Boston University
Department of Electrical and Computer Engineering

EC505 STOCHASTIC PROCESSES

EXAM 2

November 14, 2005

NAME:____________________________________

ID NUMBER:____________________________________

Honor Code: This exam represents only my own work. I did not give or receive help on this exam.

Signature:____________________________________ Date:____________

Instructions:

- This is a closed-book exam, but two $8\frac{1}{2}$" $\times$ 11" sheets of notes (both sides) are allowed.
- There are 4 problems on the exam, approximately equally weighted.
- The problems are not necessarily in order of difficulty.
- You should concisely indicate your reasoning and **show all relevant work** for each problem. Your score will be based on a judgment of your understanding as reflected by what you have written for an answer. NO credit will be given for answers with no explanation or that are unreadable.
- All work you want graded must go in this booklet. Use the blue books for scratch work only.

(1) ________________

(2) ________________

(3) ________________

(4) ________________

Total: ________________

Problem #1:

Consider a discrete-time LTI system, $H(z)$ with input $W(n)$ and output $X(n)$. It is known that $W(n)$ is a zero-mean process with $R_{WW}(m) = \sigma^2\delta(m)$ and the system $H(z)$ is described by the relationship: $X(n) = aX(n-1) + W(n)$ for some constants a and σ^2.

(a) If $R_{XX}(0) = 2$ and $R_{XX}(3) = -1/4$, what are the values of $R_{XX}(1)$ and $R_{XX}(2)$?

(b) Find the system parameters a and σ^2.

(c) If $R_{ZZ}(m) = \left(\frac{1}{3}\right)^{|m|}$ what type of system could $G(z)$ be: AR, MA, or ARMA? Explain.

Answer #1:

Answer #1 (Continued):

Problem #2:

Say whether each of the following statements is true/possible or false/impossible and give a brief justification. One word answers will receive no credit.

(a) In a binary detection problem suppose you want to maximize P_D while constraining P_F to be below α. Then a randomized test can sometimes provide better performance than a non-randomized (deterministic) test.

(b) Consider a binary detection problem with two observations, Y, Z and suppose, Z is a function of Y, i.e., $Z = g(Y)$. Then there is a scalar sufficient statistic that does not depend on Z.

(c) If $X(t)$ is a zero-mean, wide-sense stationary process with power spectral density $S_{XX}(f) = \alpha e^{-4\pi f^2}$, then it is mean-square differentiable at $t = -1$ and ergodic in the mean.

(d) For any jointly wide-sense stationary random processes $X(t)$ and $Y(t)$, the power spectral density of the process $Z(t) = X(t) + Y(t)$ is given by $S_{ZZ}(f) = S_{XX}(f) + S_{YY}(f)$.

(e) If $X(t)$, $Y(t)$ are wide-sense stationary processes, then $Z(t) = X(t)Y(t)$ is a wide-sense stationary process.

Answer #2:

Answer #2 (Continued):

Problem #3:

Suppose we want to detect a "deep sky" object with an optical sensor pointed into space. We want to develop an optimal detector for the object. If the object is present in the field of view, the observation Y has an exponential distribution with mean $m_Y = 2$. If the object is absent, then we observe the galactic background, and Y has an exponential distribution with mean $m_Y = 1$.

(a) If it is known that there is a 50% probability of the object being present before making any observations, what decision rule will yield the minimum probability of error? Sketch the decision regions in the observation space Y.

(b) What are P_F, P_D, and Pr(error) for the decision rule of part (a)?

(c) Suppose that additional cosmological analysis reveals that there is actually only a 10% chance that the object will be present before any observations are made. How do the decision regions of part (a) change?

(d) Find the decision rule that maximizes P_D subject to the constraint that $P_F \leq 1/e^2$.

Answer #3:

Answer #3 (Continued):

Problem #4:

Consider the continuous-time digital communication system shown in the Figure. Over the interval $0 \leq t \leq T$ one of 4 equally likely signals $s_i(t)$ is transmitted:

$$s_1(t) = Ar(t) \quad s_2(t) = \tfrac{1}{3}Ar(t) \quad s_3(t) = -\tfrac{1}{3}Ar(t) \quad s_4(t) = -Ar(t)$$

where A is a constant and $r(t) = \sqrt{2/T}\sin(2\pi t/T)$. The received signal under hypothesis H_i consists of the transmitted signal corrupted by additive white Gaussian noise $W(t)$ with $R_{ww}(t) = q\delta(t)$: $Y(t) = s_i(t) + W(t)$. In this problem we will investigate design of a receiver. Note that $\int_0^T r^2(t)\,dt = 1$

(a) What is the minimum probability of error decision rule for this problem – specify the required processing of the data $y(t)$ and the associated decision regions.

(b) What is the corresponding probability of error in terms of $Q(x) = \frac{1}{\sqrt{2\pi}}\int_x^\infty e^{-\alpha^2/2}d\alpha$? Hint: Use symmetry.

(c) What is the behavior of Pr(error) as A^2/q increases? Explain.

Answer #4:

Answer #4 (Continued)