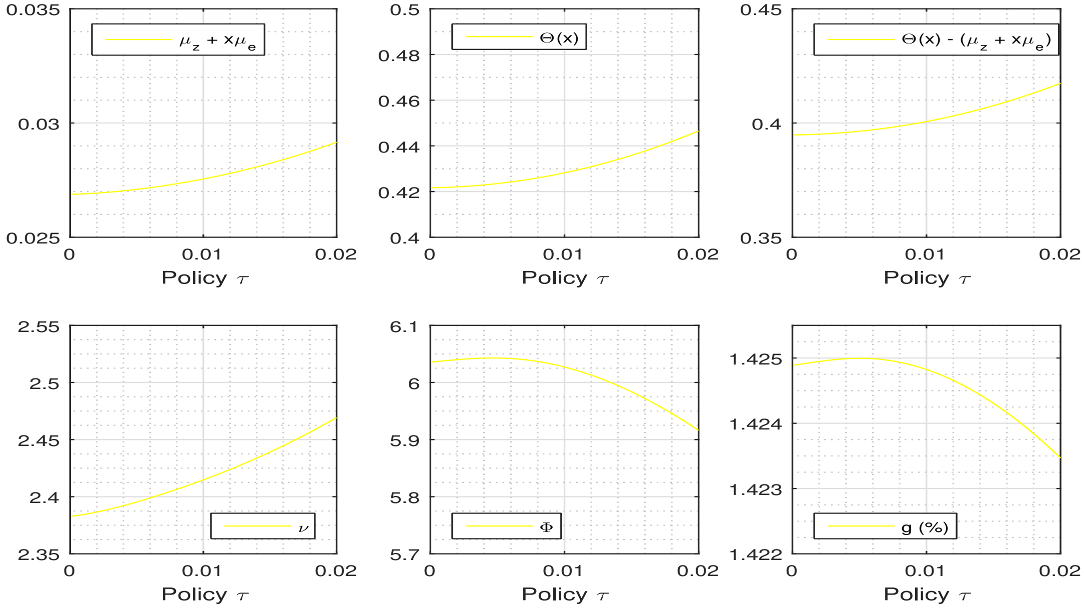

Figure 3: Response of Selected Variables to Credit-to-GDP Gap Policy

baseline model. The outside equity finance ratio x increases greatly under every policy considered, but there are small differences between them. The equity finance ratio x, growth rate g, and welfare $\mathcal{W}$ maps from the policy parameter under each rule are presented in 4 below.

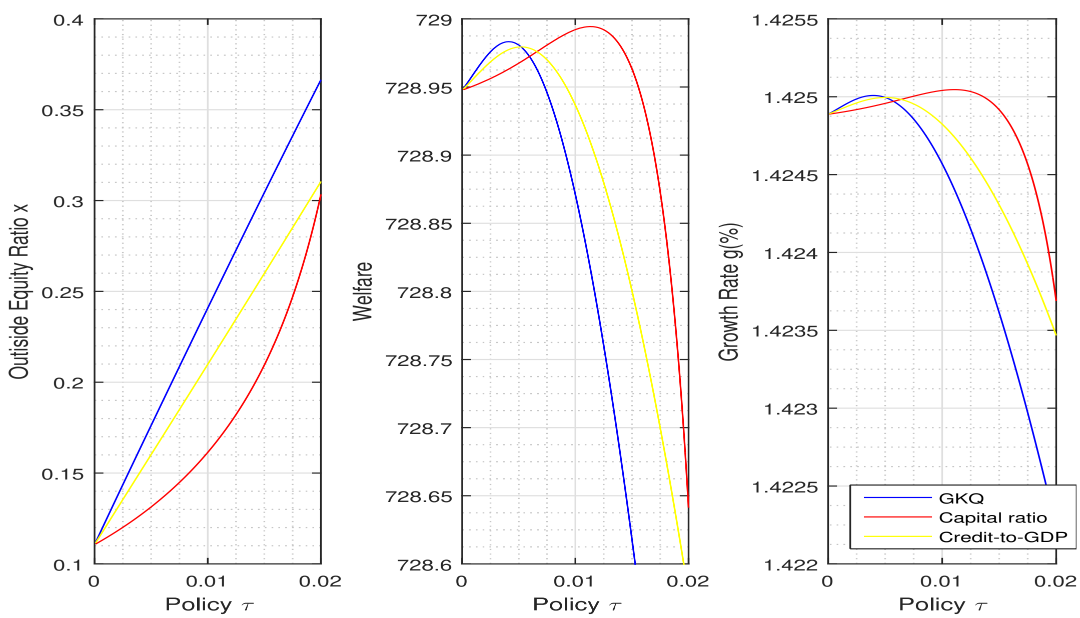

Figure 4: Comparison of different policy cases.

We start by noting that each policy modifies the risk of assets of the economy in a particular way. The subsidy to issue outside equity responds in different ways in each policy case because the automatic stabilizer τ_t^e operates differently in each case. Obviously, different responses of endogenous variables to exogenous innovations distort the risk structure of the economy in a particular manner for each policy. Therefore, this is how the macroprudential policy operates: not only does the mitigation of risk makes the economy better off, but *how* this risk reduction that modifies the structure of asset risk in the economy also matters.

Our numerical exercises suggest that the best policy is the capital ratio rule. We start by analyzing the modification of the steady-state values under this rule. The distortion in the risk profile of assets under this rule leads to a higher deposit value for bankers ν, associated with higher leverage Φ. Meanwhile, the capital structure choice x is almost a quarter after an intense policy intervention, at $\tau = 0.0113$. To be clear, this level of policy, under such a rule, leads to the highest subsidy rate (0.1939%). Leverage increases marginally, despite the banker multipliers (ν and $\mu_z + x\mu_e$) being the highest compared to other policies. The reason is that the divertable fraction $\Theta(x)$ is also the highest under the capital ratio rule, from x being the highest among the policies we considered and from equation (21). Interestingly, the positive feedback between τ^e and x explained in section 4.4 induces the value of x to be low relative to the other policies when the policy parameter τ is low. This makes sense: the positive feedback mechanism between τ^e and x allows x to adjust only gradually for a low-intensity policy parameter τ. Moreover, such a feedback mechanism allows banks to adjust their balance sheet structure more freely than its size, resulting in quite different dynamics of the economy in response to shocks. The modification of risk matters the most to this policy in outperforming the other policy rules we considered.

The main measure of cyclical fluctuation is that the aggregate leverage (the ratio of total assets to the sum of bank net worth and outside equity) is the lowest under the capital ratio rule (2.9677). Thus, the bank balance sheet is healthier under this policy rule than under the others. Leverage is virtually equal under all policies, but aggregate leverage is significantly lower under the capital ratio rule proposed initially by Liu (2016).

Medial welfare gain is observed under the GKQ rule. Although the rule induces a plausible increase in banks' total profit $\mu_z + x\mu_e$ and in deposit saving costs ν, it does so at the cost of a sizable tightening of bankers' incentive constraints. That is, $\Theta(x)$ increases quickly under this policy rule. From figure 4, the outside equity finance x increases drastically under the GKQ rule. Interestingly, the increases appear to be linear. As a result of the quickly tightening incentive constraint, leverage increases only in a small subset of the policy parameter considered. The optimal policy level ($\tau = 0.0041$) is associated with higher leverage than the no-policy regime (6.0436) and a better hedged financial sector, with an aggregate leverage of 3.0303.

