Problem 3

```
y <- c(13,52, 6,40,10, 7,66,10,10,14,16, 4, 65, 5,11,10,15, 5,76,56,88,24,51, 4, 40, 8,1
8, 5,16,50,40, 1,36, 5,10,91, 18, 1,18, 6, 1,23,15,18,12,12,17, 3)

n <- length(y)

#Distributions

w_Likelihood <- function(lambda){
  if (lambda != 0){
    return((y^lambda - 1)/lambda)
  }
  else if (lambda == 0){
    return(log(y))
  }
}

mu_condPosterior <- function(sigma2, lambda){
  rnorm(1, mean = mean(w_Likelihood(lambda)), sd = sqrt(sigma2/n))
}

sigma2_condPosterior <- function(mu, lambda){
  sigma2.hat <- sum((w_Likelihood(lambda)-mu)^2 / (n-1))
  sigma2.post      <-  (n-1) * (sigma2.hat)/rchisq(1,n-1)
  return(sigma2.post)
}

lambda_proposal <- function(lambda){
  rnorm(1, mean = lambda, sd = 0.2)
}

log_jointposterior <- function(lambda, mu, sigma2){
  log_post <- log(1/sqrt(sigma2))
  w <- w_Likelihood(lambda)
  for (i in 1:n){
    log_post <- log_post + log(dnorm(w[i],mean=mu,sd=sqrt(sigma2))) + (lambda-1)*log(y
[i])
  }
  return(log_post)
}
#Starting points 1
sims = 50000
lambda <- matrix(0,nrow = sims, ncol = 1)
lambda[1] <- c(0)
mu <- matrix(0,nrow = sims, ncol = 1)
mu[1] <- mean(w_Likelihood(lambda[1]))
sigma2 <- matrix(0,nrow = sims, ncol = 1)
sigma2[1] <- var(w_Likelihood(lambda[1]))
```

```
#MonteCarlo_Hastings
MonteCarlo_Hastings <- function(lambda, mu, sigma2) {
  for (i in 2:sims){
    mu[i] <- mu_condPosterior(sigma2[i-1], lambda[i-1])
    sigma2[i] <- sigma2_condPosterior(mu[i], lambda[i-1])
    lambda[i] <- lambda_proposal(lambda[i-1])
    r <- min(exp(log_jointposterior(lambda[i],mu[i],sigma2[i]) - log_jointposterior(lamb
da[i-1],mu[i-1],sigma2[i-1])), 1)

    if ( r >= runif(1) ) { #Accept
      lambda[i] <- lambda[i]
    } else { #Reject
      lambda[i] <- lambda[i-1]
    }
  }

  MCH_s <- cbind("lambda" = lambda[40000:sims],"mu" = mu[40000:sims], "sigma" = sqrt(sig
ma2[40000:sims]))
  print(t(apply(MCH_s,2, function(x) quantile(x, c(.025,.25,.5,.75,.975)))))
  return(list(lambda,mu,sigma2))
}
```

** Quantiles of all Parameters **

```
params_1 <- MonteCarlo_Hastings(lambda, mu, sigma2)
```

```
##               2.5%       25%       50%       75%    97.5%
## lambda -0.03569759 0.1067339 0.1879568 0.2670312 0.470359
## mu      2.40546032 3.0741507 3.5499201 4.1524320 6.276021
## sigma   1.01849796 1.4917521 1.8570577 2.3504916 4.331329
```

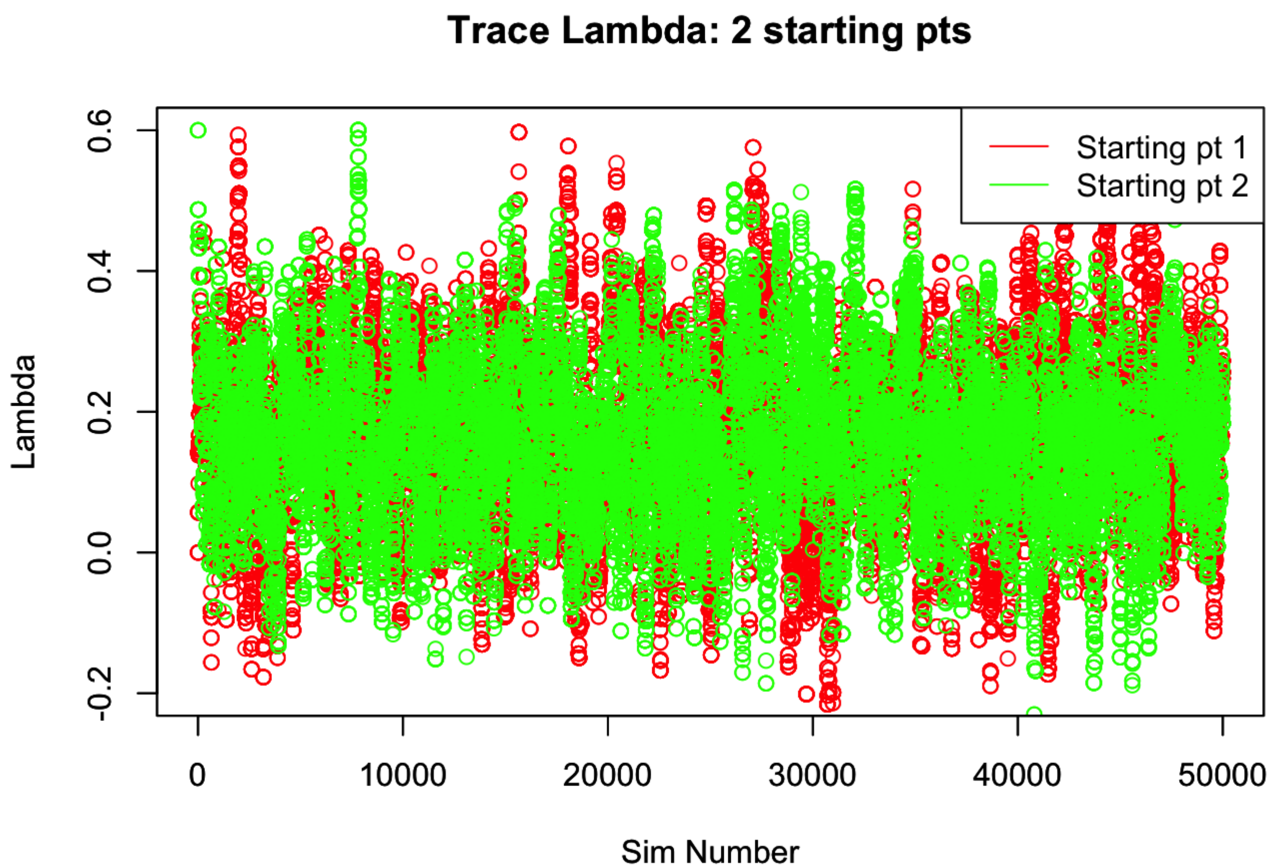
```
#Starting points 2
sims = 50000
lambda <- matrix(0,nrow = sims, ncol = 1)
lambda[1] <- c(0.6)
mu <- matrix(0,nrow = sims, ncol = 1)
mu[1] <- mean(w_Likelihood(lambda[1]))
sigma2 <- matrix(0,nrow = sims, ncol = 1)
sigma2[1] <- var(w_Likelihood(lambda[1]))

params_2 <- MonteCarlo_Hastings(lambda, mu, sigma2)
```

