

## Day 3 Notes MA660

### More Shifting and Heaviside Function

#### Practice 1

Solve the following problem using L.T.

$$u'' + 2u' + 2u = t \text{ when } u(0) = 1 \text{ and } u'(0) = -1$$

$$\text{Final Answer: } u(t) = 1/2(t + 3e^{-t} \cos(t) - 1)$$

#### Introduction to the Heaviside Function

Let's look at a problem with a discontinuity.

Students solve:

Find the Laplace Transform of:

$$f(t) = \begin{cases} t & 0 < t < 5 \\ t + 10 & t \geq 5 \end{cases}$$

$$\text{The answer to this } F(s) = \frac{1}{s^2} + \frac{10e^{-5s}}{s}$$

Define the Heaviside function

$$H(t) = \begin{cases} 0 & t < 0 \\ 1 & t \geq 0 \end{cases}$$

Then find the Laplace Transform for  $H(t)$ .

Then find the Laplace Transform for  $H(t - c)$ .

#### Second shifting Lemma

$$\mathcal{L}(H(t - c)f(t - c)) = e^{-cs} \cdot F(s)$$

#### Intuitions of the Dirac Delta Function