The credit-to-GDP ratio also improves welfare, but the increase is the smallest among the policies considered. We recall that the rule normalizes the policy parameter τ by 10. As shown in 3 in the upper-middle chart, the divertable fraction $\Theta(x)$ increases greatly when parameter τ increases. The increase in the outside equity finance ratio x appears to be linear upon increases on τ. Optimal policy intensity occurs at $\tau = 0.0052$. The rule is strongly effective in increasing total bank profit ($\mu_z + x\mu_e$), together with the increase in banks' savings in deposit costs ν, leaving the leverage (Φ) at 6.0428. Under such a rule, the outside equity finance ratio x is the lowest among the policy rules we considered, yet quite far from the baseline model. Aggregate leverage is the highest, suggesting that the banking sector is effectively less hedged against headwinds.

So far, we have discussed the role of banking system risk mitigation in the model, but we have not yet provided any evidence. To illustrate the risk channel for banking lending (and therefore for growth), consider the output response to a one-standard-deviation shock in intermediary productivity. In figure 5 in the right panel, we plot the response of output (Y) for such a shock for the baseline model and for each policy we considered. We constructed a counterfactual no-shock path for the baseline model and compared the output response for each policy design in proportional terms. There are two main forces at play here. First, TFP growth is higher under each policy we considered compared to the no-policy regime. By itself, such a higher growth rate makes the economy achieve and surpass – at some time not shown in the figure – the no-policy, no-shock counterfactual. Second, the dynamic response of endogenous variables differs from policy to policy, so over time, their accumulated divergence marks a clear distinction in the operation of each policy.

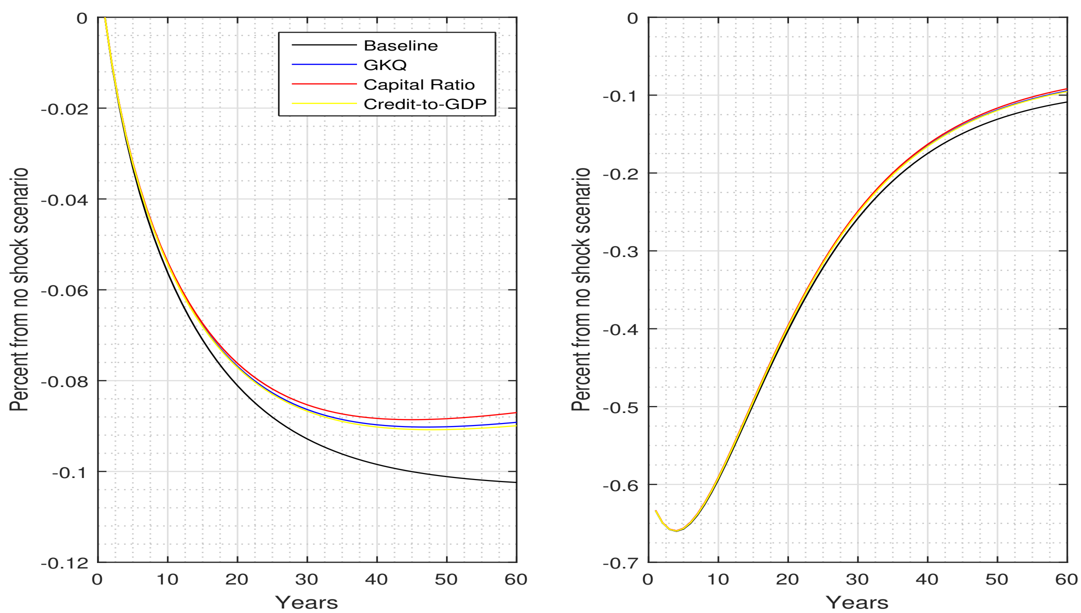

Figure 5: Output response for a one-standard-deviation shock in entrepreneurial activity (left panel) and in intermediary productivity (right panel).

Now, consider the response of output for a shock in entrepreneurial productivity. Such a response is shown in figure 5 in the left panel. Because of the endogenous nature of TFP growth, an initial decline in entrepreneurial productivity leads to permanent effects on output. This implication is clarified if we observe that in either case, whenever a negative shock affects the economy, output never reaches the trend-adjusted output (see the black lines in figure 5 and 7).

In the baseline model, banks finance their activities with a low level of loss-absorbing debt. When a negative productivity shock affects the economy, banks suffer from severe balance sheet shrinkage, as they do not see any further room to adjust their balance sheet structure. For each policy we considered, the scenario is quite different: banks' balance sheet structure *before* the shock can accommodate a change towards more noncontingent debt because of the countercyclical nature of the equity issuance subsidy rate. Such a change allows banks to generate more profits from their activities during a downturn than in the baseline no-policy case. In response, their balance sheet size shrinkage is alleviated, and thus, the TFP growth rate is sustained.

To illustrate how such a balance sheet composition change is at play, consider figure 7. We simulated the model for 1,000,000 periods and then plotted histograms on key bank variables for the baseline model and for each policy we analyzed. A striking observation comes from the distribution of x at the bottom in the middle of figure 6. First, banks back their activity with more equity under macroprudential rules. Second, the balance sheet structure is more volatile and is good for the economy. By adjusting their balance sheet structure, banks need to adjust their balance sheet size less.

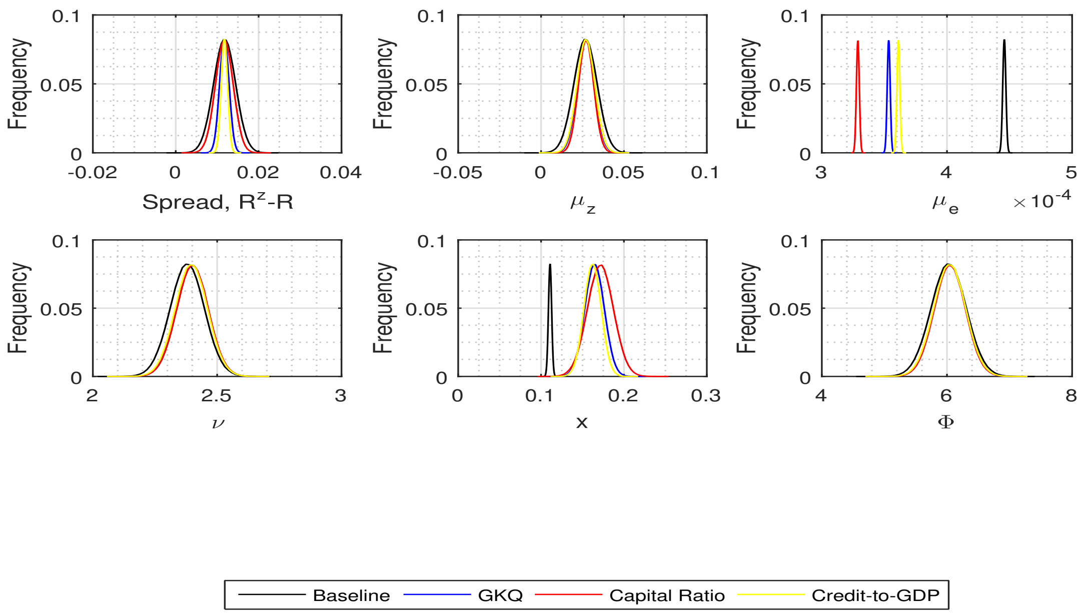

Figure 6: Distribution of key bank variables for 1,000,000 periods of simulated models.

