Here is a comprehensive guide to the equations, properties, and concepts required for the specific GRE Quantitative Reasoning sections you listed.

Section 1: Number Properties

1. Divisibility, Factors and Multiples, and Prime Numbers

- **Definitions:**
  - **Factor:** An integer that divides another integer with no remainder.
  - **Multiple:** The product of an integer and another integer.
  - **Prime Number:** An integer greater than 1 that has exactly two factors: 1 and itself. (Note: **2** is the only even prime number; **1** is not prime).
- **Divisibility Rules:**
  - **2:** Last digit is even.
  - **3:** Sum of digits is divisible by 3.
  - **4:** Last two digits form a number divisible by 4.
  - **5:** Last digit is 0 or 5.
  - **6:** Divisible by both 2 and 3.
  - **9:** Sum of digits is divisible by 9.
  - **10:** Ends in 0.
  - **12:** Divisible by both 3 and 4.
- **Least Common Multiple (LCM) & Greatest Common Factor (GCF):**
  - Property: $LCM(a, b) \times GCF(a, b) = a \times b$.
- **Prime Factorization:** Breaking a number down into the product of its prime factors (e.g., $12 = 2^2 \times 3$).
- **Remainders:** Dividend = (Divisor $\times$ Quotient) + Remainder. (Where $0 \leq \text{Remainder} < \text{Divisor}$).

2. Arithmetic Operations

- **Order of Operations (PEMDAS):**
  1. **P**arentheses
  2. **E**xponents
  3. **M**ultiplication / **D**ivision (Left to Right)
  4. **A**ddition / **S**ubtraction (Left to Right)
- **Even/Odd Operations:**

- Even $\pm$ Even = **Even**
- Odd $\pm$ Odd = **Even**
- Even $\pm$ Odd = **Odd**
- Even $\times$ Even = **Even**
- Odd $\times$ Odd = **Odd**
- Even $\times$ Odd = **Even**

- **Positives/Negatives:**
  - Pos $\times$ Pos = Pos
  - Neg $\times$ Neg = Pos
  - Pos $\times$ Neg = Neg

3. Exponents and Radicals

- **Exponent Rules:**
  - $x^a \cdot x^b = x^{a+b}$
  - $\frac{x^a}{x^b} = x^{a-b}$
  - $(x^a)^b = x^{a \cdot b}$
  - $(xy)^a = x^a y^a$
  - $x^0 = 1$ (for $x \neq 0$)
  - $x^{-a} = \frac{1}{x^a}$
- **Radical Rules:**
  - $\sqrt{xy} = \sqrt{x}\sqrt{y}$
  - $\sqrt{\frac{x}{y}} = \frac{\sqrt{x}}{\sqrt{y}}$
  - $x^{\frac{1}{n}} = \sqrt[n]{x}$
  - $x^{\frac{a}{b}} = \sqrt[b]{x^a}$

4. Percents, Ratios, and Proportions

- **Percents:**
  - $\text{Percent} = \frac{\text{Part}}{\text{Whole}} \times 100\%$
  - $\text{Percent Change} = \frac{\text{New} - \text{Original}}{\text{Original}} \times 100\%$
  - *Increase by x%:* Multiply by $(1 + \frac{x}{100})$.
  - *Decrease by x%:* Multiply by $(1 - \frac{x}{100})$.
- **Compound Interest:** $A = P(1 + \frac{r}{n})^{nt}$ (where P is principal, r is rate, n is compounding frequency, t is time).
- **Ratios:**
  - If ratio is $a : b$, the total parts are $a + b$.
  - Actual amounts are usually ax and bx.

- **Proportions:**
  - $\frac{a}{b} = \frac{c}{d} \Rightarrow ad = bc$ (Cross-multiplication).

5. Absolute Value

- **Definition:** $|x|$ is the distance of x from 0 on the number line. Distance is always non-negative.
- **Solving:**
  - If $|x| = k$, then $x = k$ or $x = -k$.
  - If $|x| < k$, then $-k < x < k$.
  - If $|x| > k$, then $x > k$ or $x < -k$.

6. Fractions and Decimals

- **Fractions:**
  - Adding/Subtracting: Requires a common denominator.
  - Multiplying: Multiply straight across (top $\times$ top, bottom $\times$ bottom).
  - Dividing: "Keep, Change, Flip" (Multiply by the reciprocal).
- **Decimals:**
  - **Terminating Decimals:** Only occur if the denominator's prime factors are only 2s and/or 5s (when simplified).
  - **Scientific Notation:** $a \times 10^n$ where $1 \leq a < 10$.