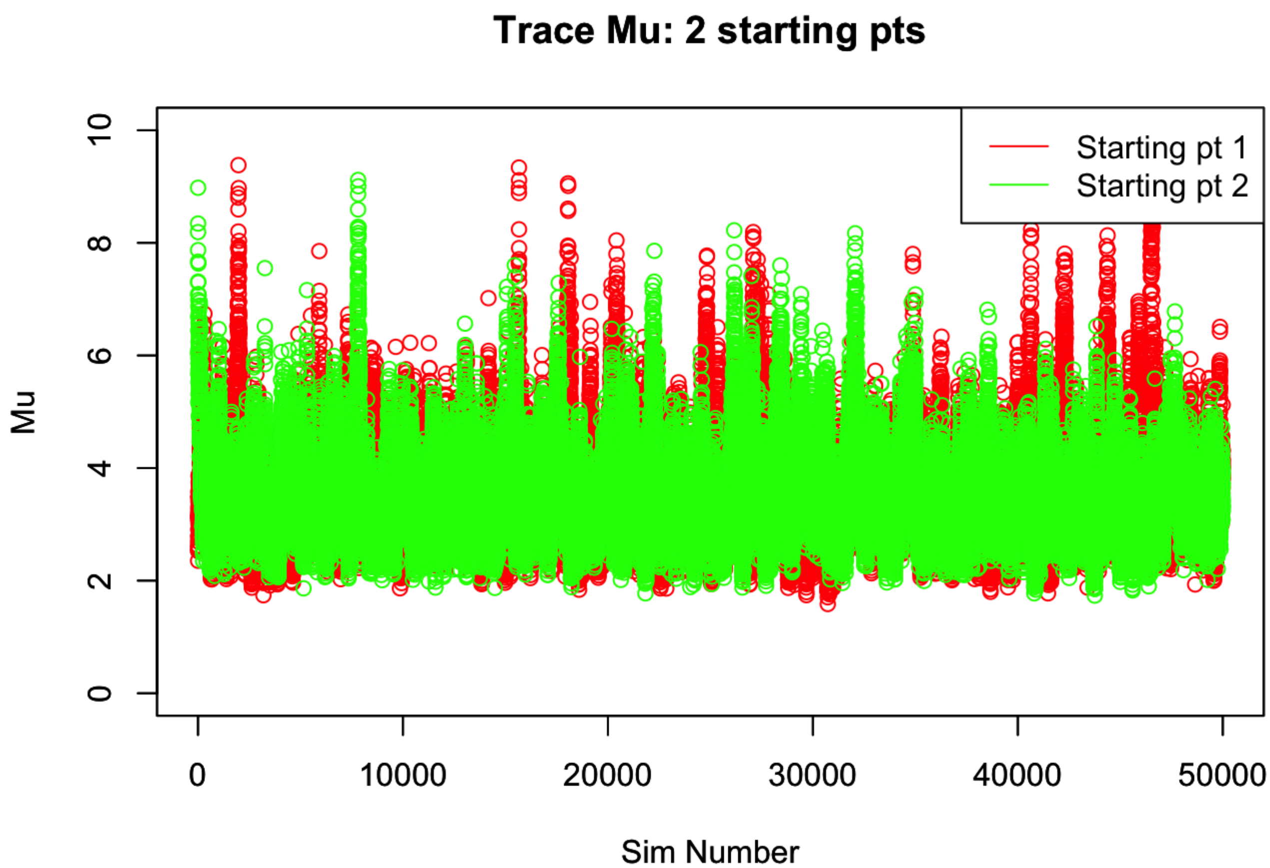
```
##               2.5%        25%       50%       75%     97.5%
## lambda -0.07377194 0.07331497 0.1522486 0.2142988 0.3331535
## mu      2.29528154 2.91633047 3.3244069 3.7752825 4.8465564
## sigma   0.94296485 1.37433018 1.6804519 2.0311106 2.9520996
```

** Trace Plots for all Parameters **

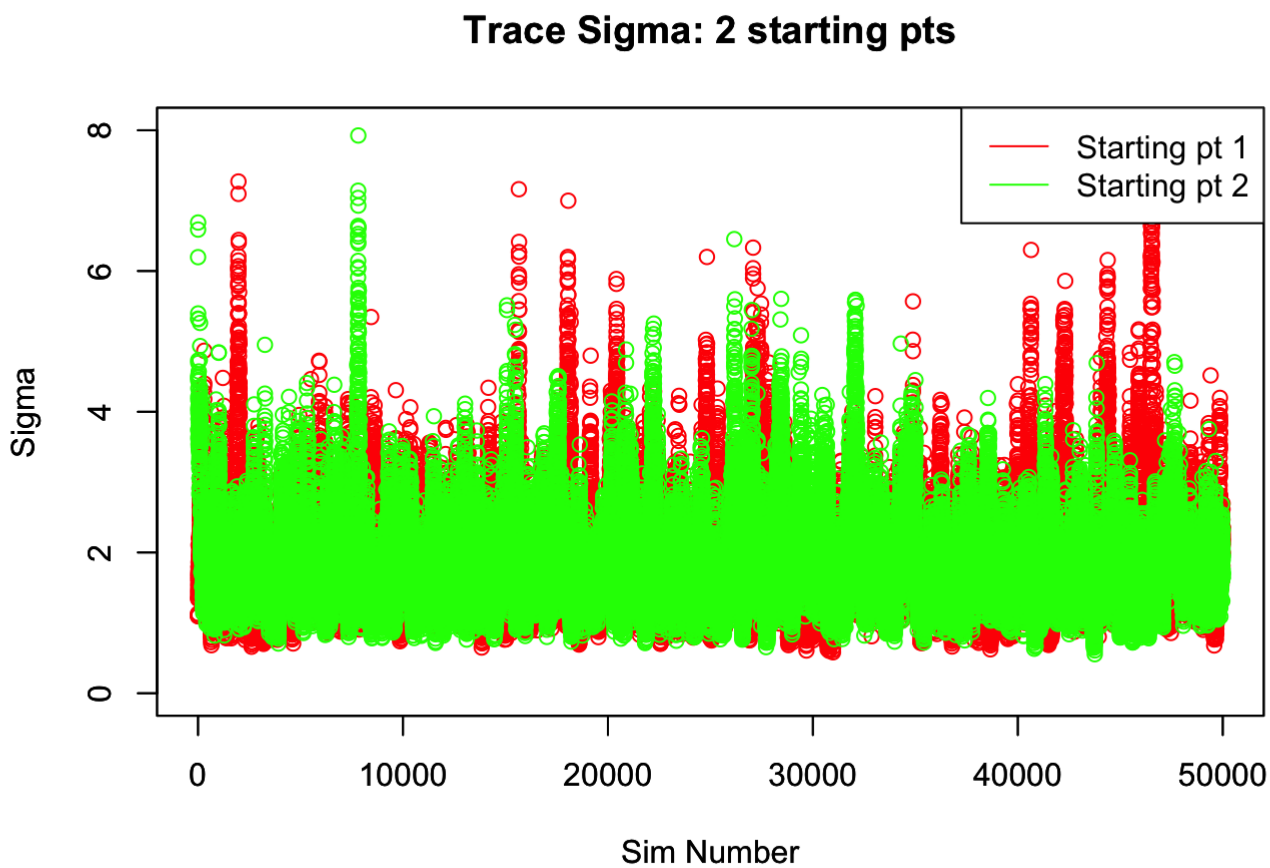
```
plot(params_1[[1]], col = "red", main = "Trace Lambda: 2 starting pts", xlab = "Sim Numb
er", ylab = "Lambda", ylim = c(-0.2,0.6))
par(new=TRUE)
plot(params_2[[1]], col = "green",main = "Trace Lambda: 2 starting pts", xlab = "Sim Num
ber", ylab = "Lambda", ylim = c(-0.2,0.6))
legend("topright",col = c("red","green"),
       legend=c("Starting pt 1","Starting pt 2"),
       lty = c(1,1,1))
```




```
plot(params_1[[2]], col = "red", main = "Trace Mu: 2 starting pts", xlab = "Sim Number",
ylab = "Mu", ylim = c(0,10))
par(new=TRUE)
plot(params_2[[2]], col = "green", main = "Trace Mu: 2 starting pts", xlab = "Sim Numbe
r", ylab = "Mu", ylim = c(0,10))
legend("topright",col = c("red","green"),
       legend=c("Starting pt 1","Starting pt 2"),
       lty = c(1,1,1))
```