Bank leverage is higher (on average) and less volatile under each macroprudential policy. Additionally, the credit spread is lower and less volatile. Both of these are very beneficial for the economy. On the one hand, higher leverage means greater finance capacity given a level of banking net worth. On the other hand, a lower credit spread is beneficial, as more new varieties are financed in every period. Essentially, a lower spread translates into more abundant credit.

Prudential rules also make the demand for new varieties less volatile. Before turning to demand, let us analyze the supply of new varieties. Consider the stationary version of equation (15)

$$J_t = \left[\frac{1}{\Upsilon_t^z}\right]^{\frac{1}{\eta}} \left[\frac{1}{\eta}\right] \left[\frac{Z_t}{\bar{L}_S}\right]^{\frac{1-\eta}{\eta}}$$

which we interpreted as an aggregate supply curve of new varieties. This supply curve moves only in response to exogenous forcing processes. Accordingly, there is nothing that macroprudential policy can do to make this supply curve ”less” volatile when a shock strikes the economy.

Now, consider the stationary aggregate demand curve for new varieties, given by equation (34).

$$J_t = \frac{\Phi_t N_t}{(1 + Z_t)}$$

This demand curve now depends on endogenous elements, both the leverage ratio and aggregate bank net worth. As can be inferred from figure 6, macroprudential policies can modify both the stationary values of Φ and N as well as their respective volatility. Aiming to illustrate such mechanisms, we took the simulations for 1,000,000 periods again, but this time, we conducted two exercises. We computed the lower and upper bounds for the demand curve of new varieties within the simulation period. The results are presented in figure 7.

Aggregate demand for new varieties shifts upwards under each macroprudential policy. Moreover, the demand range is much lower under these policies than under the baseline model. For every result and illustration we show,

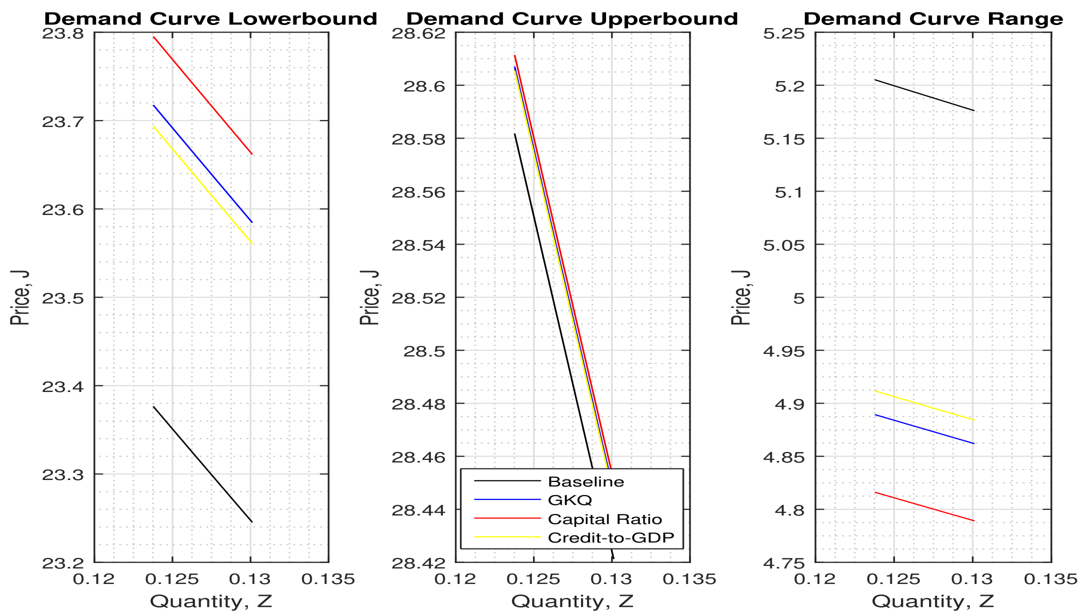

Figure 7: Lower and upper bounds for aggregate demands of new varieties for 1,000,000 periods of the simulated model.

there is a clear option for the best welfare. In all cases, the baseline has worse results than each policy we considered. However, among the policies we considered, it is not only in welfare that the capital-ratio rule outperformed the other policies. First, output loss occurs more quickly from the outset after any negative shock. Second, banks adjust their balance sheet composition more severely and thus need to slightly adjust their balance sheet size. Third, and connected to the second point, the demand range over a large simulation exercise shows that the demand for new varieties is more stable under the capital-ratio rule. Also important is the fact that the GKQ rule is also better in each metric than the credit-to-GDP ratio rule, including for welfare.

We also conducted an interesting counterfactual exercise. What if a uniform regulator exogenously sets $\tau = 0.02$ for each policy above? Intuitively, there would be better hedging against headwinds so that the output loss after a negative shock would be dampened. However, this is not the result. The super subsidized equity issuance would raise x so much that it would make the incentive constraint much tighter, lowering the leverage ratio. The growth rate declines as a result even if the banking system is better hedged. As an illustration, we show in figure 8 below the analog to figure 5 but under $\tau = 0.02$.

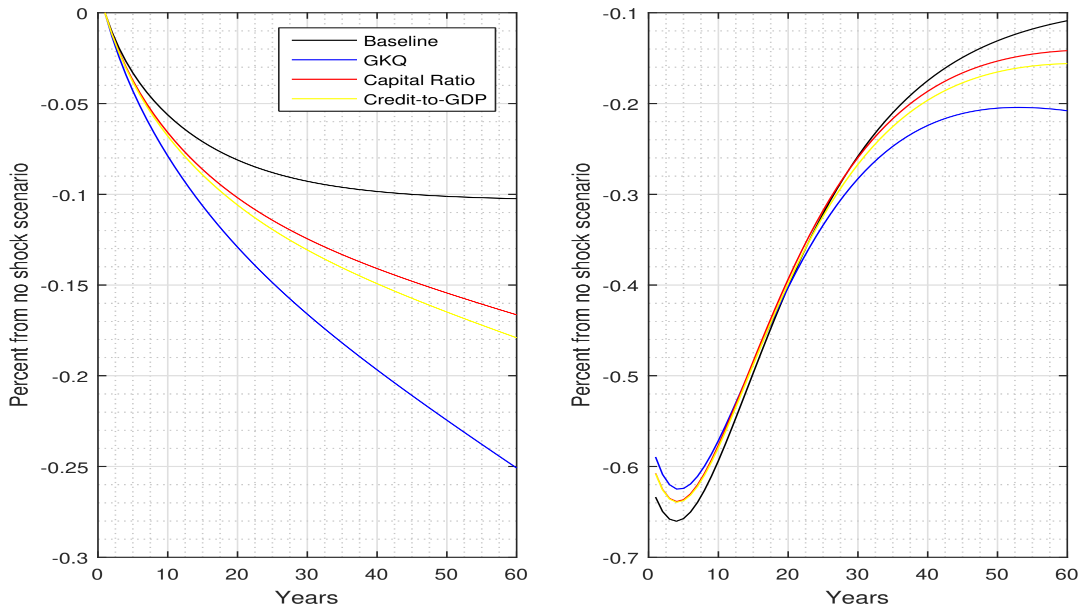

Figure 8: Output response for a one-standard-deviation shock in entrepreneurial activity (left panel) and in intermediary productivity (right panel) for $\tau = 0.02$.

