Parity Asymmetry in the CMB

UF REU under Dr. Zachary Slepian

Michael Bartlett

University of Oklahoma

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Outline

1. Overview of the Problem
2. Initial Calculations
3. Solving for Coefficients

Intitial Calculations

Start by expanding the two correlation functions in terms of isotropic basis functions:

$$\begin{aligned} \zeta_{x_0}(\vec{r}_1, \vec{r}_2, \vec{r}_3) &= \sum_{\Lambda} \tilde{\zeta}_{\Lambda}(r_1, r_2, r_3) \mathcal{P}_{\Lambda}(\hat{r}_1, \hat{r}_2, \hat{r}_3) \\ \zeta_E(\vec{x}_0, \vec{x}_1, \vec{x}_2, \vec{x}_3) &= \sum_{\Lambda'} \tilde{\zeta}_{\Lambda'}(x_0, x_1, x_2, x_3) \mathcal{P}_{\Lambda'}(\hat{x}_0, \hat{x}_1, \hat{x}_2, \hat{x}_3) \end{aligned}$$

Where Λ is the set $\{l_1, l_2, l_3\}$ and Λ' is the set $\{l'_0, l'_1, l'_{01}, l'_2, l'_3\}$

Intitial Calculations

We can then set the two functions equal since they are correlation functions describing the same points

$$
\begin{aligned}
\zeta_{x_0}(\vec{r}_1, \vec{r}_2, \vec{r}_3) &= \zeta_E(\vec{x}_0, \vec{x}_1, \vec{x}_2, \vec{x}_3) \\
&\Downarrow \\
\sum_{\Lambda} \zeta_\Lambda(r_1, r_2, r_3)\mathcal{P}_\Lambda(\hat{r}_1, \hat{r}_2, \hat{r}_3) &= \sum_{\Lambda'} \zeta_{\Lambda'}(x_0, x_1, x_2, x_3)\mathcal{P}_{\Lambda'}(\hat{x}_0, \hat{x}_1, \hat{x}_2, \hat{x}_3)
\end{aligned}
$$

Intitial Calculations

We now multiply both sides by $\mathcal{P}^*_{\Lambda''}(\hat{x}_0, \hat{x}_1, \hat{x}_2, \hat{x}_3)$ and then integrate over all orientations $d\hat{x}_0 d\hat{x}_1 d\hat{x}_2 d\hat{x}_3 = d\hat{x}$

$$\int d\hat{x} \sum_{\Lambda} \zeta_\Lambda(r_1, r_2, r_3)\mathcal{P}_\Lambda(\hat{r}_1, \hat{r}_2, \hat{r}_3)P^*_{\Lambda''}(\hat{x}_0, \hat{x}_1, \hat{x}_2, \hat{x}_3) =$$

$$\int d\hat{x} \sum_{\Lambda'} \zeta_{\Lambda'}(x_0, x_1, x_2, x_3)\mathcal{P}_{\Lambda'}(\hat{x}_0, \hat{x}_1, \hat{x}_2, \hat{x}_3)P^*_{\Lambda''}(\hat{x}_0, \hat{x}_1, \hat{x}_2, \hat{x}_3)$$

Intitial Calculations

Using the orthogonality of isotropic basis functions which states

$$\int d\hat{R}\ \mathcal{P}_{\Lambda}(\hat{R})\mathcal{P}^{*}_{\Lambda'}(\hat{R}) = \delta_{\Lambda_1\Lambda_1'}\delta_{\Lambda_2\Lambda_2'}\cdots$$

we can simplify the right hand side of the previous equation:

$$\int d\hat{x} \sum_{\Lambda'} \zeta_{\Lambda'}(x_0, x_1, x_2, x_3)\mathcal{P}_{\Lambda'}(\hat{x}_0, \hat{x}_1, \hat{x}_2, \hat{x}_3)P_{\Lambda''}(\hat{x}_0, \hat{x}_1, \hat{x}_2, \hat{x}_3)$$
$$= \zeta_{\Lambda''}(x_0, x_1, x_2, x_3)$$

