Day 6 – "Catch up"

1. Find the average rate of change of $f(t)$ between $t = 20 \ min$ and $t = 30 \ min$. Then find the instantaneous rate of change of $f(t)$ at $t = 24 \ min$.

t	20	22	24	26	28	30
$f(t)$	5	7	11	18	29	45

2. Approximate the instantaneous velocity of $g(x) = \frac{e^x + 1}{x - 2}$ at $g(3)$. Explain the steps you used.

3. Find the equation of the tangent line to the function $f(x) = \sqrt{x - 2}$ at $x - 4$.

4. Find all vertical and horizontal asymptotes for the function $g(x) = \frac{x-2}{x^3 - 8}$. Show all work and use limits as appropriate.

5. Find the following limits:

   a. $\lim_{x \to \frac{\pi}{2}^-} \frac{|cos(x)|}{x - \frac{\pi}{2}}$

   b. $\lim_{x \to 2} \frac{\frac{3}{x-1} - 3}{x - 2}$

   c. $\lim_{x \to 1} \ sin^{-1}(2x - 1)$

   d. $\lim_{x \to 1} \ sin^{-1}\left(2x - x^2\right)$

6. Sketch a possible function with the given characteristics:

   a. $\lim_{x \to 1} f(x) \neq f(x)$

   b. $\lim_{x \to -5} f(x) = d.n.e.$

   c. $\lim_{x \to \infty} f(x) = 3$

   d. $f(-1)$ is undefined

   e. $\lim_{x \to 4^+} f(x) = 1$

   f. $\lim_{x \to 4^-} f(x) = -\infty$

7. Use the graph of $f(x)$ to answer the questions below it.

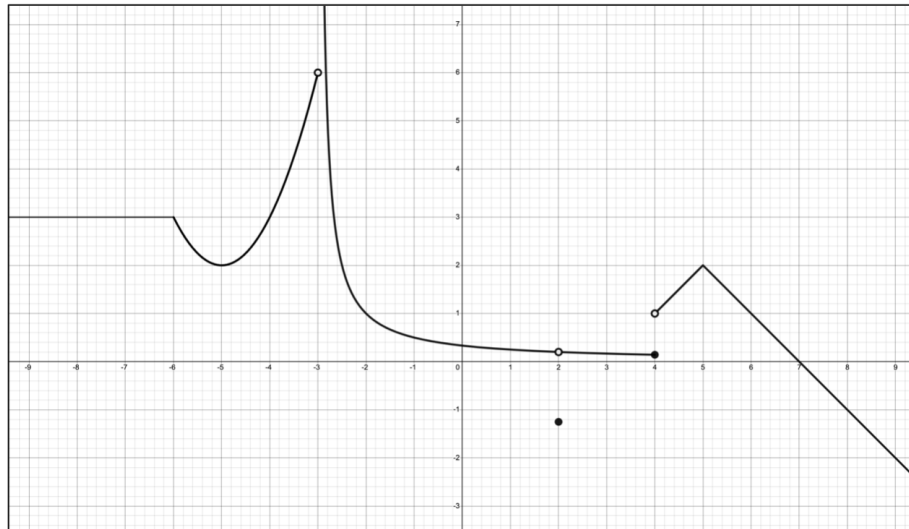Figure 1: Piecewise Graph

a. $\lim_{x \to -3^-} f(x) =$ _____

b. $\lim_{x \to -3^+} f(x) =$ _____

c. $\lim_{x \to -3} f(x) =$ _____

d. $\lim_{x \to 2^-} f(x) =$ _____

e. $\lim_{x \to 2^+} f(x) =$ _____

f. $\lim_{x \to 2} f(x) =$ _____

g. $\lim_{x \to \infty} f(x) =$ _____

h. $\lim_{x \to -\infty} f(x) =$ _____

i. $f(x)$ has jump discontinuity at: _____

j. $f(x)$ has infinite discontinuity at: _____

k. $f(x)$ has removable discontinuity at: _____

l. The horizontal asymptotes of $f(x)$ are: _____

m. The vertical asymptotes of $f(x)$ are: _____

2

8. Find k so that $f(x)$ is continuous for all real numbers, given:

$$f(x) = \begin{cases} e^x, & x > 0 \\ -(x+1)^2 + k, & x \leq 0 \end{cases}$$

9. Find: $\lim_{x \to 0} \left(\ln \left(\tan^2 x \right) \right) = $ _____

10. #37 from 2.4 Homework:

Use the Intermediate Value Theorem (IVT) to show that there is a root of the given equation in the specified interval. $x^4 + x - 3 = 0$ *on* $(1, 2)$

11. #39 from 2.4 Homework:

Use the Intermediate Value Theorem (IVT) to show that there is a root of the given equation in the specified interval. $\cos x = x$ *on* $(0, 1)$