The result is striking. Even with a more resilient aggregate bank liability, the lowered growth rate of the economy makes the intensified policy a calamity. The output response is mild in the first periods, as shown in the right panel of figure 8; however, as the economy returns to its ”normal” growth rate, the gap from the baseline model increases. As periods accumulate, the lower ”normal” growth rate makes a strong difference. Output losses are not only permanent but also widening over time.

6 Conclusion

We extended a standard quantitative macroeconomic model to incorporate endogenous bank liability choice and endogenous productivity change. Within our model, banks finance innovations and choose the structure of their balance sheet endogenously. A meaningful trade-off between risk and private return for the banker induces financial institutions to hedge against adverse shocks optimally from their individual point of view but suboptimally from a social perspective. The interaction between innovations and financial frictions is not a novelty of this work, but there is plenty of work to be done in the field yet.

We showed that macroprudential policies can induce bankers to individually internalize the pecuniary externality within the financial system. To address risk correctly with our solution technique, we considered a risk-adjusted balanced growth path. In this case, macroprudential policy affects not only the dynamics of the economy but also the steady state of endogenous variables. This is because future risk matters and macroprudential policy has an important role in the economy. We explored in detail the fact that mitigating future risk is important, but *how* this mitigation occurs is key. Inducing bankers to back their assets with more loss-absorbing liabilities is fundamental to improving welfare while modifying the response of the economy to adverse shocks is the core role of macroprudential policy within our framework. A key result is that the welfare-maximizing policy parameter is not the growth-maximizing parameter for each policy considered. Growth is a fundamental part of welfare in the long run but is also a risk factor.

To compare different macroprudential schemes, we properly defined a welfare function. We then tested different rules proposed elsewhere in the literature on macroprudential policy. A capital ratio rule in line with the discussion in Hanson et al. (2011) achieved the highest welfare among the policy rules we considered. Positive feedback between the policy action and banks' balance sheet structure appears to be the main cause of such success. Although the credit-to-GDP gap measure receives very close attention within the macroprudential literature, the rule based on such an indicator was outperformed by all other rules we considered. By conducting a counterfactual experiment, we investigated what kind of damage excessive policy intervention can lead to.

Our analysis is bounded by some important limitations. First, there is no role for liquidity in our model. Although evidence in the literature suggests that capital requirements are key in the design of macroprudential policy, there is growing evidence that lowering liquidity and maturity mismatches improves welfare. Second, our banking system does not feature an interbank market or consider the direct interconnectedness of the financial system. Third, there is no bank failure in our model. A macroprudential policy may be important to mitigate bankruptcy within the financial sector, as explored in Miles et al. (2013). Finally, we abstract from both fiscal and monetary policies. There is plenty of evidence that the design of fiscal and monetary rules affects the responsiveness of the economy to shocks. It may thus be important to consider the interplay of such policies with a macroprudential policy to mitigate future risk. Our results are also bounded by the parameter values we chose to solve the model. Estimation was not feasible due to the technicalities used in the solution of the model, so we either calibrated parameters to reach targets found elsewhere in the literature or adopted standard values.

These limitations should certainly not overshadow our results. For future work, a deeper understanding of the link between capital markets and innovation can be a primary target of macroprudential policy analysis. It would also be interesting to allow banks to finance multiple sectors of the economy and track how risk-mitigating policies alter the economy's dynamics and the steady state under this modeling approach. In our modeling choice, banks share their own risk with households, but such risk only indirectly dampens output and consumption volatility. The reason is that banks do finance the creation of new firms, not the production of existing firms. Extending the model to incorporate liquidity shocks would also link our model to other branches of the literature and explore different macroprudential rules based on liquidity requirements.

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A Calculations

This appendix contains calculations to obtain some of the equations in the text.

A.1 Asset Price J_t

To obtain this expression, consider the following cost minimization problem:

$$\min_{\mathcal{R}_t(i), L_{S,t}(i)} \mathcal{R}_t(i) + W_{S,t} L_{S,t}(i)$$

subject to

$$\Upsilon_t^z \mathcal{R}_t^\eta (A_t L_{S,t})^{1-\eta} \geqslant 1$$

The marginal cost MC_t is equal to the multiplier associated with the constraint. To see this, observe that the first-order necessary conditions imply the following optimal choice of inputs:

$$L_{S,t} = \frac{\mathcal{R}_t(i)}{W_{S,t}(i)} \frac{1-\eta}{\eta}$$

Entering this into the minimization problem and taking the derivative with respect to $\mathcal{R}_t(i)$, we find

$$1 + \frac{1-\eta}{\eta} - \theta_t \Upsilon_t^z \left[\frac{A_t}{W_{S,t}(i)} \frac{1-\eta}{\eta} \right]^{1-\eta} = 0$$

and with some algebra,

$$MC_t = \theta_t = \frac{1}{\Upsilon_t^z} \left(\frac{1}{\eta} \right)^\eta \left(\frac{W_{S,t}/A_t}{1-\eta} \right)^{1-\eta} = \left[\frac{1}{\Upsilon_t^z} \right] \left[\frac{1}{\eta} \right]^\eta \left[\frac{1}{1-\eta} \right]^{1-\eta} \left[\frac{W_{S,t}}{A_t} \right]^{1-\eta}$$

We can use the free entry of entrepreneurs to say

$$J_t = MC_t$$

A.2 Aggregate Supply of New Varieties

For this, observe that we can rewrite Z_t as

$$Z_t = \Upsilon_t^z \left[\frac{\mathcal{R}_t(i)}{A_t \bar{L}_S} \right]^\eta A_t \bar{L}_S$$

For the entrepreneur, the marginal cost of labor must equal its marginal benefit, which leads us to

$$W_{S,t} = J_t \cdot \Upsilon_t^z \mathcal{R}_t(i)^{\eta}(A_t\bar{L}_S)^{1-\eta}(A_t\bar{L}_S)^{-1}A_t \quad \Rightarrow \quad \frac{W_{S,t}}{A_t} = J_t \cdot (1-\eta)\frac{Z_t}{A_t\bar{L}_S}$$

Entering this result into (13) and substituting (14), we find

$$J_t = \left[\frac{1}{\Upsilon_t^z}\right]\left[\frac{1}{\eta}\right]^{\eta}\left[\frac{1}{1-\eta}\right]^{1-\eta}\left[\frac{Z_t}{A_t\bar{L}_S}\right]^{1-\eta}[1-\eta]^{1-\eta}[J_t]^{1-\eta}$$

Slightly simplifying, we have

$$J_t = \left[\frac{1}{\Upsilon_t^z}\right]^{\frac{1}{\eta}}\left[\frac{1}{\eta}\right]\left[\frac{1}{\bar{L}_S}\right]^{\frac{1-\eta}{\eta}}\left[\frac{Z_t}{A_t}\right]^{\frac{1-\eta}{\eta}}$$

A.3 Banks' Value Function

We conjecture that henceforth, the bank's value V_t is a function of *the current* bank's asset value and its net worth. The economic rationale for this *guess* is the following. As the banker's objective is to maximize the terminal dividend, it does so by accumulating earnings through investing in banks' net worth and collecting a positive excess return of assets over liabilities.

