6.) (a) when $p=1$, (6.12) becomes:

$$\chi = (y_1-\beta_1)^2 + \lambda\cdot\beta_1^2$$

$$\frac{\partial\chi}{\partial\beta_1} = 2(y_1-\beta_1)(-1)+\lambda\cdot 2\beta_1 = 0$$

$$-2y_1+2\beta_1+2\lambda\beta_1=0$$

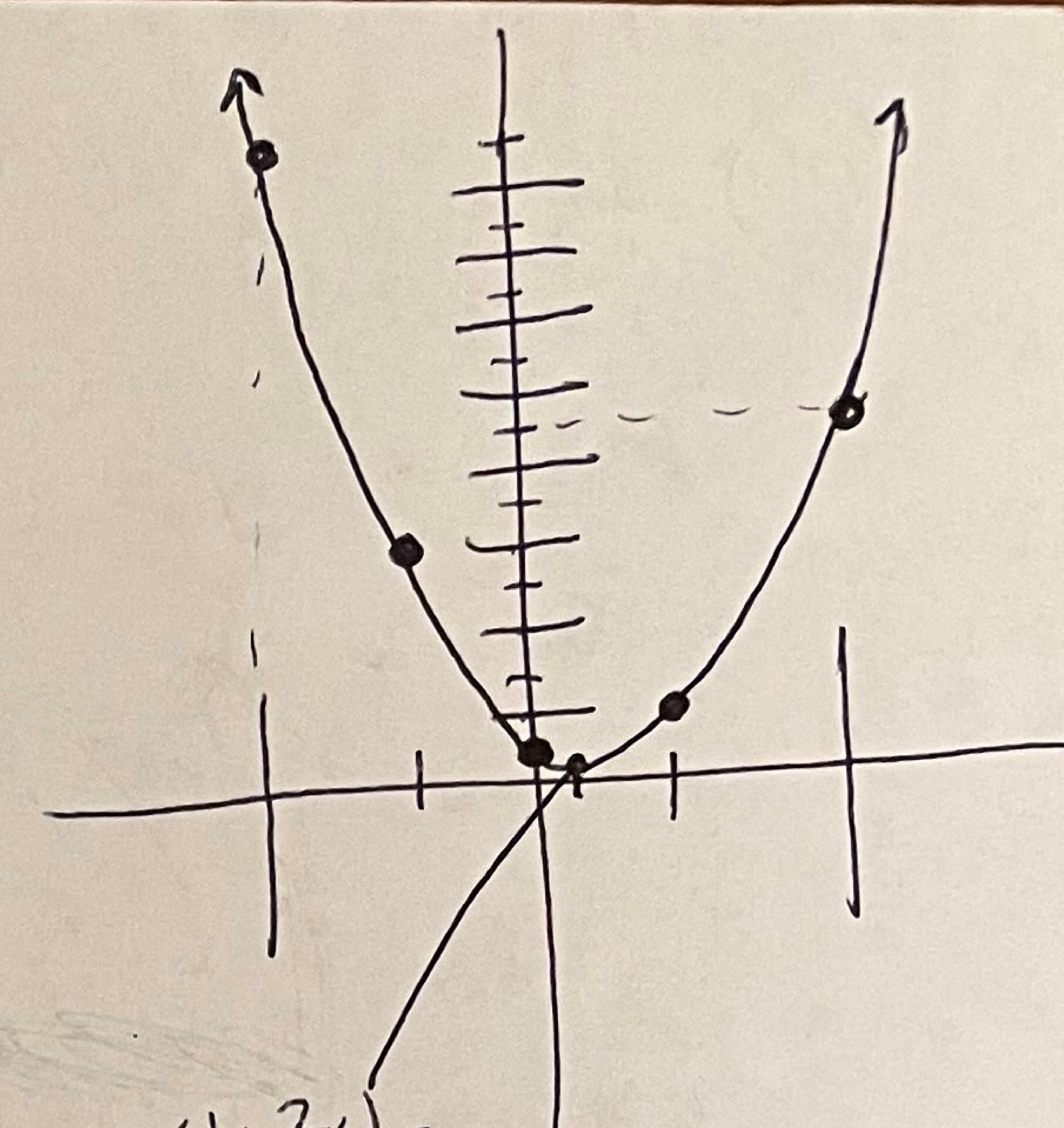
$$-2y_1+\beta_1(2\cancel{2}(1+\lambda))=0$$

$$-y_1+\beta_1(1+\lambda)=0$$

$$\beta_1(1+\lambda)=y_1 \Rightarrow \boxed{\hat\beta_1 = \frac{y_1}{(1+\lambda)}}$$

$= \frac{1}{1+2} = 1/3$ ✓

which shows (6.12) is solved by (6.14)



