Università degli Studi di Milano

Data Science For Economics

DEM: Problem Set 1

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Exercise 1

An economy is composed of identical individuals. Each individual lives for 2 periods (you may imagine them as adulthood and old age). Individuals may work during the first period of their life for a proportion L of the day, for an income equal to wL. In the second period they retire and consume their remaining lifetime savings. Their lifetime utility is given by:

$$U = \log(C_1) + \log(C_2) + \log(1 - L), \tag{1}$$

where C_i is consumption in period i.

(a)

If the rate of interest on savings is R, write down the individual's budget constraints for both periods, and then combine them in a lifetime (inter-temporal) budget constraint.

Solution

The budget constraints for the two periods are

$$\begin{aligned} C_1 + S_1 &= wL \\ C_2 + S_2 &= (1 + R)S_1, \end{aligned}$$

where $S_2 = 0$ obviously. Therefore, solving for S_1, the inter-temporal budget constraint (IBC) is given by

$$C_1 + \frac{C_2}{(1 + R)} = wL. \tag{2}$$

(b)

Solve for optimal consumption each period and the optimal work effort. Comment on what you find.

Solution

By plugging (2) in (1) we obtain

$$U = \log\left(wL - \frac{C_2}{1 + R}\right) + \log(C_2) + \log(1 - L). \tag{3}$$

We now derive FOCs by computing the first order partial derivatives and setting them equal to zero.

$$\frac{\partial U}{\partial C_2} = -\frac{1+R}{(1+R)wL - C_2}\left(\frac{1}{1+R}\right) + \frac{1}{C_2} = 0$$

when

$$(1+R)wL - 2C_2 = 0,$$

giving

$$C_2 = \frac{(1+R)wL}{2}. \tag{4}$$

On the other hand

$$\frac{\partial U}{\partial L} = \frac{(1+R)w}{(1+R)wL - C_2} - \frac{1}{1-L} = 0$$

when

$$(1+R)w - 2(1+R)wL + C_2 = 0,$$

giving

$$L = \frac{1}{2} + \frac{C_2}{2w(1+R)} \tag{5}$$

By substituting (5) into (4) we get

$$\begin{aligned} C_2^* =& \frac{w}{3}(1+R), \\ L^* =& \frac{2}{3} \end{aligned} \tag{6}$$

and

$$C_1^* = w\left(L^* - \frac{1}{3}\right) = \frac{w}{3}.$$

From the obtained result we can see that the individual is willing to work a constant fraction of his day, independently of other factors. We could interpret this fact as a high value given to leisure by the individual, even though he works the majority of his day: in fact, if the wage increased, he would not work more, but he would consume more in both periods. Let w be fixed. We can also notice that there is a direct relation between consumption in the second period and the interest rate R. In fact, if R increases consumption in period one gets more expensive and, since L is constant, the individual will consume less in 1 to save more and be able to consume more in period 2. Last but not least, notice that this attitude strongly depends on the utility the agent has.

(c)

The government introduces a fixed pension paid to individuals in the second period of their lives, funded by a lump sum tax paid by those who work. Re-write the inter-temporal budget constraint. What will be the impact of the pension on consumption and labour supply decisions of the young? What about the old when the pension is introduced? Give intuition for your answers.

Solution

Let T indicate both the lump sum tax and the fixed pension. The IBC is given by

$$C_2 = (wL - T - C_1)(1 + R) + T$$

We can substitute it into the utility function and derive FOCs, bearing in mind that we want to study the impact on the young individual's behaviour.

$$\frac{\partial U}{\partial C_1} = \frac{1}{C_1} - \frac{1 + R}{(wL - T - C_1)(1 + R) + T} = 0$$

when

$$(wL - T)(1 + R) + T - 2C_1(1 + R) = 0,$$

that gives

$$C_1 = \frac{1}{2} - \left(wL - \frac{RT}{1 + R}\right). \tag{7}$$

As regards the labour supply,

$$\frac{\partial U}{\partial L} = \frac{w(1 + R)}{(wL - T - C_1)(1 + R) + T} - \frac{1}{1 - L} = 0$$

when

$$(w + T + C_1)(1 + R) - T - 2wL(1 + R) = 0,$$

thus

$$L = \frac{1}{2w}\left(C_1 + \frac{RT}{(1 + R)}\right). \tag{8}$$

By substitution we can solve the system and obtain

$$\begin{aligned} C_1^* &= \frac{w}{3} - \frac{RT}{3(1 + R)}, \\ L^* &= \frac{2}{3} + \frac{RT}{3w(1 + R)}. \end{aligned} \tag{9}$$

Regarding the young behaviour, it is quite obvious from (9) that the pension funded by a lump sum tax makes C_1^* decrease and labour supply increase with respect to the previous point. In other words, the young individual works more and consume less in presence of a tax. The rationale behind this might resides in the fact that T is lump sum. In fact, the more he works, the higher his net income is. Then, for consumption smoothing, C_2^* decreases in T as well (see (10) below). Furthermore, we can note that if the wage increases the additional fraction of labour supply decreases. Recall that before the young was working a constant portion of the day, no matter what; now as $w \to \infty$ (which is unrealistic, of course), the agent does not care about the tax anymore.

