4.7 Excercises

1.) Let $e^{\beta_0+\beta_1 X} = e^{\vec{\beta}X}$

(4.2) says $p(X) = \dfrac{e^{\vec{\beta}X}}{1+e^{\vec{\beta}X}} \Rightarrow (1+e^{\vec{\beta}X})p(X) = e^{\vec{\beta}X}$

$$p(X)+p(X)e^{\vec{\beta}X} = e^{\vec{\beta}X}$$

$$p(X) = e^{\vec{\beta}X} - e^{\vec{\beta}X}p(X)$$

$$p(X) = e^{\vec{\beta}X}(1-p(X))$$

$$\boxed{e^{\vec{\beta}X} = \frac{p(X)}{1-p(X)}} \quad (4.3)$$

2.) Assume K is class w/ largest (4.12). Thus, for all $l \in (1,K)$, must be that:

$$\oplus \; p_K(x) \geq p_l(x)$$

denom will cancel, so

$$\frac{\pi_K \cdot \frac{1}{\sqrt{2\pi}\sigma}\exp\left(-\frac{1}{2\sigma^2}(x-\mu_K)^2\right)}{\sum_{j=1}^{K}\pi_{j}\frac{1}{\sqrt{2\pi}\sigma}\exp\left(-\frac{1}{2\sigma^2}(x-\mu_{j})^2\right)} \geq \frac{\pi_l \frac{1}{\sqrt{2\pi}\sigma}\exp\left(\frac{1}{2\sigma^2}(x-\mu_l)^2\right)}{\sum_{l=1}^{K}\pi_{j}\frac{1}{\sqrt{2\pi}\sigma}\exp\left(\frac{-1}{2\sigma^2}(x-\mu_j)^2\right)}$$

$$\rightarrow \pi_K \cdot \exp\left(\frac{-1}{2\sigma^2}(x-\mu_K)^2\right) \geq \pi_l \exp\left(-\frac{1}{2\sigma^2}(x-\mu_l)^2\right)$$

log

$$\log(\pi_K) - \frac{1}{2\sigma^2}(x-\mu_K)^2 \geq \log(\pi_l) - \frac{1}{2\sigma^2}(x-\mu_l)^2$$

$$\left[\log(\pi_K) - \cancel{\frac{x^2}{2\sigma^2}} + \frac{2x\mu_K}{2\sigma^2} - \frac{\mu_K^2}{2\sigma^2}\right] \geq \left[\log(\pi_l) - \cancel{\frac{x^2}{2\sigma^2}} + \frac{2x\mu_l}{2\sigma^2} - \frac{\mu_l^2}{2\sigma^2}\right]$$

Thus max $\oplus$ equiv to max discrim fn!

$$(\ast)\;\left[\log(\pi_K) + \frac{x\cdot\mu_K}{\sigma^2} - \frac{\mu_K^2}{2\sigma^2}\right] \geq \left[\log(\pi_l) + \frac{x\cdot\mu_l}{\sigma^2} - \frac{\mu_l^2}{2\sigma^2}\right]$$