The guess takes the form of

$$V_t(n_t, x_t, s_t) = a_t J_t s_t + v_t n_t$$

The Bellman equation is as follows:

$$V_t(n_t, x_t, s_t) = \mathbb{E}_t\left[\Lambda_{t,t+1}\left\{(1-\sigma)n_{t+1} + \sigma \max_{s_t, x_t}[V_{t+1}]\right\}\right]$$

Our strategy is then to insert the guess and verify under which conditions for a_t and v_t, the guess satisfies the Bellman conditions.

We then have, using this guess,

$$a_t J_t s_t + v_t n_t = \mathbb{E}_t\left[\Lambda_{t,t+1}\left\{(1-\sigma)n_{t+1} + \sigma\left[\frac{a_{t+1}J_{t+1}s_{t+1} + v_{t+1}n_{t+1}}{n_{t+1}}\right]n_{t+1}\right\}\right]$$

Recalling that the leverage ratio is given by

$$\Phi_t = \frac{J_t s_t}{n_t}$$

we can write

$$a_t J_t s_t + v_t n_t = \mathbb{E}_t\left[\Lambda_{t,t+1}\left\{\left[(1-\sigma) + \sigma\left[a_{t+1}\Phi_{t+1} + v_{t+1}\right]\right]n_{t+1}\right\}\right]$$

Now, moving (23) one period forward, we can define n_{t+1} as

$$n_{t+1} = T_{t+1}J_t s_t + R_{t+1}n_t$$

with

$$T_{t+1} = R^z_{t+1} - x_t R^e_{t+1} - (1 - x_t)R_{t+1}$$

Thus, we have

$$a_t J_t s_t + v_t n_t = \mathbb{E}_t \left[\Lambda_{t,t+1} \{(1-\sigma) + \sigma \left[a_{t+1}\Phi_{t+1} + v_{t+1}\right]\} \{T_{t+1}J_t s_t + R_{t+1}n_t\}\right]$$

We might define the shadow value of an extra unit of net worth as

$$\Omega_{t+1} = (1-\sigma) + \sigma[a_{t+1}\Phi_{t+1} + v_{t+1}]$$

so that we have

$$a_t J_t s_t + v_t n_t = \mathbb{E}_t \left[\Lambda_{t,t+1}\Omega_{t+1} \{T_{t+1}J_t s_t + R_{t+1}n_t\}\right]$$

We then conclude that

$$a_t = \mathbb{E}_t[\Lambda_{t,t+1}\Omega_{t+1}T_{t+1}]$$
$$v_t = \mathbb{E}_t[\Lambda_{t,t+1}\Omega_{t+1}R_{t+1}]$$

We can then recall the definition of T_{t+1} and divide it into two economically reasonable parts, defining

$$\mu_{z,t} = \mathbb{E}_t[\Lambda_{t,t+1}\Omega_{t+1}(R^z_{t+1} - R_{t+1})]$$
$$\mu_{e,t} = \mathbb{E}_t[\Lambda_{t,t+1}\Omega_{t+1}(R_{t+1} - R^e_{t+1})]$$

Given these definitions, we have

$$a_t = \mu_{z,t} + x_t \mu_{e,t}$$

and

$$\Omega_{t+1} = 1 - \sigma + \sigma[\Phi_{t+1}(\mu_{z,t+1} + x_{t+1}\mu_{e,t+1}) + v_{t+1}]$$

The value function has the following format:

$$V_t(n_t, x_t, s_t) = (\mu_{z,t} + x_t\mu_{e,t})J_t s_t + v_t n_t$$

A.4 Intermediate Profits

For this, observe that in any symmetric equilibrium, $K_t(s) = K_t/A_t$ and $L_t(s) = L_t/A_t$ for each s. Thus, we have

$$Y_t(s) = \Upsilon_t^y \left[\frac{K_t}{A_t}\right]^{\alpha} \left[\frac{L_t}{A_t}\right]^{1-\alpha}$$

Implying that

$$Y_t(s) = \Upsilon_t^y \frac{[K_t]^{\alpha} L_t^{1-\alpha}}{A_t}$$

In (35), we have

$$Y_t = \left[\int_0^{A_t} \left(\Upsilon_t^y \frac{[K_t]^{\alpha} L_t^{1-\alpha}}{A_t}\right)^{\frac{v-1}{v}} ds\right]^{\frac{v}{v-1}}$$

Then,

$$Y_t = \Upsilon_t^y A_t^{\frac{v}{v-1}} \frac{[K_t]^{\alpha} L_t^{1-\alpha}}{A_t}$$

Simplifying to (42)

$$Y_t = \Upsilon_t^y A_t^{\frac{1}{v-1}} (K_t)^{\alpha} L_t^{1-\alpha}$$

B Welfare Function

This appendix presents the calculations to obtain the welfare function of the model. Let us start with the utility function

$$U(C_t, L_t) = \frac{(C_t + \frac{\Psi}{1+\varepsilon} A_t L_t^{1+\varepsilon})^{1-\rho}}{1-\rho} \tag{1}$$

For any $t > t_o$ if no shock materializes, the economy continues along its balanced growth path, i.e.,

$$A_t = A_{t_o} g^{t-t_o} \tag{2}$$

where g is the growth rate of the economy. In that case, labor is level stationary, i.e., $L_t = L$, $\forall t$. For consumption, there is a growth trend, so we have to normalize it

$$\tilde{C}_t = \frac{C_t}{A_t} \tag{3}$$

We can then write

$$U(C_{t+\tau}, L_{t+\tau}) = \frac{(\tilde{C}_t A_t g^{\tau} + \frac{\Psi}{1+\varepsilon} A_t g^{\tau} L_t^{1+\varepsilon})^{1-\rho}}{1-\rho} \tag{4}$$

Simplifying to

$$U(C_{t+\tau}, L_{t+\tau}) = g^{\tau(1-\rho)} U(\tilde{C}_t, L_t) \tag{5}$$

Since the future is risky, we take a second-order Taylor expansion around the risk-adjusted steady state so that the volatilities of labor and consumption are considered within our welfare function.