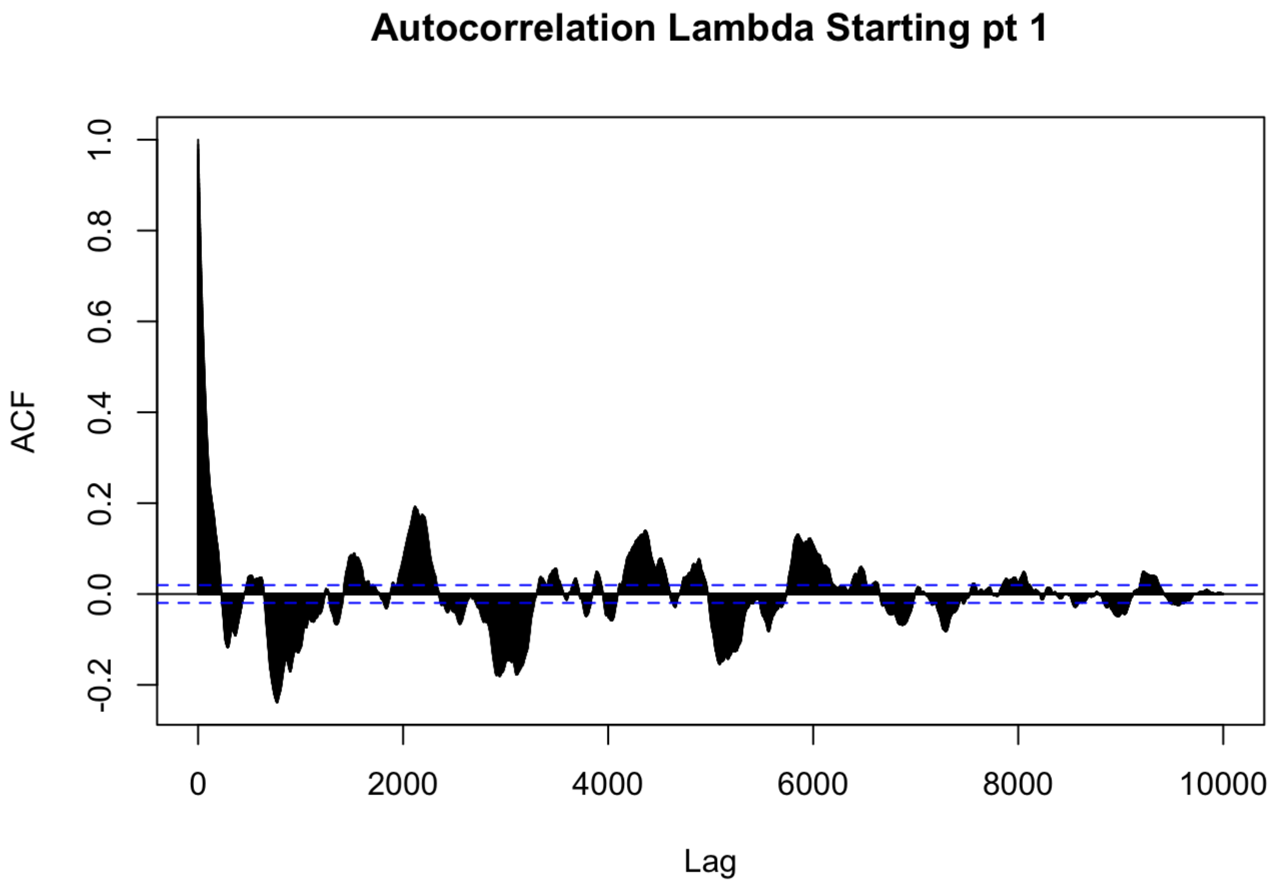



```
plot(sqrt(params_1[[3]]), col = "red", main = "Trace Sigma: 2 starting pts", xlab = "Sim
Number", ylab = "Sigma", ylim= c(0,8))
par(new=TRUE)
plot(sqrt(params_2[[3]]), col = "green", main = "Trace Sigma: 2 starting pts", xlab = "S
im Number", ylab = "Sigma", ylim = c(0,8))
legend("topright",col = c("red","green"),
       legend=c("Starting pt 1","Starting pt 2"),
       lty = c(1,1,1))
```

