

2.) (a)  $\lambda = \infty; m = 0$

Soln:  $\hat{g} = 0$ .

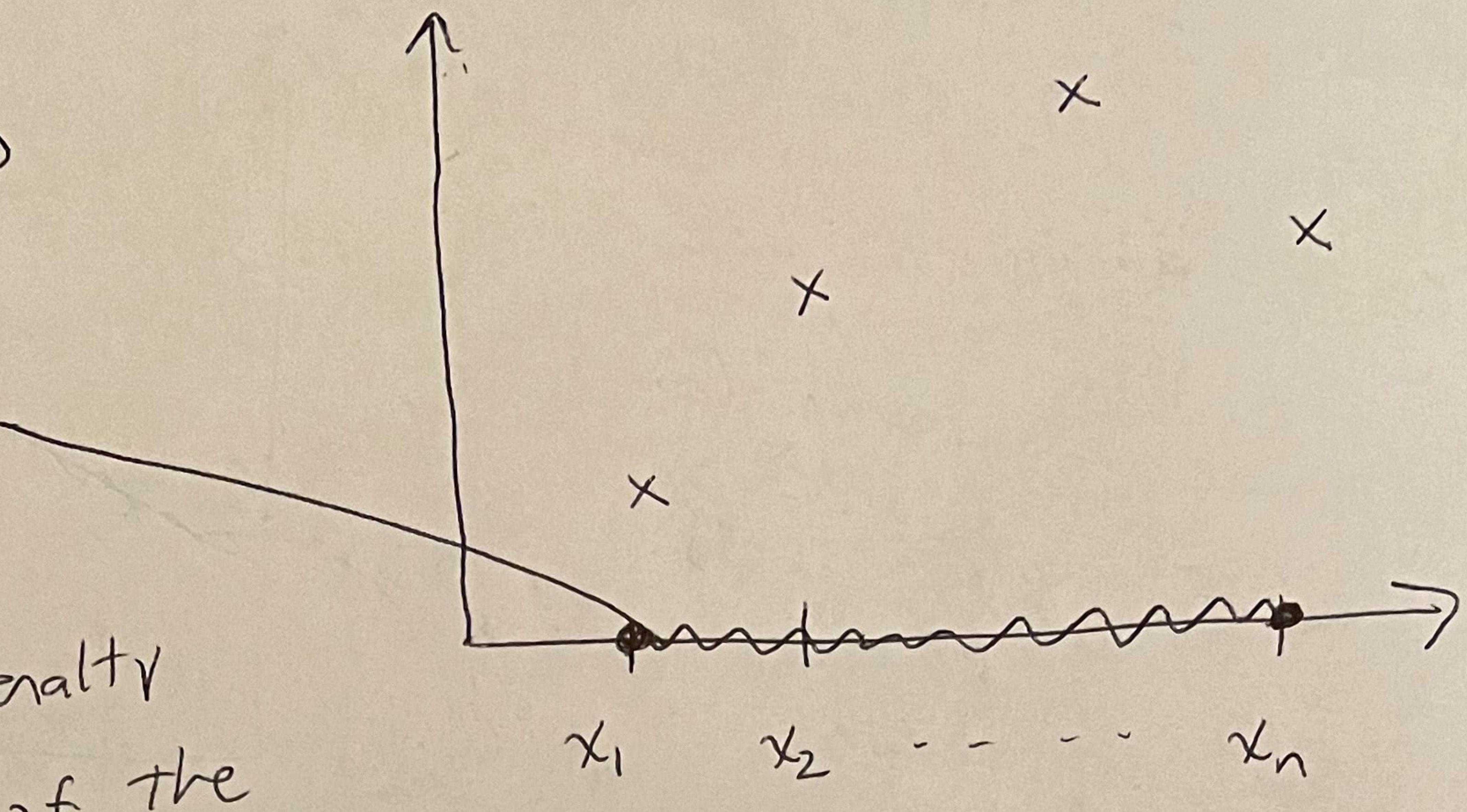
Reason: when  $\lambda \rightarrow \infty$ ,

the second term = penalty

dominates, regardless of the non-zero value of the integral.