We begin by normalizing without loss of generality the initial level of technology A_{t_o} to 1. Next, we define the following auxiliary variable $\mathcal{Z}$ to reduce notation:

$$\mathcal{Z}_t = \tilde{C}_t - \frac{\Psi}{1+\varepsilon} L_t^{1+\varepsilon}, \quad \forall t \tag{6}$$

We observe that given the parameter choice (1 in the text), $\mathcal{Z} > 0$. Given this definition, utility might be expressed as

$$U(C_{t+\tau}, L_{t+\tau}) = g^{\tau(1-\rho)} \frac{\mathcal{Z}^{1-\rho}}{1-\rho} < 0$$

A second-order Taylor expansion around the risk-adjusted steady state is then performed from equation (5). Growth between t and $t-1$ was already decided at $t-1$ because of the choice of Z_{t-1}, so tomorrow's technological level is known when *today*'s shock is realized. Therefore, we need not consider the volatility of the growth rate g. Since no shock materializes today by the definition of the risk-adjusted steady state, we need not be concerned about the volatility of growth.

The first-order terms with respect to $\tilde{C}$ and L are

$$\frac{\partial U}{\partial \tilde{C}} = g^{\tau(1-\rho)} \mathcal{Z}^{-\rho} \tag{7}$$

$$\frac{\partial U}{\partial L} = g^{\tau(1-\rho)} \mathcal{Z}^{-\rho} (-\Psi) L^{\varepsilon} \tag{8}$$

The second-order terms with respect to C and L are

$$\frac{\partial^2 U}{\partial \tilde{C}^2} = g^{\tau(1-\rho)} (-\rho) \mathcal{Z}^{-\rho-1} \tag{9}$$

$$\frac{\partial^2 U}{\partial L^2} = g^{\tau(1-\rho)} \left\{ (-\rho) \mathcal{Z}^{-\rho-1} \Psi^2 L^{2\varepsilon} - \mathcal{Z}^{-\rho} \Psi \epsilon L^{\varepsilon-1} \right\} \tag{10}$$

$$\frac{\partial^2 U}{\partial L \partial \tilde{C}} = g^{\tau(1-\rho)} \left\{ (-\rho) \mathcal{Z}^{-\rho-1} (-\Psi) L^{\varepsilon} \right\} \tag{11}$$

Putting this all together, we have the following approximation up to a second order

$$\begin{aligned} U(C_{t+\tau}, L_{t+\tau}) \approx g^{\tau(1-\rho)} & \left\{ \frac{\mathcal{Z}^{1-\rho}}{1-\rho} + \frac{1}{2}(-\rho) \mathcal{Z}^{-\rho-1} \mathcal{V}(\tilde{C}) + \rho \mathcal{Z}^{-\rho-1} \Psi L^{\epsilon} \mathcal{C}(\tilde{C}, L) \right\} \\ & + g^{\tau(1-\rho)} \left\{ \frac{1}{2} (-\rho Z^{-\rho-1} \Psi^2 L^{2\varepsilon} + \mathcal{Z}^{-\rho} (-\Psi) \epsilon L^{\epsilon-1}) \mathcal{V}(L) \right\} \end{aligned} \tag{12}$$

where $\mathcal{V}(\tilde{C})$ is the variance of de-trended consumption, $\mathcal{V}(L)$ is the variance of labor and $\mathcal{C}(\tilde{C}, L)$ is the covariance between de-trended consumption and labor. The first-order terms should appear in this equation, but we suppressed them because the second-order Taylor expansion is taken within the expectations operator and thus the first terms would vanish.

Slightly simplifying notation, we have

$$U(C_{t+\tau}, L_{t+\tau}) \approx g^{\tau(1-\rho)} \left\{ \frac{\mathcal{Z}^{1-\rho}}{1-\rho} + \mathcal{M}(\tilde{C}, L) \right\} \tag{13}$$

where $\mathcal{M}(\tilde{C}, L)$ are the second moments of $\tilde{C}$ and L and are given by

$$\mathcal{M}(\tilde{C}, L) = \frac{1}{2}(-\rho) \mathcal{Z}^{-\rho-1} \mathcal{V}(\tilde{C}) + \frac{1}{2} \left\{ -\rho Z^{-\rho-1} \Psi^2 L^{2\varepsilon} + \mathcal{Z}^{-\rho} (-\Psi) \epsilon L^{\epsilon-1} \right\} \mathcal{V}(L) + \rho \mathcal{Z}^{-\rho-1} \Psi L^{\epsilon} \mathcal{C}(\tilde{C}, L) \tag{14}$$

Interpretation. The first term in $\mathcal{M}$ is negative, which follows from the diminishing marginal utility of consumption. The second term is also negative and follows from the fact that variance in labor implies variance in leisure, and since utility is concave in leisure, the household negatively discounts this source of risk. The last term is positive: since both labor and consumption are procyclical, the covariance between these variables is positive. The overall risk evaluation turns out to be negative: the variance terms dominate the covariance force, so the household prefers no net risk.

We then proceed to express welfare.

$$\mathcal{W}_t = \mathbb{E}_t \left\{ \sum_{\tau=0}^{\infty} \beta^{\tau} U(C_{t+\tau}, L_{t+\tau}) \right\} \tag{15}$$

Using equation (13),

$$\mathcal{W} = \mathbb{E}_t \left\{ \frac{\mathcal{Z}^{1-\rho}}{1-\rho} + \beta g^{1-\rho} \left\{ \frac{\mathcal{Z}^{1-\rho}}{1-\rho} + \mathcal{M}(\tilde{C}, L) \right\} + \beta^2 g^{2(1-\rho)} \left\{ \frac{\mathcal{Z}^{1-\rho}}{1-\rho} + \mathcal{M}(\tilde{C}, L) \right\} + \dots \right\} \tag{16}$$

In period t, there is no risk in deed, so the second moments appear only from $t+1$ onward. Now, since $\rho > 1$, $\beta^{\tau} g^{\tau(1-\rho)} < 1, \quad \forall \tau \in \mathbb{N}$, we can sum all the terms within brackets to obtain our welfare function:

$$\mathcal{W} = \frac{\mathcal{Z}^{1-\rho}}{1-\rho} + \frac{\beta g^{1-\rho}}{1-\beta g^{1-\rho}} \left\{ \frac{\mathcal{Z}^{1-\rho}}{1-\rho} + \mathcal{M}(\tilde{C}, L) \right\}$$

This term can be further simplified to

$$\mathcal{W} = \frac{1}{1-\beta g^{1-\rho}} \left\{ \frac{\mathcal{Z}^{1-\rho}}{1-\rho} \right\} + \frac{\beta g^{1-\rho}}{1-\beta g^{1-\rho}} \left\{ \mathcal{M}(\tilde{C}, L) \right\} \tag{17}$$

Log utility. For the case $\rho \to 1$, i.e., log utility, welfare is given by

$$\mathcal{W} = \log(g) \left\{ \sum_{t=1}^{\infty} \beta^t t \right\} + \frac{1}{1-\beta} \left\{ \log(\mathcal{Z}) + \beta \mathcal{M}(\tilde{C}, L) \right\} \tag{18}$$

$\sum_{t=1}^{\infty} \beta^t t$ is trivially convergent for any $0 < \beta < 1$. For our numerical exercises, we took the first 100,000 periods as representative of the whole series. After that, each period adds less than 5.5455×10^{-244} to the series.

