Day 2

Calculating Limits from Limit Laws

Class Goals

- There are rules to limits that help us compute the limit
- Limits are linear (it works well with sums/differences and scalar multiplication)
- Limits work well with products and division (as long as you aren't dividing by zero)
- Limits of most common functions can be solved by plugging in. Things to watch out for:
  - $\frac{0}{0}$ This could be anything.
  - Piecewise defined functions at the values where you change definitions
- Squeeze theorem

Limit Laws Suppose that $\lim_{x\to a} f(x)$ and $\lim_{x\to a} g(x)$ exist then

Limit Law	Formula
Addition/ Subtraction	$\lim_{x\to a}[f(x) \pm g(x)] = \lim_{x\to a} f(x) \pm \lim_{x\to a} g(x)$
Scalar Multiplication	$\lim_{x\to a} cf(x) = c\lim_{x\to a} f(x)$
Function Multiplication	$\lim_{x\to a} f(x)g(x) = \lim_{x\to a} f(x) \cdot \lim_{x\to a} g(x)$
Function Division	$\lim_{x\to a} \frac{f(x)}{g(x)} = \frac{\lim_{x\to a} f(x)}{\lim_{x\to a} g(x)}$ if $\lim_{x\to a} g(x) \neq 0$
Product Law	$\lim_{x\to a}[f(x)]^n = [\lim_{x\to a} f(x)]^n$ n is a positive integer
Root Law	$\lim_{x\to a} \sqrt[n]{f(x)} = \sqrt[n]{\lim_{x\to a} f(x)}$ n is a positive integer and if n is even $f(x) > 0$

Consequences of Limit Laws For every function that is a polynomial or rational function, if you are trying to evaluate the limit by substitution then you can do so as long as the value is in the domain.

Squeeze Theorem First we look at a supporting idea:

If

$$f(x) \leq g(x)$$

when x is near a (except possibly a) and the limits of both f and g exist as x approaches a then

$$\lim_{x \to a} f(x) \leq \lim_{x \to a} g(x)$$

Squeeze Theorem

If

$$f(x) \leq g(x) \leq h(x)$$

and if

$$\lim_{x \to a} f(x) = \lim_{x \to a} h(x) = L$$

Then

$$\lim_{x \to a} g(x) = L$$

Example Problem Find the limit of

$$\lim_{x \to 0} x^2 \sin(\frac{1}{x})$$

More Examples for Students to work on TK