when  $g = 0 \Rightarrow g^2 = 0 \Rightarrow \int g^2 = 0$ ,

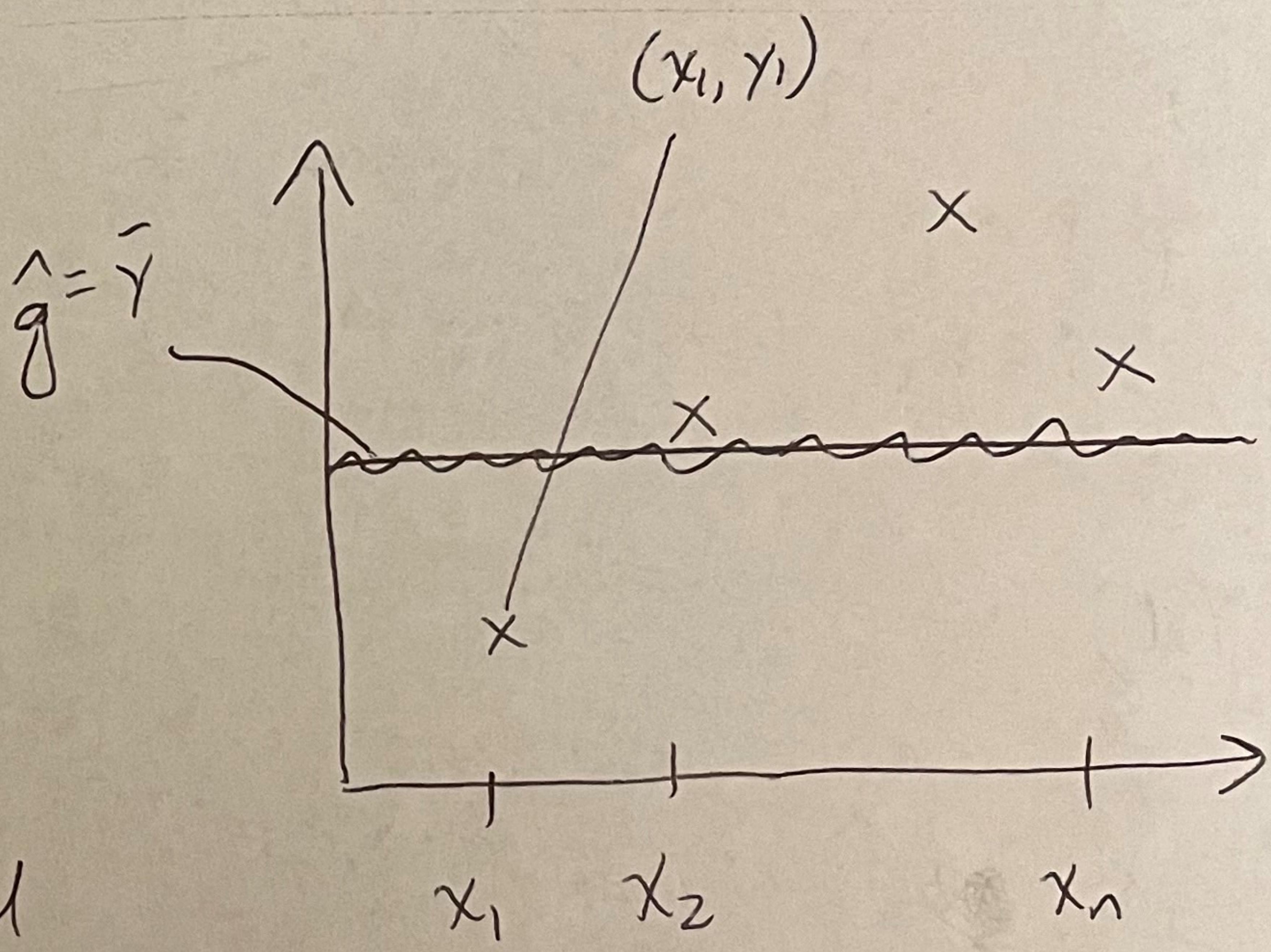
the minimum value of the argument is achieved.



(b)  $\lambda = \infty; m = 1$

Soln:

$$\hat{g} = \bar{y} = \left( \frac{1}{n} \sum_{i=1}^n y_i \right)$$



Reason: Again, when  $\lambda \rightarrow \infty$ , the second term dominates. we can set the

2nd term to zero if

$$g' = 0 \Rightarrow (g')^2 = 0 \Rightarrow \int (g')^2 = 0$$

Thus a constant function will do this.

