Given: $\hat{\beta} = \dfrac{\sum x_i y_i}{\sum x_i^2}$ (11d)

Proof: $$t = \frac{\hat{\beta}}{se(\hat{\beta})} = \left(\frac{\sum x_i y_i}{\sum x_i^2}\right) \cdot \sqrt{\frac{(n-1)\sum x_i^2}{\sum (y_i - x_i \hat{\beta})^2}}$$

$$= \left(\frac{\sqrt{(n-1)} \cdot \sum x_i y_i}{1}\right) \cdot \left(\frac{1}{\sum x_i^2}\right) \sqrt{\frac{\sum x_i^2}{\sum (y_i - x_i \hat{\beta})^2}}$$

$$= \text{''} \cdot \sqrt{\frac{\sum x_i^2}{(\sum x_i^2)^2 \sum (y_i - x_i \hat{\beta})^2}}$$

$$= \text{''} \cdot \sqrt{\frac{1}{\sum x_i^2 \cdot \sum (y_i^2 - 2 x_i y_i \hat{\beta} + x_i^2 \cdot \hat{\beta}^2)}}$$

$$\cancel{= \cdot \sqrt{\frac{1}{\sum x_i^2 \cdot y_i^2 - \sum 2 x_i y_i \sum x_i y_i + \sum x_i^2 \cdot (\sum x_i y_i)^2}}}$$

$$\cancel{= \text{''} \cdot \sqrt{\frac{1}{\sum x_i^2 \cdot \sum y_i^2 - \sum x_i y_i (}}}$$

$$= \text{''} \cdot \sqrt{\frac{1}{\sum x_i^2 \sum y_i^2 - \sum x_i^2 \cdot 2 \sum x y \hat{\beta} + \sum x_i^2 \cdot \sum x_i^2 \cdot \hat{\beta}^2}}$$

$$= \text{''} \cdot \sqrt{\frac{1}{\sum x_i^2 \sum y_i^2 + \hat{\beta} \cdot \sum x_i^2 [-2 \cdot \sum x_i y_i + \sum x_i^2 \hat{\beta}]}}$$

$$= \text{''} \cdot \sqrt{\frac{1}{\sum x_i^2 \cdot \sum y_i^2 + \sum x_i y_i [-2 \cdot \sum x_i y_i + \sum x_i y_i}}$$

$$= \text{''} \cdot \sqrt{\frac{1}{\sum x_i^2 \cdot \sum y_i^2 + \sum x_i y_i [- \sum x_i y_i]}}$$

$$\frac{\hat{\beta}}{se(\hat{\beta})} = \frac{\sqrt{(n-1)} \cdot \sum x_i y_i}{\sqrt{(\sum x_i^2) \cdot (\sum y_i^2) - (\sum x_i \cdot y_i)^2}}$$

Q.E.D.