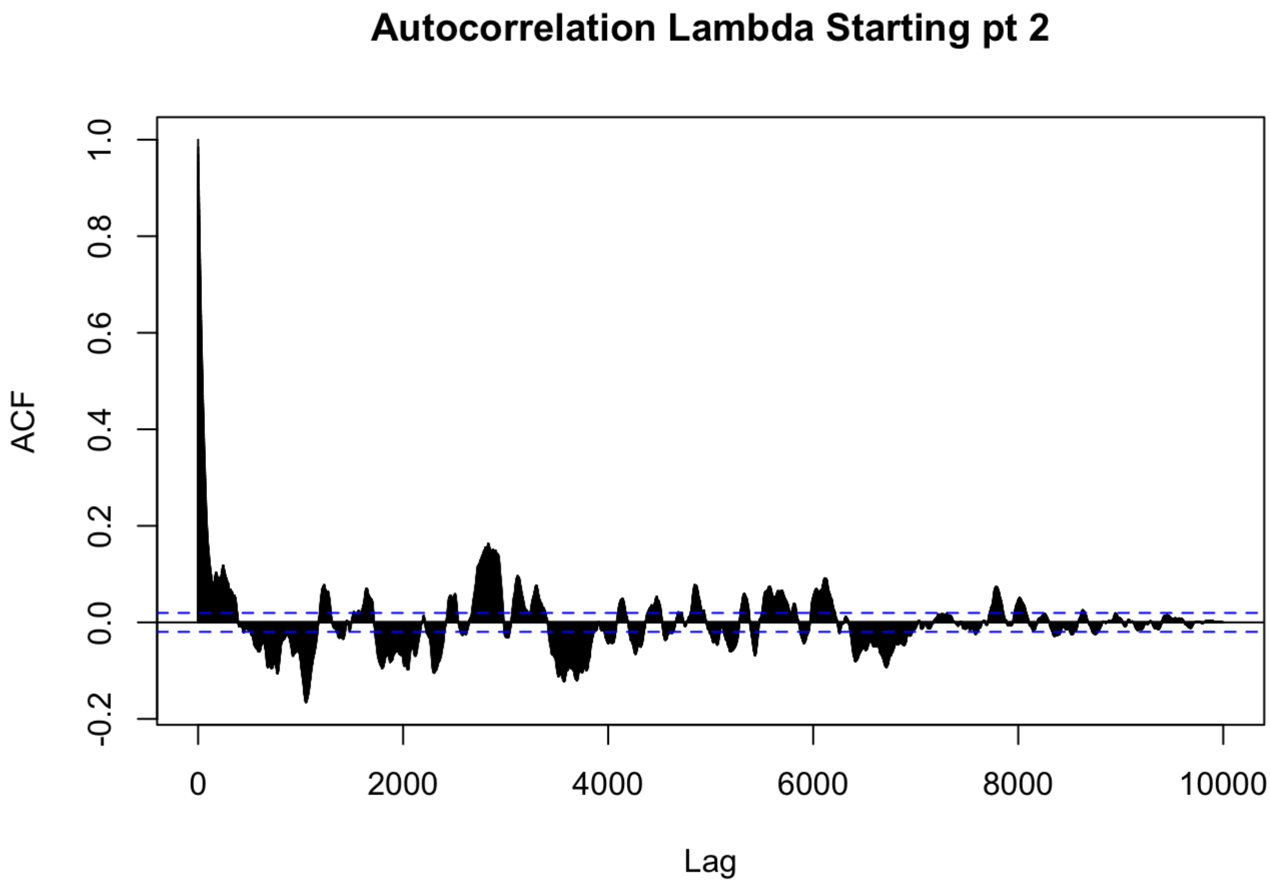


** Autocorrelation Plots **

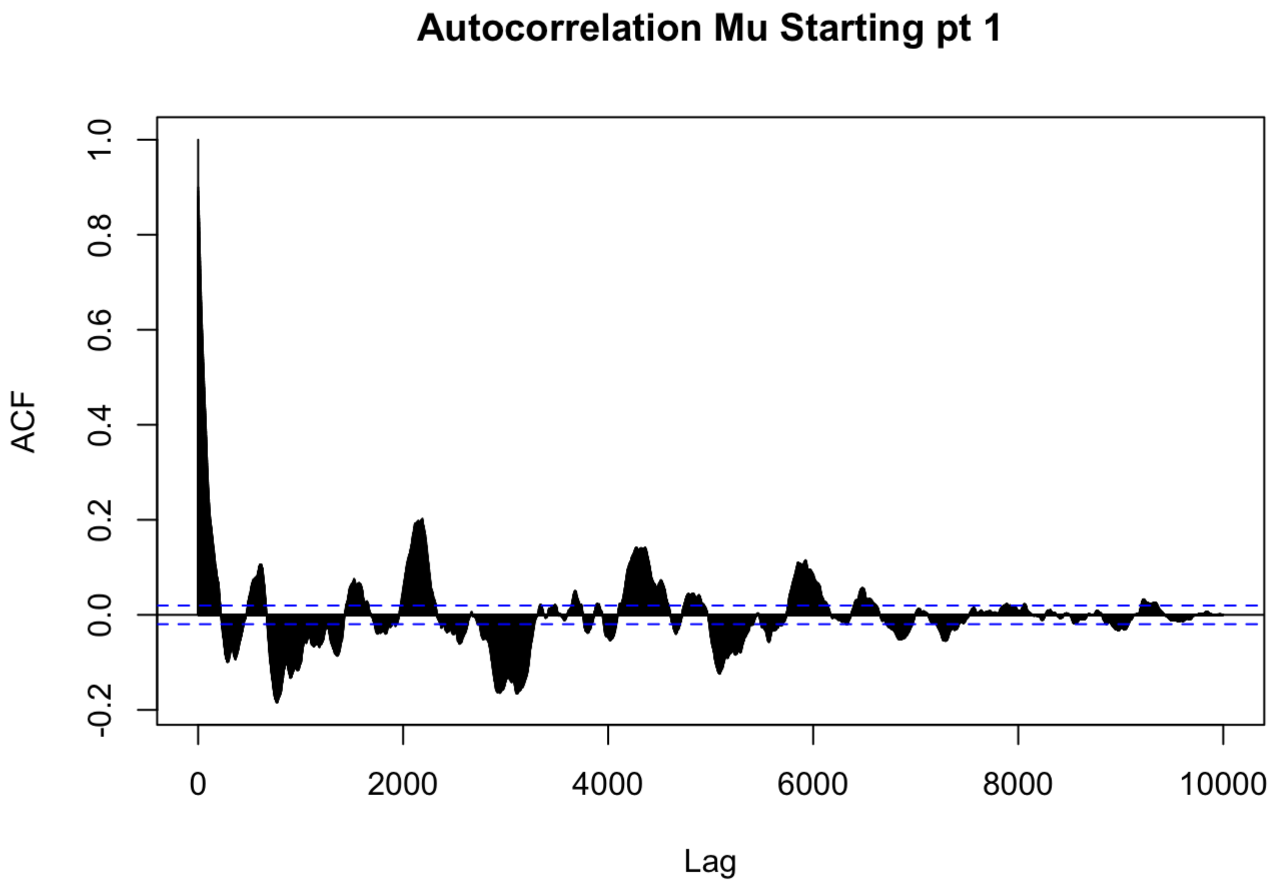
```
Autocor_l1 <- acf(params_1[[1]][40000:50000], lag.max = 10000, plot = FALSE)
plot(Autocor_l1, main = "Autocorrelation Lambda Starting pt 1")
```



