7.) Prove: $R^2 = (\rho_{XY})^2$

$$= \left(\frac{\Sigma x_i \cdot y_i}{\sqrt{\Sigma x_i^2}\ \sqrt{\Sigma y_i^2}} \right)$$

$$= \frac{(\Sigma x_i y_i)(\Sigma x_i y_i)}{(\Sigma x_i^2)(\Sigma y_i^2)}$$

$$R^2 = \frac{(\Sigma y_i^2) - \Sigma (y_i - \hat{y}_i)^2}{\Sigma y_i^2}$$

$\bar{x} = \bar{y} = 0 \Rightarrow \hat{\beta}_0 = 0$

$\hat{\beta}_0 = \bar{y} - \bar{x}\hat{\beta}_1$

$\hat{y} = \hat{\beta}_1 \cdot x_i$

$$= \frac{\Sigma y_i^2 - \Sigma [y_i^2 - 2 y_i \hat{y}_i + \hat{y}_i^2]}{\Sigma y_i^2}$$

$$= \frac{\Sigma y_i^2 - \Sigma y_i^2 + 2 \Sigma y_i \cdot \hat{y}_i - \Sigma \hat{y}_i^2}{\Sigma y_i^2}$$

$$= \frac{2 \Sigma y_i \cdot (\hat{\beta} \cdot x_i) - \Sigma (\hat{\beta} \cdot x_i)^2}{\Sigma y_i^2}$$

$$= \frac{2\hat{\beta} \Sigma x_i y_i - \hat{\beta}^2 \Sigma x_i^2}{\Sigma y_i^2} = \frac{\left(\frac{\Sigma x_i y_i}{\Sigma x_i^2}\right)\left(2 \Sigma x_i y_i - \left(\frac{\Sigma xy}{\Sigma x_i^2}\right) \Sigma x_i^2\right)}{\Sigma y_i^2}$$

$$= \frac{\Sigma x_i y_i \left(2 \Sigma x_i y_i - \Sigma x_i y_i\right)}{\Sigma x_i^2 \Sigma y_i^2}$$

$$= \boxed{\frac{\Sigma x_i y_i \Sigma x_i y_i}{\Sigma x_i^2 \Sigma y_i^2}}$$