Probability and Statistics

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Sampling Distributions of Estimators

The Sampling Distribution of a Statistic

A statistic is a function of some observable random variables, and hence is itself a random variable with a distribution. Its distribution is called *sampling distribution* and gives us what values the statistic is likely to assume and the probabilities associated with these values *prior to* observing our data.

Statistics and Estimators

Sampling Distribution A statistic $T = f(X_1, ..., X_{n,}\theta)$ is also a random variable and its distribution given θ is called *sampling distribution.* The notation $\mathbb{E}_\theta[T]$ denotes the mean of T calculated from its sampling distribution. "Sampling" comes from the fact that the distribution of T is derived from a sample.

An example $X_1, ..., X_n$ sample of normal distributions (μ, σ^2). $\overline{X_n}$ sample mean is MLE for μ. $\overline{X_n}$ sampling distribution is normal distribution $(\mu, \frac{\sigma^2}{n})$. What about distribution of other statistics like sample variance or functions of sample mean and variance?

Purpose of Sampling Distribution

The Chi-Square Distributions

Definition of the Distributions

Properties of the Distributions

The t Distributions

Overview Let $X_1, \ldots, X_n$ be a random sample from the normal distribution with mean μ and variance σ^2. Let denote $\overline{X_n} = \frac{1}{n}\sum_{i=1}^n X_i$ the sample mean and $\sigma' = \sqrt{\frac{1}{n-1}\sum_{i=1}^n (X_i - \overline{X_n})^2}$ the sample standard deviation. The distribution of $Z = \sqrt{n}\frac{(\overline{X_n} - \mu)}{\sigma'}$ is the t distribution with $n-1$ degrees of freedom. It is useful to make inference about the value of μ when both μ and σ^2 are unknown.

Definition of the Distributions

Relation to Random Samples from a Normal Distribution

Relation to the Cauchy Distribution and to the Standard Normal Distribution