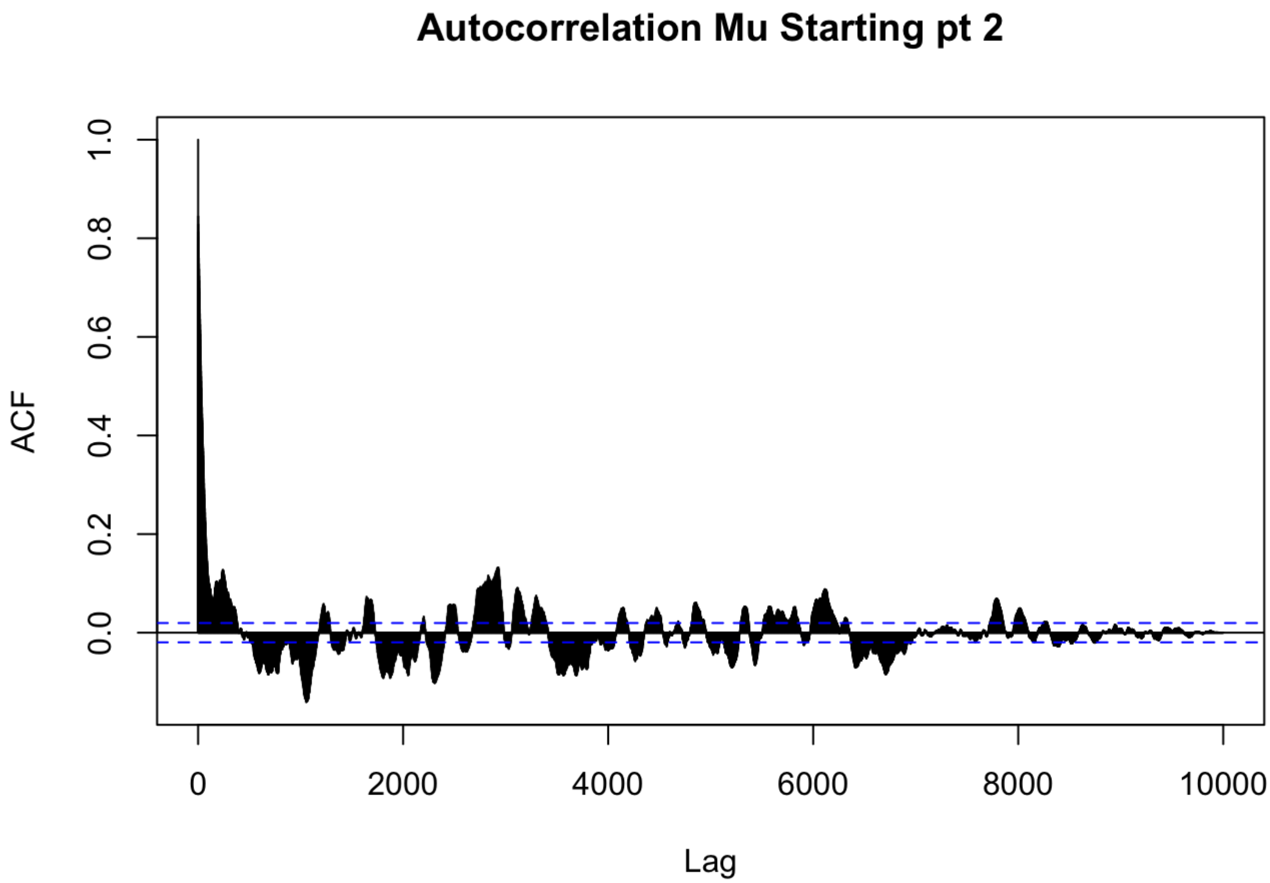

```
Autocor_l2 <- acf(params_2[[1]][40000:50000], lag.max = 10000, plot = FALSE)
plot(Autocor_l2, main = "Autocorrelation Lambda Starting pt 2")
```



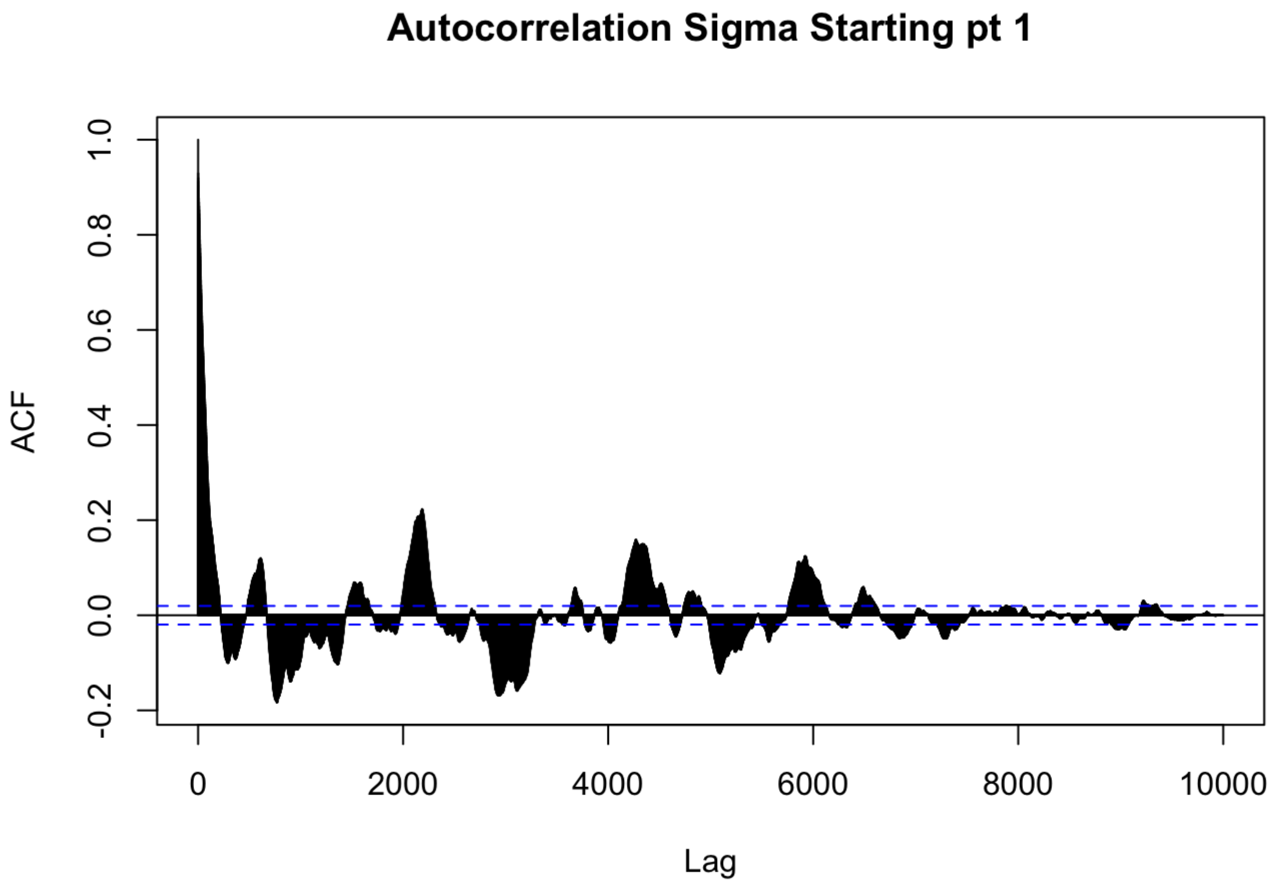

```
Autocor_Mu1 <- acf(params_1[[2]][40000:50000], lag.max = 10000, plot = FALSE)
plot(Autocor_Mu1, main = "Autocorrelation Mu Starting pt 1")
```