$(1/3, 2/3)$ = min

Let $\lambda=2$ and $y_1=1$, then

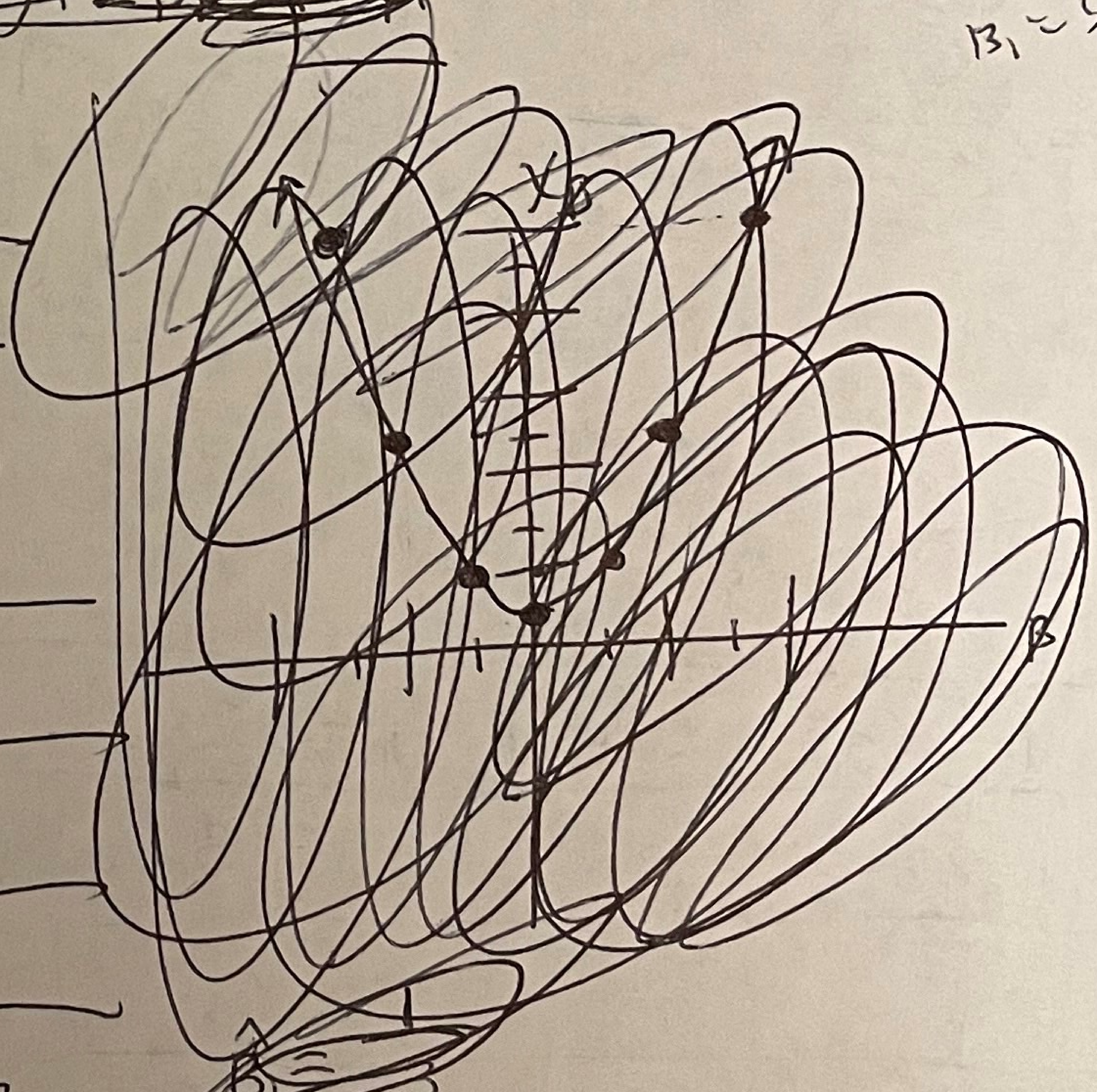
$X_\beta = (1-\beta_1)^2 + 2\beta_1^2 = (1-2\beta_1+\beta_1^2)+2\beta_1^2 = 1-2\beta_1+3\beta_1^2$

$-2+6\beta_1=0$

$6\beta_1=2$

$\beta_1=1/3$

β_1	X_β
-3	$1-2(-3)+3(9)=34$
-2	$=17$
-1	$=6$
0	$=1$
1	$=2$
2	$=9$
3	22



$X_{1/3} = 1-2(1/3)+3(1/3)^2 = 1-2/3+3(1/9) = 1-2/3+3/9 = 1-2/3+1/3$

$= 2/3$

(b) when $p=1$, (6.13) becomes:

$$X = (y_1-\beta_1)^2 + \lambda|\beta_1|$$

~~$\frac{\partial X}{\partial \beta_1} = 2(y_1-\beta_1)(-1) + \lambda \cdot \begin{cases}\beta_1 & \text{if } \beta_1>0 \\ -\beta_1 & \text{if } \beta_1<0\end{cases}$~~

~~$0 = -2y_1 + 2\beta_1 + \lambda \cdot (\ldots)$~~

~~$2y_1 = 2\beta_1 + \lambda \ldots$~~

~~If $\beta_1 \neq 0$, then~~

~~$2y_1 = 2\beta_1 + \lambda\beta_1$~~

~~$2y_1 = (2+\lambda)\beta_1$~~

~~$\hat{\beta}_1 = \frac{2y_1}{(2+\lambda)} = \frac{y_1}{1+\lambda/2}$~~

$$X = y_1^2 - 2\beta_1 y_1 + \beta_1^2 + \lambda|\beta_1|$$

Let $\lambda=2$ and $y_1=1$,

$$X_\beta = 1 - 2(1)\beta_1 + \beta_1^2 + 2|\beta_1|$$

$$X_\beta = 1 - 2\beta_1 + \beta_1^2 + 2|\beta_1|$$

Since

$|y_1| = \frac{\lambda}{2}$

$|1| = \frac{2}{2}$,

$\hat{\beta} = 0$

$$\frac{\partial X}{\partial \beta} = -2y_1 + 2\hat{\beta} + \frac{\hat{\beta}}{|\hat{\beta}|}\cdot\lambda$$

$$2y_1 = \hat{\beta}\left(2 + \frac{\lambda}{|\hat{\beta}|}\right)$$

which confirms (6.13) is solved by (6.15).

β	X_β
-3	$1-2(-3)+9+2\lvert -3\rvert = 22$
-2	$1-2(-2)+4+4 = 13$
-1	$= 6$
0	$= 1$
1	$= 2$
2	$= 5$
3	$= 10$