C Computation Strategy

Let X be the vector of endogenous variables. The solution of the model can be expressed as

$$\mathbb{E}_t[f(X_{t+1}, X_t, X_{t-1}, \varepsilon_t)] = 0$$

where ε_t is the vector of the current realization of exogenous variables. f is the function that describes the optimal plan for each endogenous variable at equilibrium.

As defined by Coeurdacier et al. (2011), the risky steady state is the point where agents choose to stay on a given date if they expect future risk and if the realization of shock is 0 on this date. The economic rationale is the following: the risk-adjusted steady state is the point in the state space where agents choose to be if all shock has dissipated and if agents expect the future to be risky.

To incorporate risk as above, one might use second-order approximations around the risky steady state as follows:

$$0 = f(E_t(X_{t+1})) + \mathbb{E}_t[f^{''}(\cdot)[X_{t+1} - \mathbb{E}_t(X_{t+1})]^2]$$

where the second-order derivatives are taken at the point $\mathbb{E}_t(V_{t+1})$.

To properly incorporate risk considerations in the steady state, one can use the following procedure to estimate the model:

1. Log-linearize the model around the deterministic steady state.
2. Simulate the model, and compute the second moments provided from this exercise. (The $\mathcal{M}$s in the next appendix).
3. Use the second moments found in step 2 to compute the risky steady state.
4. Compute the resulting moments from simulating the moments around the risk-adjusted steady state found in step 3.
5. Iterating up to the second moments used to compute the risky steady state is consistent with the moments generated by it.

Precisely, the risky steady state combines a vector of endogenous variables X together with the second moments M. The resulting solution requires that both steady-state values and second moments be found jointly.

There is a mapping from M to X; that is,

$$X = G_x(M)$$

Conversely, there is a mapping from X to M; that is,

$$M = G_m(X)$$

The algorithm described above looks for a fixed point; e.g.,

$$M^* = g_m(g_x(M^*))$$

Once the fixed point is achieved, we find the risk-adjusted steady state with

$$X^* = g_x(M^*)$$

D Equilibrium Conditions and Risk-Adjusted Balanced Growth Path

This appendix is divided into two subsections. The first shows the equilibrium conditions of the model. We explain how we have turned the model into a stationarized version and group equations used to simulate the model. The second subsection makes explicit the risk-adjusted steady state.

D.1 Equilibrium Conditions

The model features long-run growth. To simulate the model, we first need to stationarize it. Therefore, we divided by the technological level A_t the appropriate set of variables featuring long-run growth. We defined $\mathcal{Q}_{d,t} \equiv \frac{\mathcal{Q}_t}{A_t}$ for $\mathcal{Q}_t = \{Y_t, C_t, H_t, I_t, K_{t+1}, \mathcal{R}_t, A_{t+1}, Z_t, N_t, E_t, D_t\}$. The remaining variables are stationary $\{R_{t+1}$, $R^e_{t+1}, R^z_{t+1}, \mu_{z,t}, \mu_{e,t}, \nu_t, \Phi_t, \Lambda_{t,t+1}\Omega_t, \Theta_t, x_t, J_t, q_t, \pi_t, L_t\}$ and therefore need not be stationarized.

By stationarizing the system of equations, we transform the model so that the gross growth rate of technological level $g_{t+1} = \frac{A_{t+1}}{A_t}$ is an endogenous variable.

The system of equilibrium conditions is described below:

- Household Stochastic Discount Factor

$$\Lambda_{t,t+1} = \beta \frac{U_{c,t+1}}{U_{c,t}} g_{t+1}^{-\rho} \tag{19}$$

- Marginal Utility of Consumption

$$U_{c,t} = \left(C_t - \Psi \frac{1}{1+\varepsilon} L_t^{1+\varepsilon} \right)^{-\rho} \tag{20}$$

- Euler Equation for Bond Holdings

$$\mathbb{E}_t \left[\Lambda_{t,t+1} \right] R_{t+1} = 1 \tag{21}$$

- Euler Equation for Equity Holdings

$$\mathbb{E}_t \left[\Lambda_{t,t+1} R^e_{t+1} \right] = 1 \tag{22}$$

- Euler Equation for Capital Holdings

$$\mathbb{E}_t \left[\Lambda_{t,t+1} (R^k_{t+1} + (1-\delta)) \right] = 1 \tag{23}$$

- Optimality of Labor Supply

$$\Psi L_t^{\varepsilon+1} = \frac{\upsilon - 1}{\upsilon} (1-\alpha) Y_t \tag{24}$$

- Law of Motion of Mass of Varieties

$$g_{t+1} = \phi(1 + Z_t) \tag{25}$$

- Aggregate Supply of Varieties

$$J_t = \left(\frac{1}{\eta}\right)\left(\frac{1}{\Upsilon_t^z}\right)^{\frac{1}{\eta}}\left(\frac{Z_t}{\bar{L_S}}\right)^{\frac{1-\eta}{\eta}} \tag{26}$$

- Real Sector Production

$$Y_t = \Upsilon_t^y K_t^\alpha L_t^{1-\alpha} \tag{27}$$

- Intermediary Firm Profits

$$\pi_t = \frac{1}{\eta} Y_t \tag{28}$$

- Return on Capital

$$R_t^k = \frac{\upsilon - 1}{\upsilon}\alpha\frac{Y_t}{K_t} \tag{29}$$

- Economy-wide Resource Constraint

$$Y_t = C_t + I_t + \mathcal{R}_t \tag{30}$$

- Law of Motion of Capital

$$K_{t+1}g_{t+1} = I_t + (1-\delta)K_t \tag{31}$$

- Return on Equity

$$R_{t+1}^e = \phi\frac{q_{t+1} + \pi_{t+1}}{q_t} \tag{32}$$

- Return on Varieties

$$R_{t+1}^z = \phi\frac{J_{t+1} + \pi_{t+1}}{J_t} \tag{33}$$

- Balance Sheet

$$D_t = J_t(1 + Z_t) - q_t E_t + N_t \tag{34}$$

- Aggregate Net Worth Evolution

$$N_t g_t = \sigma\left(R_t^z - R_t + x_t(R_t - R_t^e)\right)J_{t-1}(1 + Z_{t-1}) + \sigma R_t N_{t-1} + \xi(1-\sigma)J_{t-1}(1 + Z_{t-1}) \tag{35}$$

- Excess Bank Profit

$$\mu_{z,t} = \mathbb{E}_t\left[\Lambda_{t,t+1}\Omega_{t+1}(R_{t+1}^z - R_{t+1})\right] \tag{36}$$

- Excess Bank Value of Substituting Debt for Equity

$$\mu_{e,t} = \mathbb{E}_t\left[\Lambda_{t,t+1}\Omega_{t+1}(R_{t+1} - R_{t+1}^e)\right] \tag{37}$$

- Bank Deposit Cost

$$\nu_t = \mathbb{E}_t\left[\Lambda_{t,t+1}\Omega_{t+1}R_{t+1}\right] \tag{38}$$

- Value of Net Worth Tomorrow

$$\Omega_{t+1} = (1-\sigma) + \sigma(\Phi_{t+1}(\mu_{z,t+1} + x_{t+1}\mu_{e,t+1}) + \nu_{t+1}) \tag{39}$$