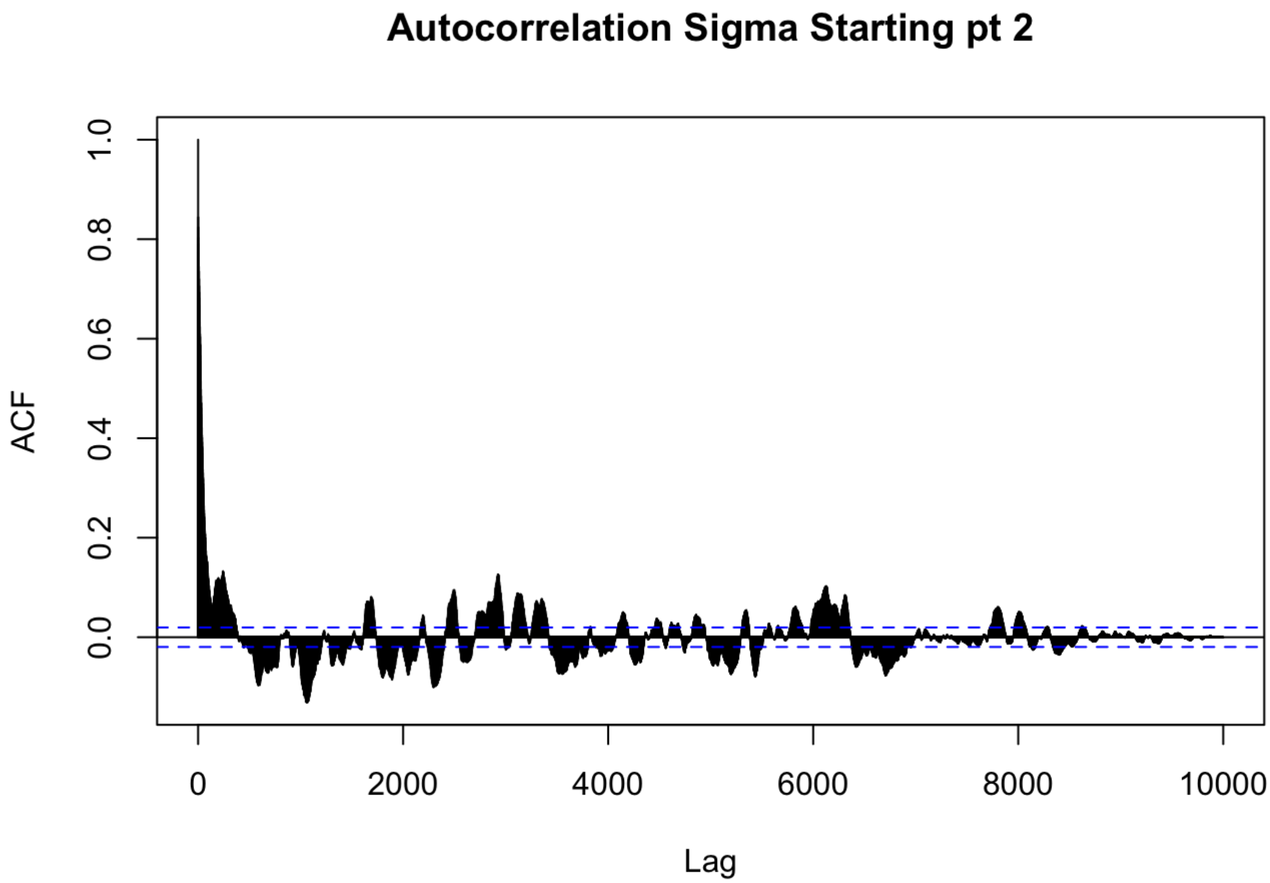

```
Autocor_Mu2 <- acf(params_2[[2]][40000:50000], lag.max = 10000, plot = FALSE)
plot(Autocor_Mu2, main = "Autocorrelation Mu Starting pt 2")
```




```
Autocor_Sig1 <- acf(sqrt(params_1[[3]][40000:50000]), lag.max = 10000, plot = FALSE)
plot(Autocor_Sig1, main = "Autocorrelation Sigma Starting pt 1")
```

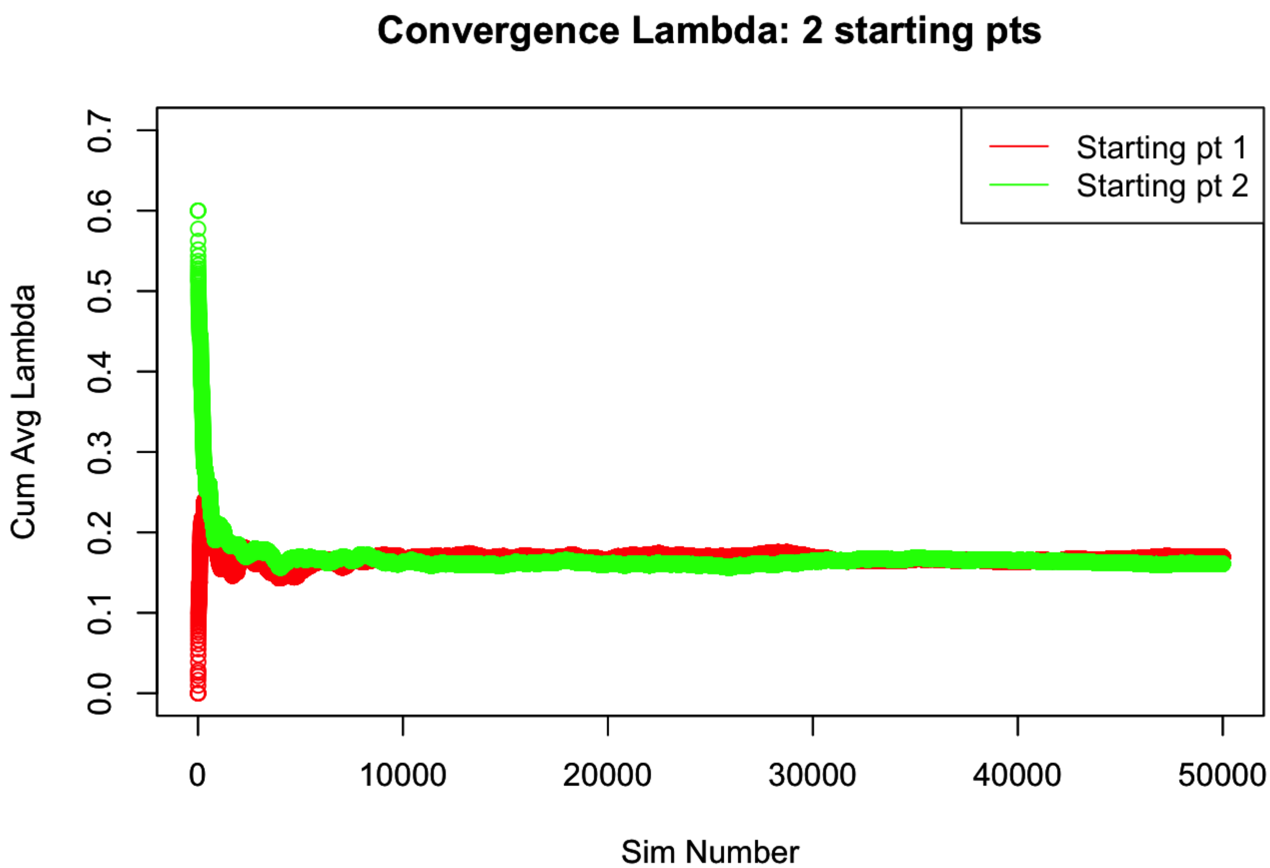



```
Autocor_Sig2 <- acf(params_2[[3]][40000:50000], lag.max = 10000, plot = FALSE)
plot(Autocor_Sig2, main = "Autocorrelation Sigma Starting pt 2")
```