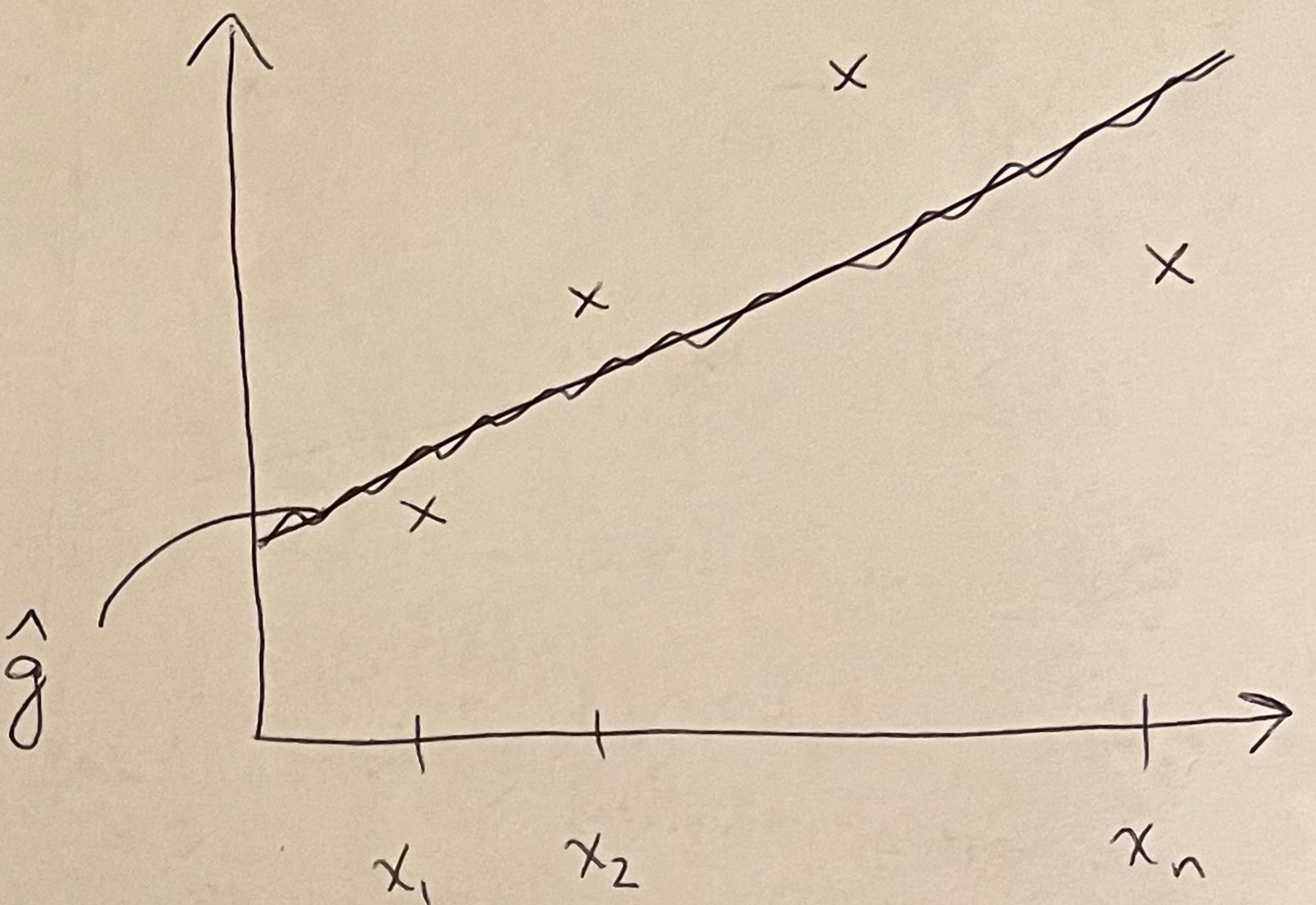
All that is left to do is minimize the first term:  $\sum_{i=1}^n (y_i - g(x_i))^2$ .

Assuming unwtl obs, this will be accomplished by setting

$\hat{g}$  equal to the unwtl avg of  $\{y_i\}$ , namely  $\hat{g} = \bar{y}$ .