- Optimal Outside Equity Finance Ratio

$$\theta(\mu_{z,t} + x_t\mu_{e,t})(\epsilon + \kappa x_t) = \theta(1 + \epsilon x_t + \frac{1}{2}\kappa x_t^2)(\mu_{e,t}) \tag{40}$$

- Maximum Leverage Ratio

$$\Phi_t = \frac{\nu_t}{\Theta(x_t) - (\mu_{z,t} + x_t\mu_{e,t})} \tag{41}$$

- Leverage Constraint

$$N_t\Phi_t = J_t(1 + Z_t) \tag{42}$$

- Divertable Fraction

$$\Theta(x_t) = \theta(1 + \epsilon x_t + \frac{1}{2}\kappa x_t^2) \tag{43}$$

- Law of Motion of Exogenous TFP of Production of Varieties

$$\log(\Upsilon_t^z) = \rho_z \log(\Upsilon_{t-1}^z) + \epsilon_t^z \tag{44}$$

- Law of Motion of Exogenous TFP of Real Production

$$\log(\Upsilon_t^y) = \rho_y \log(\Upsilon_{t-1}^y) + \epsilon_t^y \tag{45}$$

D.2 Risk-Adjusted Balanced Growth Path

We let $\hat{b}_t = \frac{b_t - b}{b}$ be the log deviation of variable b on date t from the risk-adjusted steady state b. Observe that the covariance between the two variables M and V is equal to $\text{Cov}(M_{t+1}, V_{t+1}) = MV\text{Cov}(\hat{M}_{t+1}, \hat{V}_{t+1})$. The stationarized model has the following equations for the risk-adjusted steady state. We exclude time subscripts and adjust for future risk in the lines of appendix 3.

$$\Lambda = \beta g^{-\rho}\mathcal{M}_1 \tag{46}$$

$$\Lambda R = 1 \tag{47}$$

$$\Lambda R^e + \mathrm{Cov}(\Lambda_{t,t+1}, R^e_{t+1}) = 1 \tag{48}$$

$$\Lambda R^k(R^k + (1-\delta)) + \mathrm{Cov}(\Lambda_{t,t+1}, R^k_{t+1}) = 1 \tag{49}$$

$$\Psi L^{\varepsilon+1} = \frac{\upsilon - 1}{\upsilon}(1-\alpha)Y \tag{50}$$

$$g = \phi(1+Z) \tag{51}$$

$$J = \left(\frac{1}{\eta}\right)\left(\frac{Z}{\bar{L_S}}\right)^{\frac{1-\eta}{\eta}} \tag{52}$$

$$Y = K^{\alpha}L^{1-\alpha} \tag{53}$$

$$\pi = \frac{1}{\eta}Y \tag{54}$$

$$R^k_{\nabla} = \frac{\upsilon - 1}{\upsilon}\frac{Y}{K} \tag{55}$$

$$R^k = \frac{\upsilon - 1}{\upsilon}\frac{Y}{K}\mathcal{M}_2 \tag{56}$$

$$Y = C + I + \mathcal{R} \tag{57}$$

$$Kg = I + (1-\delta)K \tag{58}$$

$$R^e_{\nabla} = \phi\frac{q+\pi}{q} \tag{59}$$

$$R^e = \phi + \phi \frac{\pi}{q} \mathcal{M}_2 \tag{60}$$

$$R^z_\nabla = \phi \frac{J + \pi}{J} \tag{61}$$

$$R^z = \phi \mathcal{M}_3 + \phi \frac{\pi}{J} \mathcal{M}_2 \tag{62}$$

$$D = J(1 + Z) - qE - N \tag{63}$$

$$g = \sigma \left[(R^z_\nabla - R + x(R - R^e_\nabla)\Phi + R \right] + \xi(1 - \sigma)\Phi \tag{64}$$

$$\mu_z = \Lambda \Omega R^z \mathcal{M}_4 - \Omega \mathcal{M}_5 \tag{65}$$

$$\mu_e = \Omega \mathcal{M}_5 - \Lambda \Omega R^e \mathcal{M}_6 \tag{66}$$

$$\nu = \Omega \mathcal{M}_5 \tag{67}$$

$$\Omega = 1 - \sigma + \sigma(\Phi(\mu_z + x\mu_e) + \nu) \tag{68}$$

$$\theta(\epsilon + \kappa x)(\mu_z + x\mu_e) = \theta(1 + \epsilon x + \frac{1}{2}\kappa x^2)\mu_e \tag{69}$$

$$\Phi = \frac{\nu}{\Theta - (\mu_z + x\mu_e)} \tag{70}$$

$$N\Phi = J(1 + Z) \tag{71}$$

$$\Theta = \theta(1 + \epsilon x + \frac{1}{2}\kappa x^2) \tag{72}$$

Above, R^e_∇ is the *realized* rate of return of outside equity, and R^z_∇ is the *realized* rate of return of holding varieties. These realized rates of return enter into the net worth accumulation of the aggregate banking system, as in equation (63). Let us define an auxiliary variable

$$\mathcal{Z}_t = C_t - \Psi \frac{1}{1+\varepsilon} L_t^{1+\varepsilon}$$

The second moments above are represented by the $\mathcal{M}$s. They are the following:

$$\mathcal{M}_1 = 1 + \frac{1}{2}\rho(1+\rho)\mathrm{Var}(\hat{\mathcal{Z}}_t) \tag{73}$$

$$\mathcal{M}_2 = 1 + (1-\alpha)\mathrm{Cov}(\hat{\Upsilon}^y_{t+1}, \hat{L}_{t+1}) + \frac{1}{2}(-\alpha)(1-\alpha)\mathrm{Var}(\hat{L}_{t+1}) \tag{74}$$

$$\mathcal{M}_3 = 1 + \left(\frac{1}{\eta\sqrt{2}}\right)^2 \left[(-2)(1-\eta)\mathrm{Cov}(\hat{\Upsilon}^z_{t+1}, \hat{J}_{t+1}) + (1+\eta)\mathrm{Var}(\hat{\Upsilon}^z_{t+1}) + (1-\eta)(1-2\eta)\mathrm{Var}(\hat{J}_{t+1})\right] \tag{75}$$

$$\mathcal{M}_4 = 1 + \mathrm{Cov}(\hat{\Omega}_{t+1}, \hat{\Lambda}_{t,t+1}) + \mathrm{Cov}(\hat{\Omega}_{t+1}, \hat{R^z}_{t+1}) + \mathrm{Cov}(\hat{R^z}_{t+1}, \hat{\Lambda}_{t,t+1}) \tag{76}$$

$$\mathcal{M}_5 = 1 + \mathrm{Cov}(\hat{\Omega}_{t+1}, \hat{\Lambda}_{t,t+1}) \tag{77}$$

$$\mathcal{M}_6 = 1 + \mathrm{Cov}(\hat{\Omega}_{t+1}, \hat{\Lambda}_{t,t+1}) + \mathrm{Cov}(\hat{\Omega}_{t+1}, \hat{R^e}_{t+1}) + \mathrm{Cov}(\hat{R^e}_{t+1}, \hat{\Lambda}_{t,t+1}) \tag{78}$$