Section 2: Algebra

7. Functions and Symbolism

- **Function Notation:** $f(x)$ represents the "output" for a specific input x.
- **Domain:** All possible valid inputs for x (look for division by zero or negative square roots).
- **Symbolism:** The GRE often invents operators (e.g., $x \diamondsuit y = 2x + y$). Simply substitute the numbers provided into the definition given.

8. Linear Equations

- **Slope-Intercept Form:** $y = mx + b$
  - $m =$ slope
  - $b =$ y-intercept
- **Slope Formula:** $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\text{rise}}{\text{run}}$
- **Lines:**

- **Parallel Lines:** Have equal slopes ($m_1 = m_2$).
- **Perpendicular Lines:** Slopes are negative reciprocals ($m_1 = -\frac{1}{m_2}$ or $m_1 \cdot m_2 = -1$).

9. Quadratic Equations

- **Standard Form:** $ax^2 + bx + c = 0$
- **Common Identities (Memorize these):**
  - $(x + y)^2 = x^2 + 2xy + y^2$
  - $(x - y)^2 = x^2 - 2xy + y^2$
  - $(x + y)(x - y) = x^2 - y^2$ (Difference of Squares)
- **Quadratic Formula:** $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
- **Discriminant:** $b^2 - 4ac$
  - If > 0, two real solutions.
  - If $= 0$, one real solution.
  - If < 0, no real solutions.

10. Systems of Linear Equations

- **Substitution Method:** Solve one equation for one variable, plug it into the other.
- **Elimination (Combination) Method:** Add or subtract equations to eliminate a variable.
- **Solutions:**
  - **One Solution:** Lines intersect at one point.
  - **No Solution:** Lines are parallel (same slope, different y-intercepts).
  - **Infinite Solutions:** Lines are identical.

11. Inequalities

- **Golden Rule:** When multiplying or dividing an inequality by a **negative** number, you **must flip** the inequality sign.
  - Example: $-2x > 4 \rightarrow x < -2$.
- **Compound Inequalities:**
  - $a < x < b$ means x is between a and b.

Section 3: Geometry

12. Coordinate Geometry

- **Distance Formula:** $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

- **Midpoint Formula:** $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$
- **Quadrants:**
  - I (+, +), II (-, +), III (-, -), IV (+, -).

13. Lines and Angles

- **Straight Line:** Angles sum to 180°.
- **Angles around a point:** Sum to 360°.
- **Vertical Angles:** Opposite angles formed by intersecting lines are equal.
- **Parallel Lines cut by a Transversal:**
  - Alternate Interior Angles are equal.
  - Corresponding Angles are equal.
  - Same-side Interior Angles sum to 180°.

14. Triangles

- **General Properties:**
  - Sum of interior angles $= 180^\circ$.
  - **Area:** $\frac{1}{2} \times \text{base} \times \text{height}$.
  - **Triangle Inequality Theorem:** The sum of any two sides must be greater than the third side ($a + b > c$).
  - **Exterior Angle Theorem:** An exterior angle is equal to the sum of the two opposite interior angles.
- **Right Triangles:**
  - **Pythagorean Theorem:** $a^2 + b^2 = c^2$ (where c is the hypotenuse).
  - **Common Triples:** 3-4-5, 5-12-13, 6-8-10.
- **Special Right Triangles:**
  - **45-45-90:** Sides are $x, x, x\sqrt{2}$. (Isosceles Right Triangle).
  - **30-60-90:** Sides are x (opposite 30), $x\sqrt{3}$ (opposite 60), $2x$ (hypotenuse).

15. Quadrilaterals and Other Polygons

- **Polygon Angle Formulas:**
  - **Sum of Interior Angles:** $(n-2) \times 180^\circ$ (where n is number of sides).
  - **Sum of Exterior Angles:** Always 360° regardless of n.
- **Specific Areas:**
  - **Rectangle:** $\text{length} \times \text{width}$
  - **Square:** side^2 or $\frac{\text{diagonal}^2}{2}$
  - **Parallelogram:** $\text{base} \times \text{vertical height}$

- **Trapezoid:** $\frac{\text{base}_1+\text{base}_2}{2} \times \text{height}$ (Average of bases $\times$ height).

16. Circles

- **Circumference:** $C = 2\pi r$ or $C = \pi d$
- **Area:** $A = \pi r^2$
- **Arcs and Sectors:**
  - **Arc Length:** $\frac{\text{central angle}}{360} \times 2\pi r$
  - **Sector Area:** $\frac{\text{central angle}}{360} \times \pi r^2$
- **Properties:**
  - A tangent line is perpendicular to the radius at the point of contact.
  - Inscribed angles holding the same arc are equal.
  - An angle inscribed in a semicircle is 90°.

