Boston University
Department of Electrical and Computer Engineering

SC505 STOCHASTIC PROCESSES

EXAM 1

October 7, 2002

NAME:______________________________

ID NUMBER:______________________________

Honor Code: This exam represents only my own work. I did not give or receive help on this exam.

Signature:______________________________ Date:__________

Instructions:

- This is a closed-book exam, but one $8\frac{1}{2}$" $\times$ 11" sheet of notes (both sides) is allowed.
- There are 4 problems on the exam, approximately equally weighted.
- The problems are not necessarily in order of difficulty.
- You should concisely indicate your reasoning and **show all relevant work** for each problem. Your score will be based on a judgment of your understanding as reflected by what you have written for an answer. NO credit will be given for answers with no explanation or that are unreadable.
- All work you want graded must go in this booklet. Use the blue books for scratch work only.

(1) ________________

(2) ________________

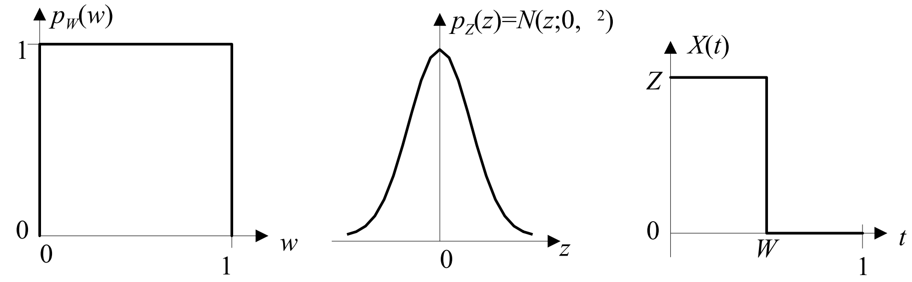
(3) ________________

(4) ________________

Total: ________________

Problem #1:

Let W and Z be independent random variables, with W uniformly distributed and Z Gaussian distributed, as illustrated in the figure below. The stochastic process $X(t)$ is defined on the interval $[0, 1]$ in terms of the random variables W and Z as shown on the right in the figure below.



$$X(t) = \begin{cases} Z & t < W \\ 0 & t \geq W \end{cases}$$

(a) Find the mean function $m_X(t)$ and autocovariance function $K_{XX}(t_1, t_2)$.

(b) Find the first order probability density function $p_{X(t)}(x)$.

(c) Is $X(t)$ a Markov process? Explain.

(d) Is $X(t)$ strict-sense stationary process? Is $X(t)$ a Gaussian random process? Explain.

(e) Is $X(t)$ an independent increments processes? Explain.

Answer #1:

Answer #1 (Continued):

Problem #2:

Say whether each of the following statements is true or false and give a brief justification. One word answers will receive no credit.

(a) If $X(t)$ and $Y(t)$ are independent increments processes that are independent of each other, then $Z(t) = X(t) + Y(t)$ is an independent increments process.

(b) If $X(t)$ is a wide-sense stationary process, then the mean square integral process $Y(t) = \int_0^t X(s)\,ds$ is always wide-sense stationary.

(c) If $X(t)$ and $Y(t)$ are independent processes then $R_{XY}(t_1, t_2) = 0$.

(d) If $X(t)$ is a Gaussian random process, then $Y(t) = X(|t|)$ is a Gaussian random process.

(e) For any jointly wide-sense stationary random processes $X(t)$ and $Y(t)$, the power spectral density of the process $Z(t) = X(t) + Y(t)$ is given by $S_{ZZ}(f) = S_{XX}(f) + S_{YY}(f)$.

Answer #2:

Answer #2 (Continued):

Problem #3:

Let $X(t)$ be a zero-mean, wide-sense stationary random process with autocovariance function given by:

$$K_{XX}(t_1, t_2) = \alpha \frac{\sin(t_2 - t_1)}{(t_2 - t_1)}$$

(a) Find the power spectral density function $S_{XX}(f)$ and the value of α if the total average power in $X(t)$ is $R_{XX}(0) = 1$.

(b) Is $X(t)$ mean-square continuous at time $t = -1$? Is $X(t)$ mean-square differentiable at time $t = 4$? Explain.

(c) Is $X(t)$ ergodic in the mean? Explain.

Answer #3:

Answer #3 (Continued):

Problem #4:

Let $N_1(t)$ and $N_2(t)$ be two independent, standard Poisson counting processes with rates $\lambda_1 = 1$ and $\lambda_2 = 3$, respectively. Define the continuous-time random process: $X(t) = 3N_1(t) - N_2(t)$.

(a) Is $X(t)$ itself a standard Poisson counting process? Explain. If it is, specify its rate λ.

(b) Find the mean process $m_X(t)$ and the autocorrelation function $R_{XX}(t_1, t_2)$. Is the process wide-sense stationary? Explain.

(c) Is $X(t)$ an independent increments process? Explain.

Answer #4:

Answer #4 (Continued)