

Probability and Statistics

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Sampling Distributions of Estimators

The Sampling Distribution of a Statistic

A statistic is a function of some observable random variables, and hence is itself a random variable with a distribution. Its distribution is called *sampling distribution* and gives us what values the statistic is likely to assume and the probabilities associated with these values *prior to* observing our data. We use the sampling distribution of an estimator $\hat{\theta}$ to calculate the probability that $\hat{\theta}$ will be close to θ .

Statistics and Estimators

Sampling Distribution A statistic $T = f(X_1, \dots, X_n, \theta)$ is also a random variable and its distribution given θ is called *sampling distribution*. The notation $\mathbb{E}_\theta[T]$ denotes the mean of T calculated from its sampling distribution. “Sampling” comes from the fact that the distribution of T is derived from a sample.

An example X_1, \dots, X_n sample of normal distributions (μ, σ^2) . \overline{X}_n sample mean is MLE for μ . \overline{X}_n sampling distribution is normal distribution $(\mu, \frac{\sigma^2}{n})$. What about distribution of other statistics like sample variance or functions of the sample mean and variance?

The Chi-Square Distributions

χ^2 distributions form a subcollection of Gamma distributions. They arise as sampling distributions of variance estimators based of random sample of normal distributions.

Definition of the Distributions

Properties of the Distributions

The t Distributions

Overview Let X_1, \dots, X_n be a random sample from the normal distribution with mean μ and variance σ^2 . Let denote $\overline{X}_n = \frac{1}{n} \sum_{i=1}^n X_i$ the sample mean and $\sigma' = \sqrt{\frac{1}{n-1} \sum_{i=1}^n (X_i - \overline{X}_n)^2}$ the sample standard deviation. The distribution of $Z = \sqrt{n} \frac{(\overline{X}_n - \mu)}{\sigma'}$ is the t distribution with $n - 1$ degrees of freedom. It is useful to make inference about the value of μ when both μ and σ^2 are unknown.

Definition of the Distributions

Relation to Random Samples from a Normal Distribution

Relation to the Cauchy Distribution and to the Standard Normal Distribution