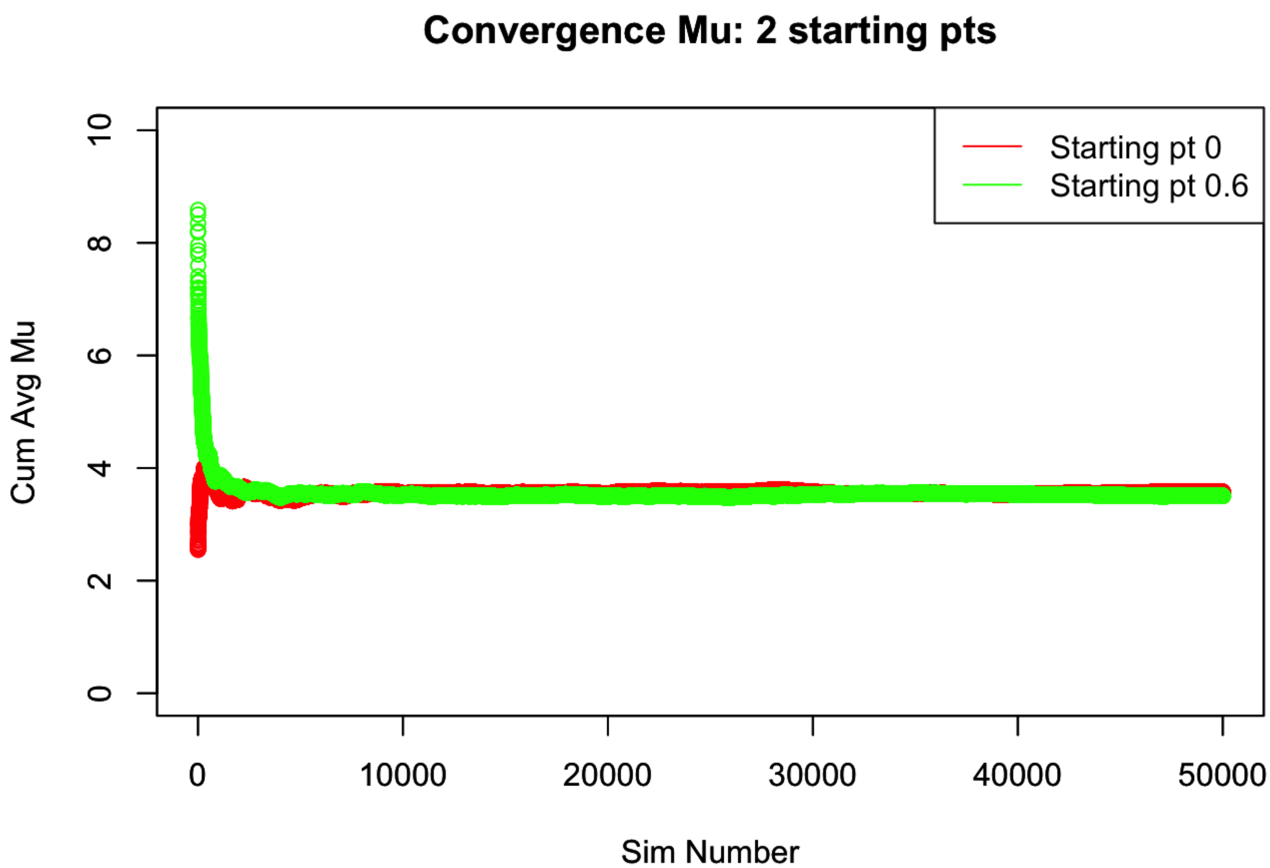


** Convergence of Cumulative Averages Diagnostics **

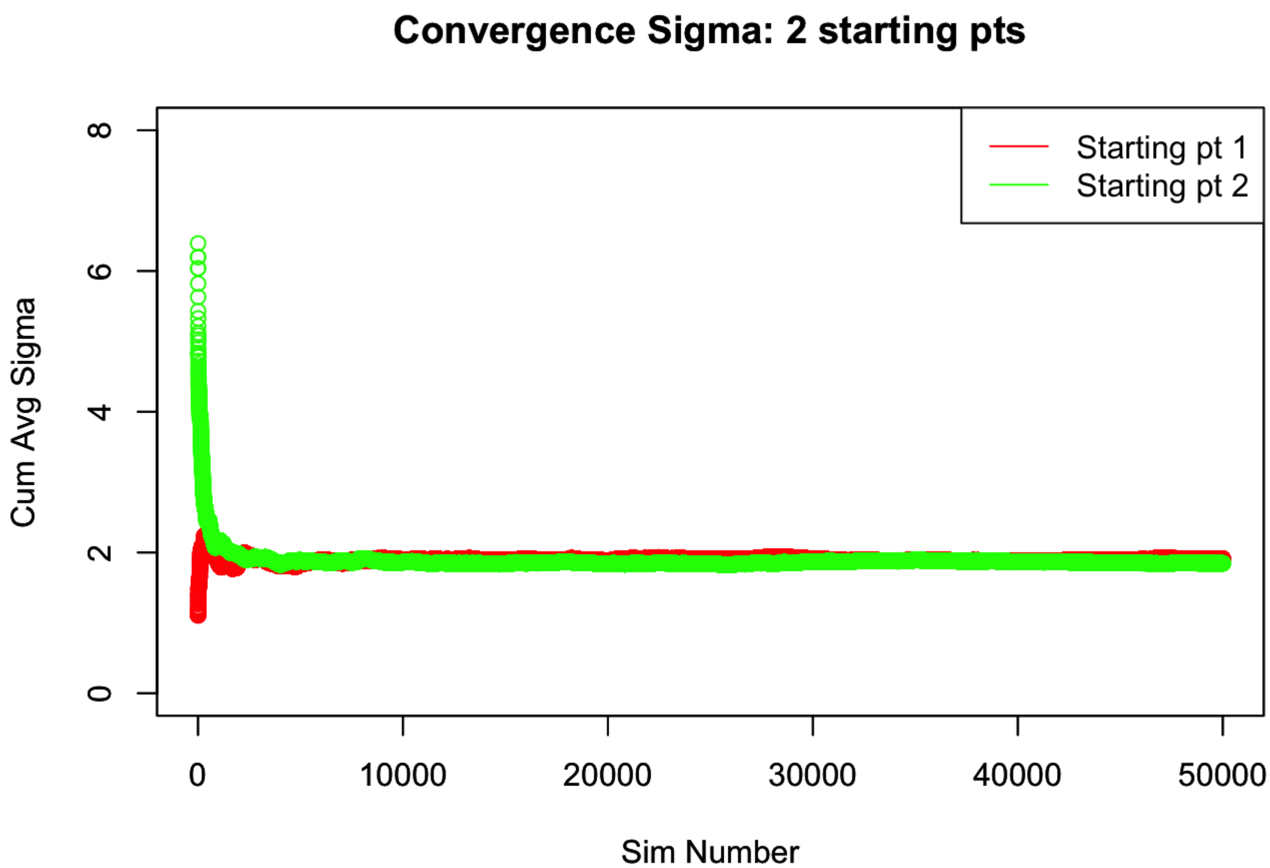
```
plot(cumsum(params_1[[1]])/c(1:sims), col = "red", ylim = c(0,0.7), main = "Convergence
 Lambda: 2 starting pts", xlab = "Sim Number", ylab = "Cum Avg Lambda")
par(new=TRUE)
plot(cumsum(params_2[[1]])/c(1:sims), col = "green", ylim = c(0,0.7), main = "Convergenc
e Lambda: 2 starting pts", xlab = "Sim Number", ylab = "Cum Avg Lambda")
legend("topright",col = c("red","green"),
       legend=c("Starting pt 1","Starting pt 2"),
       lty = c(1,1,1))
```




```
plot(cumsum(params_1[[2]])/c(1:sims), col = "red", main = "Convergence Mu: 2 starting pt
s", xlab = "Sim Number", ylab = "Cum Avg Mu", ylim = c(0,10))
par(new=TRUE)
plot(cumsum(params_2[[2]])/c(1:sims), col = "green", main = "Convergence Mu: 2 starting
 pts", xlab = "Sim Number", ylab = "Cum Avg Mu", ylim = c(0,10))
legend("topright",col = c("red","green"),
       legend=c("Starting pt 0","Starting pt 0.6"),
       lty = c(1,1,1))
```




```
plot(cumsum(sqrt(params_1[[3]]))/c(1:sims), col = "red", main = "Convergence Sigma: 2 st
arting pts", xlab = "Sim Number", ylab = "Cum Avg Sigma", ylim= c(0,8))
par(new=TRUE)
plot(cumsum(sqrt(params_2[[3]]))/c(1:sims), col = "green", main = "Convergence Sigma: 2
 starting pts", xlab = "Sim Number", ylab = "Cum Avg Sigma", ylim= c(0,8))
legend("topright",col = c("red","green"),
       legend=c("Starting pt 1","Starting pt 2"),
       lty = c(1,1,1))
```