We can plug both C_1^* and L^*into the IBC to obtain

$$C_2^* = \frac{w}{3}(1 + R) - \frac{RT}{3}. \tag{10}$$

In period 2 the individual decreases consumption with respect to the previous point due to consumption smoothing, by an amount proportional to T. Note that if the pension is introduced when an individual is already in period 2, his consumption is higher, as one can see in the IBC.

(d)

The pension is now funded by an income tax, i.e., a tax equal to τwL, where τ is the tax rate. Will the behaviour of the young change in the case where $R = 0$? Interpret this result.

Solution

If $R = 0$ the previous solutions in (9) become

$$\begin{aligned} C_1^* &= \frac{w}{3}, \\ L^* &= \frac{2}{3}. \end{aligned} \tag{11}$$

Let's see the if the behaviour of the young changes when the tax is no longer lump sum. The IBC is

$$C_2 = wL - (1+R)C_1$$

By deriving the FOCs we obtain

$$\begin{aligned} \frac{\partial U}{\partial C_1} &= 0 \quad \Rightarrow \quad C_1 = \frac{wL}{2(1+R)} \\ \frac{\partial U}{\partial L} &= 0 \quad \Rightarrow \quad L = \frac{1}{2}\left(\frac{(1+R)C_1}{2w}\right). \end{aligned}$$

Imposing $R = 0$, it follows that

$$\begin{aligned} C_1^* &= \frac{w}{3} \\ L^* &= \frac{2}{3}, \end{aligned} \tag{12}$$

which are the solutions obtained in (b). This result makes sense since, if $R = 0$, the present value of the pension (in period 1) is exactly the value of the tax, whereas in the previous point the present value of the pension is lower.

Exercise 2

A business cycle is made of an expansion (boom) and a contraction (recession). During the expansion all good things (GDP, employment, productivity, and so on) tend to go up, or grow faster than "normal", and bad things (e.g. unemployment) tend to fall. During the contraction good things go down and bad things go up.

(a)

Replicate Table 1 and 2 for the US economy from - ideally - 1950 Q1 to the newest data you can find (either 2020 or 2023). You can use real GDP instead of GNP if you want.

Solution

The recreated tables are reported below. Please notice that the data used to solve this exercise starts from 1964.

	sd%	t-4	t-3	t-2	t-1	t	t+1	t+2	t+3	t+4
gnp	1.56	0.08	0.29	0.53	0.73	1.00	0.73	0.53	0.29	0.08
cnd	1.96	0.12	0.27	0.45	0.56	0.71	0.56	0.43	0.24	0.05
ncd	3.97	-0.33	-0.21	-0.02	0.15	0.49	0.57	0.55	0.49	0.41
h	1.71	0.17	0.36	0.54	0.71	0.87	0.62	0.45	0.26	0.10
aveh	0.54	-0.28	-0.18	-0.02	0.21	0.43	0.43	0.44	0.42	0.34
l	1.54	0.22	0.36	0.49	0.67	0.81	0.46	0.30	0.13	-0.01
gnp/l	0.97	-0.22	-0.11	0.07	0.13	0.34	0.45	0.37	0.27	0.15
avew	0.66	0.14	0.16	0.15	0.10	-0.06	-0.01	-0.09	-0.13	-0.16

Table 1: Cyclical Behaviour of the US Economy (1964Q1-2023Q1)

	sd	rsd	autocorr	contcorr
y	1.57	1.00	0.74	1.00
c	1.99	1.27	0.80	0.71
i	6.31	4.02	0.81	0.80
n	1.72	1.10	0.78	0.87
y/n	0.84	0.54	0.74	0.07
w	0.67	0.42	0.79	-0.06
r	1.19	0.76	0.71	-0.18
A	0.70	0.45	0.11	-0.27

Table 2: Reconstruction of Business Cycle Statistics for the US Economy from "Resuscitating Real Business Cycles".

(b)-(c)

All the answers to the subsequent points (as well as the recreation of the tables) can be found at the following link. Questions are incorporated in the code as markdown cells. Finally, data sources are listed in bibliography below.

Bibliography

[1] Federal Reserve Economic Data website.

[2] U.S. Bureau of Economic Analysis website.