Intitial Calculations

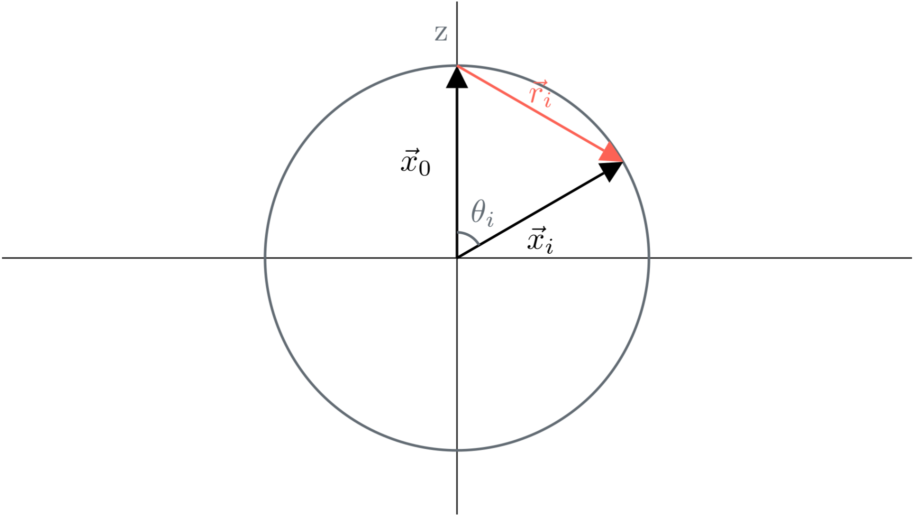
Plugging this back into the previous equation we get the key relation:

$$\boxed{\zeta_{\Lambda''}(x_0, x_1, x_2, x_3) = \int d\hat{x} \sum_{\Lambda} \zeta_{\Lambda}(r_1, r_2, r_3)\mathcal{P}_{\Lambda}(\hat{r}_1, \hat{r}_2, \hat{r}_3)P^*_{\Lambda''}(\hat{x}_0, \hat{x}_1, \hat{x}_2, \hat{x}_3)} \tag{1}$$

Solving the integral on the right will give an expression for the coefficients of the correlation function centered on earth in terms of the correlation function centered at x_0

Solving for $\vec{r}$ in terms of $\vec{x}$

Consider a great circle going through the poles at an angle ϕ_i to the x-axis where ϕ_i is the ϕ angle of vector $\vec{x}_i$ for $i = 1, 2, 3$

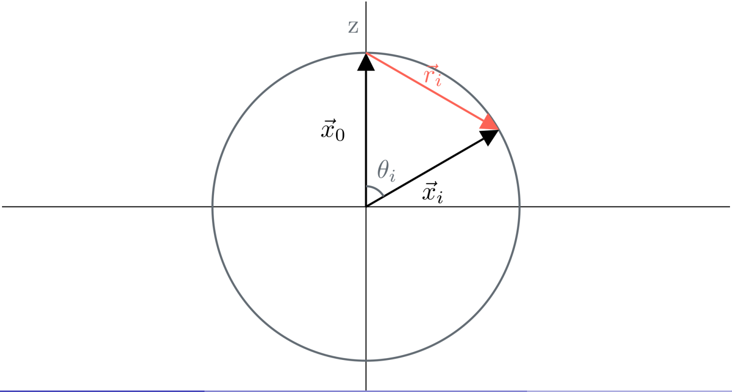


Solving for $\vec{r}$ in terms of $\vec{x}$

Using the law of Cosines and defining the distance to the cmb to be d^* we get:

