The Delta-Method applied to Sobol' indices

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Abstract

We explore the use of the Delta-method in order to estimate the Sobol' indices.

Contents

1 Convergence in distribution

Definition 1 *(*Convergence in distribution*) Assume that $X_1, X_2, ...$ is a sequence of real-valued random variables with cumulative distribution functions $\{F_n\}_{n\geq 0}$. Assume that X is a real-valued random variable with cumulative distribution function $\{F\}_{n\geq 0}$. The sequence X_n converges in distribution to X if:*

$$\lim_{n\to\infty} F_n(X_n) = F(x).$$

for any $x \in \mathbb{R}$ at which F is continuous. In this case, we write:

$$X_n \xrightarrow{D} X.$$

The following theorem gives an example of such convergence.

Theorem 1 *(*Maximum of uniform random numbers*) Assume that $X_1, X_2, ...$ are independent random numbers such that $X_n \sim U(0,1)$. Let Y_n be the maximum:*

$$Y_n = \max_{1\leq i\leq n} X_i.$$

Therefore the sequence Y_n converges in distribution to an exponential random variable, i.e.:

$$n(1-Y_n) \xrightarrow{D} \mathcal{E}(1).$$

2 Delta method

Theorem 2 *(*Delta-method*) Assume that $X_1, X_2, ...$ is a sequence of real-valued random variables so that*

$$\sqrt{n}(X_n - \theta) \xrightarrow{D} \mathcal{N}(0, \sigma^2).$$

Assume that g is a real function. Let $\theta \in \mathbb{R}$ and suppose that $g'(\theta)$ exists and that $g'(\theta) \neq 0$. Therefore,

$$\sqrt{n}(g(X_n) - g(\theta)) \xrightarrow{D} \mathcal{N}(0, \sigma^2 g'(\theta)^2).$$