** 95 % Posterior Interval **

```
lam <- sort(params_1[[1]][40000:50000])[c(250,9750)]
cat("Posterior Interval Lambda: [",lam, "]")
```

```
## Posterior Interval Lambda: [ -0.03569759 0.470359 ]
```

```
mu <- sort(params_1[[2]][40000:50000])[c(250,9750)]
cat("Posterior Interval Mu: [",mu, "]")
```

```
## Posterior Interval Mu: [ 2.404631 6.27038 ]
```

```
sig <- sort(sqrt(params_1[[3]][40000:50000]))[c(250,9750)]
cat("Posterior Interval Sigma: [",sig, "]")
```

```
## Posterior Interval Sigma: [ 1.017622 4.325593 ]
```

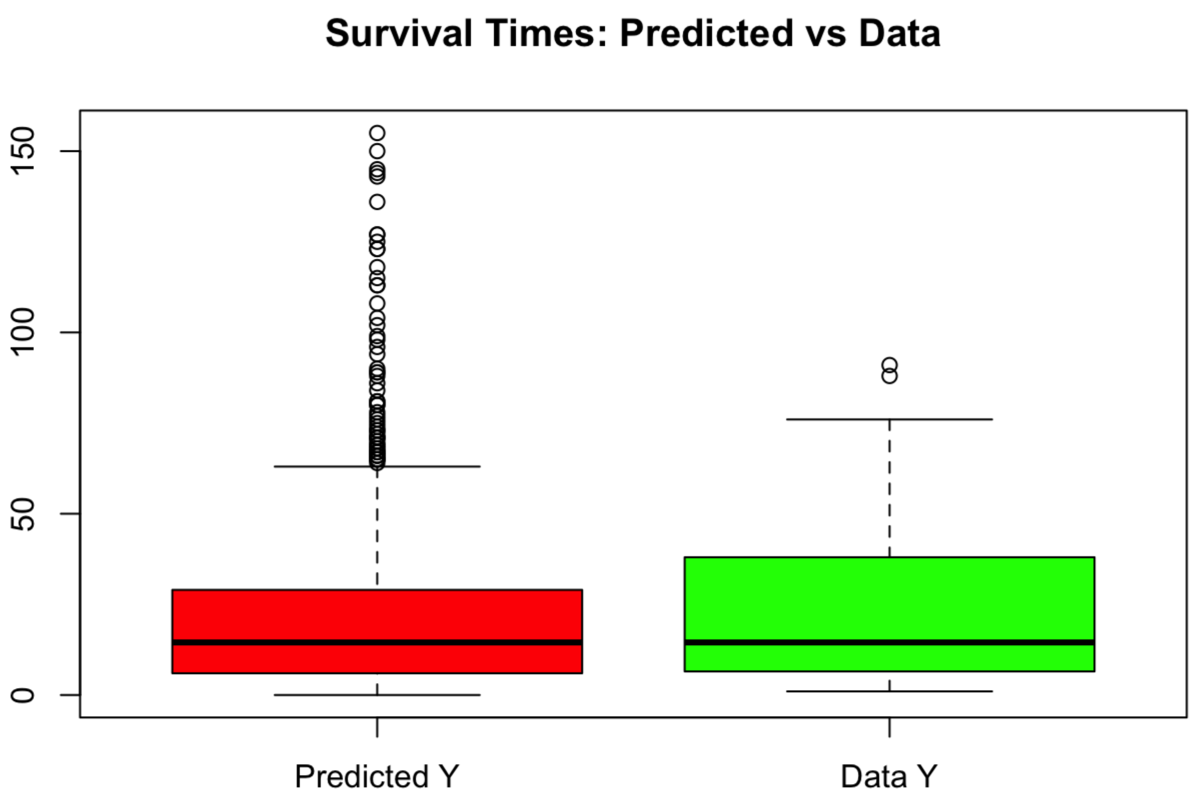
** Predictive Distribution of new Y **

```r
pred_y <- function(mu, sigma2, lambda){
    pred_w_s <- rnorm(1000, mean = mu, sd = sqrt(sigma2))
    pred_y_s <- round(exp(log(pred_w_s*lambda + 1)/lambda))

  pred_y_s
}

s_pred_y <- pred_y(params_1[[2]][50000],params_1[[3]][50000],params_1[[1]][50000])

boxplot(s_pred_y, y, col = c("red", "green"), names = c("Predicted Y", "Data Y"), main =
"Survival Times: Predicted vs Data")
```



** 95 % Posterior Interval **

```r
pred_conf <- sort(s_pred_y)[c(25, 975)]
cat("Posterior Interval Predicted Y: [", pred_conf, "]")
```

```
## Posterior Interval Predicted Y: [ 1 88 ]
```