7.9 Excercises: Conceptual

1.) (a) Since $(x-y)_+^3 = 0$ when $x \le y$, then for $x \le y$, $f(x)$ simplifies to:

$$f(x) = \beta_0 + \beta_1 \cdot x + \beta_2 \cdot x^2 + \beta_3 \cdot x_3 \; ; \; x \le y$$

Thus, for $f(x) = f_1(x)$, we simply set:

$$\boxed{\begin{array}{l} a_1 = \beta_0 \\ b_1 = \beta_1 \\ c_1 = \beta_2 \\ d_1 = \beta_3 \end{array}}$$

$(x-y)(x-y) = x^2 - 2xy + y^2$

$(x^2-2xy+y^2)(x-y) =$

$x^3 - 2x^2y + xy^2 - yx^2 + 2xy^2 - y^3$

$x^3 - 3x^2y + 3y^2x - y^3$

(b) Now for $x > y$,

$$f(x) = \beta_0 + \beta_1 \cdot x + \beta_2 \cdot x^2 + \beta_3 \cdot x^3 + \beta_4 (x-y)^3$$

$$f(x) = \beta_0 + \beta_1 \cdot x + \beta_2 x^2 + \beta_3 x^3 + \beta_4 (x^3 - 3yx^2 + 3y^2 x - y^3)$$

$$f(x) = \beta_0 + \beta_1 x + \beta_2 x^2 + \beta_3 x^3 + \beta_4 x^3 - \beta_4 \cdot 3y x^2 + \beta_4 3y^2 x - \beta_4 y^3$$

$$f(x) = (\beta_0 - \beta_4 y^3) + (\beta_1 + \beta_4 \cdot 3y^2) x + (\beta_2 - 3\beta_4 y) x^2 + (\beta_3 + \beta_4) x^3$$

Now since $f_2(x) = a_2 + b_2 x + c_2 x^2 + d_2 x^3$,

Simply set:

$$\boxed{\begin{array}{ll} a_2 = (\beta_0 - \beta_4 \cdot y^3) \; ; & c_2 = (\beta_2 - 3\beta_4 y) \\ b_2 = (\beta_1 + 3\beta_4 y^2) \; ; & d_2 = (\beta_3 + \beta_4) \end{array}}$$

(c.) $f_1(\xi) = \beta_0 + \beta_1 \cdot \xi + \beta_2 \cdot \xi^2 + \beta_3 \cdot \xi^3$

$f_2(\xi) = (\beta_0 - \beta_4 \xi^3) + (\beta_1 + 3\beta_4 \xi^2) \cdot \xi + (\beta_2 - 3\beta_4 \cdot \xi) \cdot \xi^2 + (\beta_3 + \beta_4) \cdot \xi^3$

$= \beta_0 \cancel{- \beta_4 \xi^3} + \beta_1 \cdot \xi \cancel{+ 3\beta_4 \cdot \xi^3} + \beta_2 \cdot \xi^2 \cancel{- 3\beta_4 \cdot \xi^3} + \beta_3 \cdot \xi^3 \cancel{+ \beta_4 \cdot \xi^3}$

$= \beta_0 + \beta_1 \cdot \xi + \beta_2 \cdot \xi^2 + \beta_3 \cdot \xi^3$

Thus, $f_1(\xi) = f_2(\xi) \Rightarrow f(x)$ is continous at ξ. ∎

~~(d.) $f_1'(x) = \beta_1 + 2\beta_2 \xi + 3\beta_3 \cdot \xi^2$~~

~~$f_1'(\xi) = \beta_1 + 2\beta_2$~~

(d.) $f_1'(x) = \beta_1 + 2\beta_2 \cdot x + 3\beta_3 x^2$

$\underline{f_1'(\xi) = \beta_1 + 2\beta_2 \cdot \xi + 3\beta_3 \cdot \xi^2}$

$f_2'(x) = \beta_1 + 3\beta_4 \xi^2 + (2\beta_2 - 6\beta_4 \xi) \cdot x + (3\beta_3 + 3\beta_4) \cdot x^2$

$f_2'(x) = \beta_1 + 3\beta_4 \xi^2 + 2\beta_2 x - 6\beta_4 \xi x + 3\beta_3 x^2 + 3\beta_4 x^2$

$f_2'(\xi) = \beta_1 + 3\beta_4 \xi^2 + 2\beta_2 \xi - 6\beta_4 \xi^2 + 3\beta_3 \xi^2 + 3\beta_4 \xi^2$

$\underline{f_2(\xi) = \beta_1 + 2\beta_2 \xi + 3\beta_3 \xi^2}$

Thus, $f_1'(\xi) = f_2'(\xi) \Rightarrow f'(x)$ is continous at ξ. ∎

(1) continued.

(e.) $f_1''(x) = 2\beta_2 + 3(2)\beta_3 \cdot x$

$f_1''(\xi) = 2\beta_2 + 6\beta_3 \cdot \xi$

$f_2''(x) = 2\beta_2 - 6\beta_4 \cdot \xi + 6\beta_3 x + 6\beta_4 x$

$f_2''(\xi) = 2\beta_2 - 6\beta_4 \xi + 6\beta_3 \cdot \xi + 6\beta_4 \cdot \xi$

$f_2''(\xi) = 2\beta_2 + 6\beta_3 \cdot \xi$

Thus, $f_1''(\xi) = f_2''(\xi) \Rightarrow f''(x)$ is continous at ξ. ∎