17. Three-Dimensional Shapes

- **Rectangular Solid (Box):**
  - **Volume:** $l \times w \times h$
  - **Surface Area:** $2(lw + lh + wh)$
  - **Space Diagonal:** $\sqrt{l^2 + w^2 + h^2}$
- **Cube:**
  - **Volume:** s^3
  - **Surface Area:** $6s^2$
- **Cylinder:**
  - **Volume:** $\pi r^2 h$ (Area of base $\times$ height).
  - **Lateral Surface Area:** $2\pi r h$ (Circumference $\times$ height).
  - **Total Surface Area:** $2\pi r^2 + 2\pi r h$ (Top + Bottom + Lateral).

18. Multiple Figures

- **Strategy:** There are no specific formulas for "multiple figures," but rather a strategy:
  - Break complex shapes into standard shapes (rectangles and triangles).
  - **Subtraction Method:** Find the area of the large "bounding" shape and subtract the empty/unshaded regions to find the area of the shaded region.

Here is the additional section covering Probability and Counting Methods (Combinatorics). This section is critical for the Data Analysis portion of the GRE.

Section 4: Probability and Counting Methods

19. Fundamental Counting Principle

- **The Rule:** If one event can occur in m ways and a second independent event can occur in n ways, the two events together can occur in $m \times n$ ways.
  - *Example:* If a menu has 3 appetizers and 4 entrees, there are $3 \times 4 = 12$ distinct meal combinations.

20. Factorials and Arrangements

- **Factorial Notation (!):** The product of an integer and all positive integers below it.
  - $n! = n \times (n-1) \times (n-2) \times ... \times 1$
  - **Important Identity:** $0! = 1$
- **Basic Arrangements:** The number of ways to arrange n unique items in a row is $n!$.
- **Arrangements with Repetition:** If arranging letters in a word where some letters repeat (e.g., "APPLES"), divide by the factorial of the count of the repeating items.
  - Formula: $\frac{\text{Total Letters!}}{(\text{Repeated A})! \times (\text{Repeated B})! ...}$

21. Permutations (Order Matters)

- **Definition:** Use when selecting items where the **order** of selection makes a difference (e.g., assigning specific roles like President/VP, or ranking 1st, 2nd, 3rd).
- **Formula:** $P(n, k) = \frac{n!}{(n-k)!}$
  - n = Total number of items in the pool.
  - k = Number of items being selected.

22. Combinations (Order Does NOT Matter)

- **Definition:** Use when selecting a group or a team where the order is irrelevant (e.g., picking 3 people for a committee; picking Person A then Person B is the same as picking Person B then Person A).
- **Formula:** $C(n, k) = \binom{n}{k} = \frac{n!}{k!(n-k)!}$
  - *Note:* This is the Permutation formula divided by $k!$ to remove duplicate orderings.

23. Probability Rules

- **Basic Formula:**
  - $P(E) = \frac{\text{Number of Desired Outcomes}}{\text{Total Number of Possible Outcomes}}$
  - Probability is always between 0 and 1 inclusive ($0 \le P \le 1$).

- **Complement Rule (The "Not" Rule):**
  - $P(\text{Event happens}) + P(\text{Event does NOT happen}) = 1$
  - Useful shortcut: $P(\text{Event}) = 1 - P(\text{Not Event})$.

24. Compound Events (Multiple Events)

- **"OR" Events (Addition Rule):**
  - **Mutually Exclusive (Cannot happen together):**
    - $P(A \text{ or } B) = P(A) + P(B)$
  - **Not Mutually Exclusive (Can happen together):**
    - $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$
    - *Reason:* You subtract $P(A \text{ and } B)$ so you don't double-count the overlap.
- **"AND" Events (Multiplication Rule):**
  - **Independent Events (One result does not affect the other):**
    - $P(A \text{ and } B) = P(A) \times P(B)$
    - *Example:* Flipping a coin twice.
  - **Dependent Events (The first result changes the odds for the second):**
    - $P(A \text{ and } B) = P(A) \times P(B|A)$
    - *Example:* Drawing cards from a deck **without replacement**.

25. The "At Least One" Shortcut

- This is a very common GRE trick. Instead of calculating the probability of getting 1 success, 2 successes, 3 successes, etc., use the complement.
- **Formula:** $P(\text{At least one}) = 1 - P(\text{None})$