**MA 578 — Bayesian Statistics**

# Homework 2

(Due: Tuesday, 09/24/19)

1. BDA problem 2.10.

2. BDA problem 2.13.

3. Jeffreys prior for *canonical* parameterization of binomial likelihood:

   (a) If $\theta \sim \text{Beta}(\alpha, \beta)$, show that $\lambda = \log(\theta/(1-\theta)) = \text{logit}(\theta)$ has a "Beta-logit" distribution with density
   $$\mathbb{P}(\lambda) \propto \frac{e^{\alpha\lambda}}{(1+e^{\lambda})^{\alpha+\beta}}.$$

   (b) Suppose now that $X \mid \lambda \sim \text{Binom}(n, \text{logit}^{-1}(\lambda))$, that is, that
   $$\mathbb{P}(X \mid \lambda) \propto \exp\Big\{\lambda X - n\log(1+e^{\lambda})\Big\}.$$
   Show that Jeffreys prior for $\lambda$ is Beta-logit$(\frac{1}{2}, \frac{1}{2})$ and so conclude that Jeffreys prior for $\theta = \text{logit}^{-1}(\lambda) \sim \text{Beta}(\frac{1}{2}, \frac{1}{2})$.