$$|\vec{r_i}| = \sqrt{d^{*2} + d^{*2} - 2d^*d^*cos(\theta_i)}$$

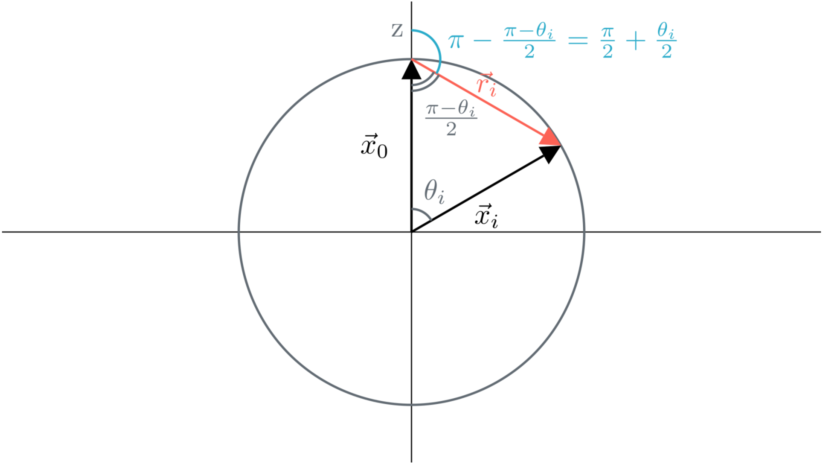
$$\boxed{r_i = \sqrt{2}d^*\sqrt{1 - cos(\theta_i)}}$$



Solving for $\vec{r}$ in terms of $\vec{x}$

Similarly, we can solve for the angular dependence of $\vec{r}_i$

$$\boxed{\theta_{ri} = \frac{\pi}{2} + \frac{\theta_i}{2};\ \phi_{ri} = \phi_i}$$



Solving for the coefficients

Plugging those back into equation 1 we get:

$$\zeta_{\Lambda''}(x_0, x_1, x_2, x_3) =$$

$$\int d\hat{x} \sum_{\Lambda} \zeta_{\Lambda}(\sqrt{2}d^* \sqrt{1 - cos(\theta_1)}, \sqrt{2}d^* \sqrt{1 - cos(\theta_2)}, \sqrt{2}d^* \sqrt{1 - cos(\theta_3)})$$

$$\times \mathcal{P}_{\Lambda}(\frac{\pi}{2} + \frac{\theta_1}{2}, \phi_1; \frac{\pi}{2} + \frac{\theta_2}{2}, \phi_2; \frac{\pi}{2} + \frac{\theta_3}{2}, \phi_3)$$

$$\times \mathcal{P}^*_{\Lambda''}(\theta_0, \phi_0; \theta_1, \phi_1; \theta_2, \phi_2; \theta_3, \phi_3)$$

Different approaches

- Directly solving the integral
- Rotating the spherical harmonics
- Using an addition formula
    - Associated Legendre Polynomials
    - Jacobi Polynomials

Directly solving the integral

We can expand the integral using the definitions of the isotropic basis functions:

$$\mathcal{P}_{l_1 l_2 l_3}(\hat{r}_1, \hat{r}_2, \hat{r}_3) = (-1)^{l_1+l_2+l_3} \sum_{m_1 m_1 m_3} \begin{pmatrix} l_1 & l_2 & l_3 \\ m_1 & m_2 & m_3 \end{pmatrix}$$
$$\times\, Y_{l_1 m_1}(\hat{r}_1) Y_{l_2 m_2}(\hat{r}_2) Y_{l_3 m_3}(\hat{r}_3)$$

$$\mathcal{P}_{l_1 l_2 (l_{12}) l_3 l_4}(\hat{r}_1, \hat{r}_2, \hat{r}_3, \hat{r}_4) = (-1)^{l_1+l_2+l_3+l_4}$$
$$\times \sum_{m_{12}} (-1)^{l_{12}-m_{12}} \sum_{m_1 m_2 m_3 m_4} \begin{pmatrix} l_1 & l_2 & l_{12} \\ m_1 & m_2 & -m_{12} \end{pmatrix} \begin{pmatrix} l_{12} & l_3 & l_4 \\ m_{12} & m_3 & -m_4 \end{pmatrix}$$
$$\times\, Y_{l_1 m_1}(\hat{r}_1) Y_{l_2 m_2}(\hat{r}_2) Y_{l_3 m_3}(\hat{r}_3) Y_{l_4 m_4}(\hat{r}_4)$$