Parity Asymmetry in the CMB

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Outline

1. Overview of the Problem

2. Initial Calculations

Intitial Calculations

Start by expanding the two correlation functions in terms of isotropic basis functions:

$$\begin{aligned}
\zeta_{x_0}(\vec{r}_1, \vec{r}_2, \vec{r}_3) &= \sum_{\Lambda} \tilde{\zeta}_{\Lambda}(r_1, r_2, r_3)\mathcal{P}_{\Lambda}(\hat{r}_1, \hat{r}_2, \hat{r}_3) \\
\zeta_{E}(\vec{x}_0, \vec{x}_1, \vec{x}_2, \vec{x}_3) &= \sum_{\Lambda'} \tilde{\zeta}_{\Lambda'}(x_0, x_1, x_2, x_3)\mathcal{P}_{\Lambda'}(\hat{x}_0, \hat{x}_1, \hat{x}_2, \hat{x}_3)
\end{aligned}$$

Where Λ is the set $\{l_1, l_2, l_3\}$ and Λ' is the set $\{l'_0, l'_1, l'_{01}, l'_2, l'_3\}$

Intitial Calculations

We can then set the two functions equal since they are correlation functions describing the same points

$$
\begin{aligned}
\zeta_{x_0}(\vec{r}_1, \vec{r}_2, \vec{r}_3) &= \zeta_E(\vec{x}_0, \vec{x}_1, \vec{x}_2, \vec{x}_3) \\
&\Downarrow \\
\sum_{\Lambda} \zeta_{\Lambda}(r_1, r_2, r_3)\mathcal{P}_{\Lambda}(\hat{r}_1, \hat{r}_2, \hat{r}_3) &= \sum_{\Lambda'} \zeta_{\Lambda'}(x_0, x_1, x_2, x_3)\mathcal{P}_{\Lambda'}(\hat{x}_0, \hat{x}_1, \hat{x}_2, \hat{x}_3)
\end{aligned}
$$

Intitial Calculations

We now multiply both sides by $\mathcal{P}^*_{\Lambda''}(\hat{x}_0, \hat{x}_1, \hat{x}_2, \hat{x}_3)$ and then integrate over all orientations $d\hat{x}_0 d\hat{x}_1 d\hat{x}_2 d\hat{x}_3 = d\hat{x}$

$$\int d\hat{x} \sum_{\Lambda} \zeta_{\Lambda}(r_1, r_2, r_3)\mathcal{P}_{\Lambda}(\hat{r}_1, \hat{r}_2, \hat{r}_3) P^*_{\Lambda''}(\hat{x}_0, \hat{x}_1, \hat{x}_2, \hat{x}_3) =$$

$$\int d\hat{x} \sum_{\Lambda'} \zeta_{\Lambda'}(x_0, x_1, x_2, x_3)\mathcal{P}_{\Lambda'}(\hat{x}_0, \hat{x}_1, \hat{x}_2, \hat{x}_3) P^*_{\Lambda''}(\hat{x}_0, \hat{x}_1, \hat{x}_2, \hat{x}_3)$$

Intitial Calculations

Using the orthogonality of isotropic basis functions which states

$$\int d\hat{R}\ \mathcal{P}_{\Lambda}(\hat{R})\mathcal{P}^*_{\Lambda'}(\hat{R}) = \delta_{\Lambda_1\Lambda'_1}\delta_{\Lambda_2\Lambda'_2}...$$

we can simplify the right hand side of the previous equation:

$$\int d\hat{x} \sum_{\Lambda'} \zeta_{\Lambda'}(x_0, x_1, x_2, x_3)\mathcal{P}_{\Lambda'}(\hat{x}_0, \hat{x}_1, \hat{x}_2, \hat{x}_3)P_{\Lambda''}(\hat{x}_0, \hat{x}_1, \hat{x}_2, \hat{x}_3)$$
$$= \zeta_{\Lambda''}(x_0, x_1, x_2, x_3)$$

Intitial Calculations

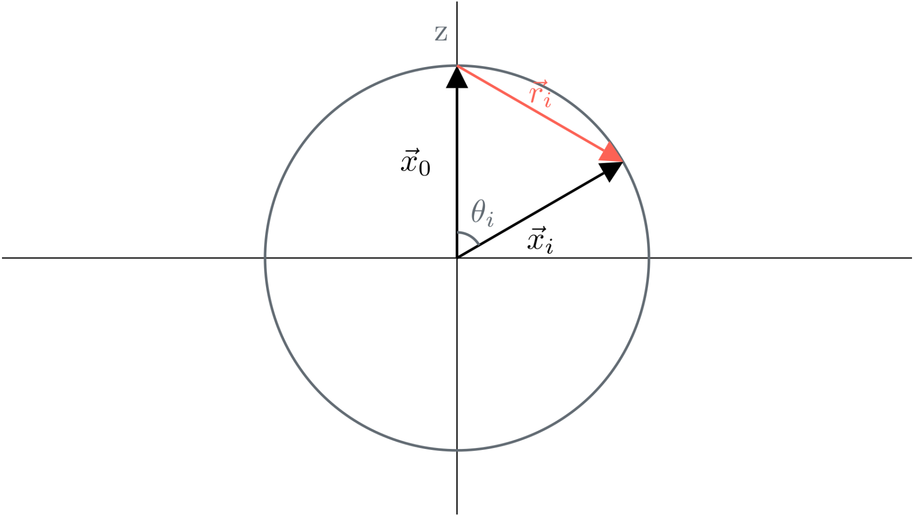
Plugging this back into the previous equation we get the key relation:

$$\boxed{\zeta_{\Lambda''}(x_0, x_1, x_2, x_3) = \int d\hat{x} \sum_{\Lambda} \zeta_{\Lambda}(r_1, r_2, r_3)\mathcal{P}_{\Lambda}(\hat{r}_1, \hat{r}_2, \hat{r}_3)P^{*}_{\Lambda''}(\hat{x}_0, \hat{x}_1, \hat{x}_2, \hat{x}_3)} \quad (1)$$

Solving the integral on the right will give an expression for the coefficients of the correlation function centered on earth in terms of the correlation function centered at x_0

Solving for $\vec{r}$ in terms of $\vec{x}$

Consider a great circle going through the poles at an angle ϕ_i to the x-axis where ϕ_i is the ϕ angle of vector $\vec{x}_i$ for $i = 1, 2, 3$



Solving for $\vec{r}$ in terms of $\vec{x}$

Using the law of Cosines and defining the distance to the cmb to be d^* we get:

$$|\vec{r}_i| = \sqrt{d^{*2} + d^{*2} - 2d^* d^* cos(\theta_i)}$$

$$\boxed{r_i = \sqrt{2} d^* \sqrt{1 - cos(\theta_i)}}$$