(c.)  $\lambda = \infty; m = 2$

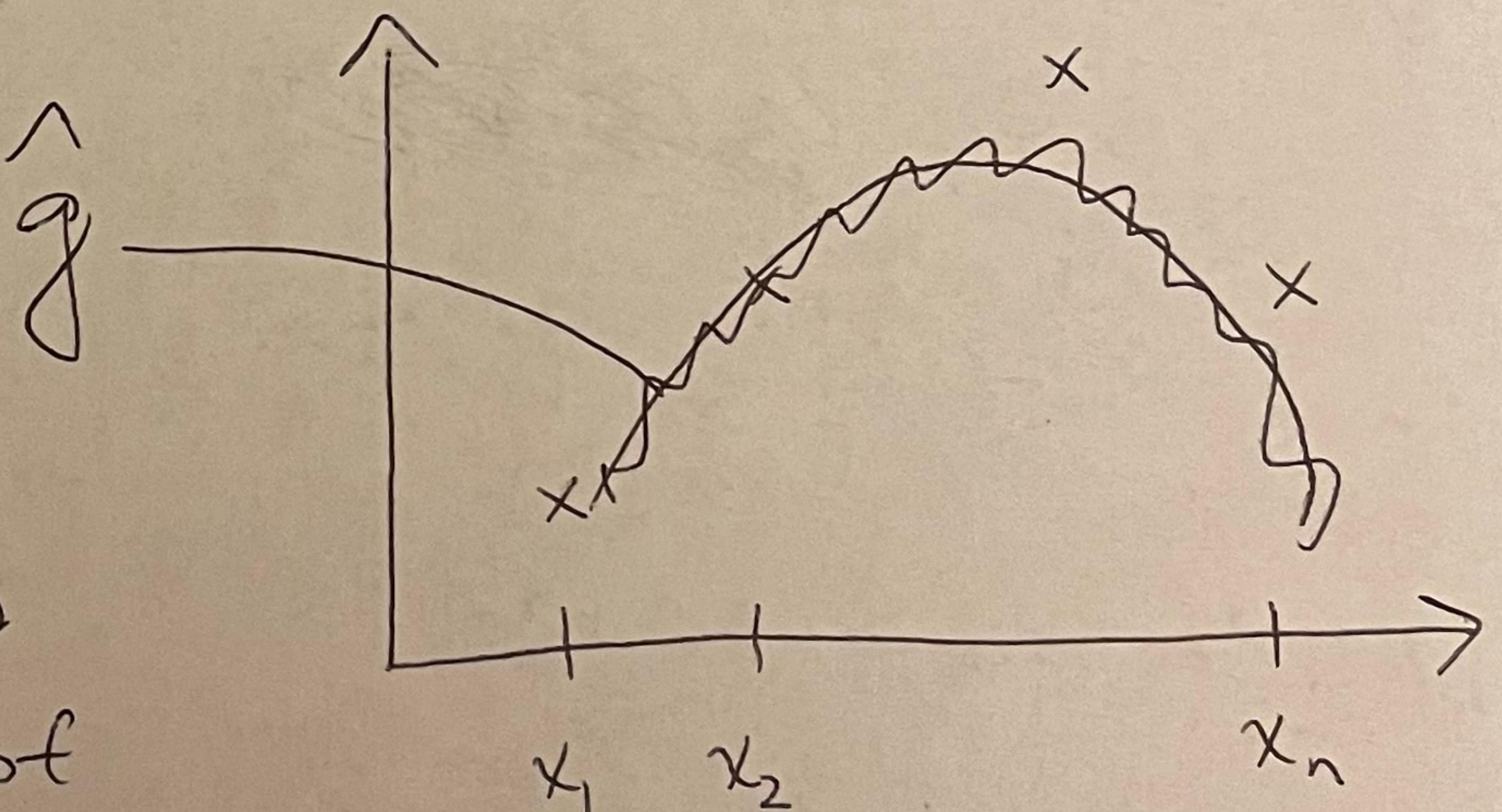
Soln:  $\hat{g}_i = \hat{\beta}_0 + \hat{\beta}_1 \cdot x_i$ , where  
 $(\hat{\beta}_0, \hat{\beta}_1)$  are OLS params from  
regression of  $\{y_i\}$  on  $\{x_i\}$ .



Reason: Again we can set the second term to 0 if we set  $g''=0$  globally. Any straight line (polynomial of order 0 or 1) will do this. Thus, all that is left to do is find the 0 or 1 order polynomial which minimizes  $\sum_i (y_i - g(x_i))^2$ , which is exactly the ordinary least squares regression line of  $\{y_i\}$  on  $\{x_i\}$ .

(d)  $\lambda = \infty; m = 3$ .

Soln:  ~~$\hat{g}(x_i) = \hat{\beta}_0 + \hat{\beta}_1 \cdot x_i + \hat{\beta}_2 \cdot x_i^2$~~ ,  
where  $(\hat{\beta}_0, \hat{\beta}_1, \hat{\beta}_2)$  are the params from the polynomial regression of  $y$  on  $(1, x, x^2)$ .



Reason: Again, set 2nd term to 0 if set  $g'''=0$  globally. Any polynomial of degree 0, 1, or 2 will have a 3rd derivative which is 0 globally, making 2nd term = 0. Thus, minimizing the first term will be accomplished by the polynomial regression of  $\{y_i\}$  on  $\{1, x_i, x_i^2\}$ .

(2) continued.

(e.)  $\lambda=0; m=3$ .

As  $\lambda=0$ , the second term = 0.

All that remains is to find  $\hat{g}$   
that minimizes the first term.

Obviously, this generally is a poor  
choice in modelling, but b/c there is no penalty/reason not to do,  
 $\hat{g}$  will be the polynomial (of degree at most  $(n-1)$ ) that  
perfectly interpolates the  $n$  points, thus setting the first  
term as well (and thus the whole expression) = 0.

It does not matter the value of  $m$ , as  $\lambda=0 \Rightarrow$  2nd term is 0;  
all we are trying to do here is minimize the train set SSE.

