Day 05 MA660

More with Dirac and Heaviside

Practice 1

A salt tank contains 100 liters of pure water at time $t = 0$,when salty water begins flowing into the tank at 2 liters per minute. The incoming liquid contains a concentration of 0.1 kg of salt per liter. The well-stirred liquid flows out of the tank at 2 liters per minute.

A. Model the situation with a first-order ODE, with x(t) as the mass of salt in the tank at time t. Solve this ODE using the Laplace transform.

B. Suppose that at time t = 20 minutes 5 kg of salt is dumped into the tank and dissolves instantaneously. Modify the ODE from part (a) appropriately (Hint: at t = 20 salt enters the tank at a high rate, for a very brief period.) Solve the resulting ODE using the Laplace transform. Plot the solution to make sure it's sensible.

Inverse laplace transform of functions with the Laplace Transform ofDirac Delta

What is the Laplace Transform of the $\delta(x)$?

Calculate it.

What about a situation where you deposit \$5000 in the bank and get 3% continuous annual interest, and then deposit \$2000 5 years later. What would be a differential equation for this situation. Solve this differential equation using the L.T.

- Be sure to see how to apply the second shifting lemma.
- Why does the solution make sense.
- What is the derivative of the solution?
- In what ways does it match our equation and what ways does it not?

Laplace transform of an integral

- What would be a reasonable guess for the laplace transform of integral? By looking at the table of transforms.
- Use FTC to find a formula for the laplace transform of an semi definite integral of a function.

Test on Monday

Topics:

- Definition of the Laplace Transform
- Using Laplace transforms to solve first and second order DE

- First shifting lemma
- Heaviside functions
- Dirac Delta functions
- Second Shifting lemma
- Table of common Laplace Transforms will be provided.
- It will be a pencil and paper test.
- Practice your partial fractions