5.) Since int=0, don't estimate it.

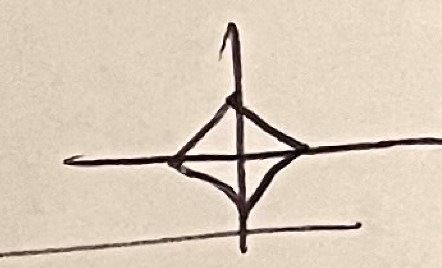
(a) $$\underset{\beta_1,\beta_2}{\arg\min} \sum_{i=1}^{2} (y_i - \beta_1 x_{i1} - \beta_2 x_{i2})^2 + \lambda \cdot \sum_{j=1}^{2} \beta_j^2$$

$$I = \underset{\beta_1,\beta_2}{\arg\min} (y_1 - \beta_1 \cdot x_{11} - \beta_2 \cdot x_{12})^2 + (y_2 - \beta_1 \cdot x_{21} - \beta_2 \cdot x_{22})^2 + \lambda\beta_1^2 + \lambda\beta_2^2$$

(b)

$$\frac{\partial I}{\partial \beta_1} = 2(y_1 - \beta_1 \cdot x_{11} - \beta_2 x_{12}) \cdot x_{11} + 2(y_2 - \beta_1 x_{21} - \beta_2 x_{22}) x_{21} + 2\lambda\beta_1$$

$(Y - X\beta)^T(Y - X\beta)$



$$0 = 2(y_1 + y_2) - 2\beta_1(x_{11} + x_{21}) - 2\beta_2(x_{12} + x_{22}) + 2\lambda\beta_1$$

$y_1 + y_2 = 0$

$0 = 2\lambda\beta_1$ analogous for $\beta_2 \Rightarrow$ $0 = 2\lambda\beta_2$

$x_1\beta_1 + x_1\beta_2$

$2\lambda\beta_1 = 2\lambda\beta_2$

$x_{11}\beta_1 + x_{12}\beta_2 +$

$x_1(\beta_1 + \beta_2) + x_2(\beta_1 + \beta_2) = 0$

$\hat{\beta}_1 = \hat{\beta}_2$

$x_1 =$

(c) $$\underset{\beta_1,\beta_2}{\arg\min} \sum_{i=1}^{2} (y_i - \beta_1 \cdot x_{i1} - \beta_2 x_{i2})^2 + \lambda \cdot \sum_{j=1}^{2} |\beta_j|$$

$$J = \underset{\beta_1,\beta_2}{\arg\min} \underbrace{(y_1 - \beta_1 x_{11} - \beta_2 x_{12})^2 + (y_2 - \beta_1 x_{21} - \beta_2 x_{22})^2}_{c} + \lambda(|\beta_1| + |\beta_2|)$$

~~(d)~~ $\cancel{\frac{\partial J}{\partial \beta_1} = 2(y_1 - \beta_1 x_{11} - \beta_2 x_{12}) + 2(y_2 - \beta_1 x_{21} - \beta_2 x_{22})}$

(5b)

$$\frac{\partial I}{\partial \beta_1} = -2(y_1 - \beta_1 x_{11} - \beta_2 x_{12})x_{11} - 2(y_2 - \beta_1 x_{21} - \beta_2 x_{22})x_{21} + 2\lambda\beta_1$$

$$0 = -y_1 x_{11} + \beta_1 x_{11}^2 + \beta_2 x_{12} x_{11} - y_2 x_{21} + \beta_1 x_{21}^2 + \beta_2 x_{22} x_{21} + \lambda \beta_1$$

$$\beta_1(x_{11}^2 + x_{21}^2 + \lambda) = y_1 x_{11} - \beta_2 x_{12} x_{11} + y_2 x_{21} - \beta_2 x_{22} x_{21}$$

$$\boxed{\hat{\beta}_1 = \frac{(y_1 x_{11} - \beta_2 x_{12} x_{11} + y_2 x_{21} - \beta_2 x_{22} x_{21})}{(x_{11}^2 + x_{21}^2 + \lambda)}} = \boxed{\frac{y_1 x_1 + y_2 x_2 - \hat{\beta}_2(x_1^2 + x_2^2)}{(x_1^2 + x_2^2 + \lambda)}}$$

symmetry suggests they are equivalent.

✓

$$\frac{\partial I}{\partial \beta_2} = \dots \Rightarrow \hat{\beta}_2 = \frac{y_1 x_1 + y_2 x_2 - \hat{\beta}_1(x_1^2 + x_2^2)}{(x_1^2 + x_2^2 + \lambda)}$$

this as well per Prince Honest blog referenced in 5d

~~Easier:~~ $\cancel{y_1 = x_{11}\beta_1 + x_{12}\beta_2}$ $\cancel{y_1 = x_1(\beta_1 + \beta_2)}$

$\cancel{y_2 = x_2(\beta_1 + \beta_2)}$

$\cancel{y_1 + y_2 = (x_1 + x_2)(\beta_1 + \beta_2)}$

$\cancel{0 =}$

(5d)

https://blog.princehonest.com/stat-learning/ch6/5.html

~~For $\beta_1, \beta_2 > 0$, then~~

$$\cancel{\frac{\partial J}{\partial \beta_1} = 2(y_1 - \beta_1 x_{11} - \beta_2 x_{12})(-x_{11}) + 2(y_2 - \beta_1 x_{21} - \beta_2 x_{22})(-x_{21}) + \lambda}$$

$$\cancel{\frac{\partial J}{\partial \beta_1} = -2y_1 x_{11} + 2\beta_1 x_{11}^2 + 2\beta_2 x_{11} x_{12} - 2y_2 x_{21} + 2\beta_1 x_{21}^2 + 2\beta_2 x_{22} x_{21} + \lambda}$$

Above per Prince Honest blog, sample soln:

Geom interpretation: $|\hat{\beta}_1| + |\hat{\beta}_2| < s$, subject to constraint C above in (c), simplified to:

min $2 \cdot (y_1 - (\beta_1 + \beta_2) x_{11})^2$

Simple soln: $\hat{\beta}_1 + \hat{\beta}_2 = \frac{y_1}{x_{11}}$, ∥ to ◇ $\beta_1 + \beta_2 = s$

Solns are contours of that touch Lasso ◇: $\beta_1 + \beta_2 = s$.

As β_1 and β_2 vary along the line they touch the LASSO diamond at diff points, thus entire edge $\beta_1 + \beta_2 = s$ is a potential soln to LASSO problem.

→ Similarly for $\beta_1 + \beta_2 = -s$, $\beta_1 \le 0$; $\beta_2 \le 0$

→ thus gen form of soln given by:

$\hat{\beta}_1 + \hat{\beta}_2 = s$ for $\hat{\beta}_1, \hat{\beta}_2 \ge 0$ AND

$\hat{\beta}_1 + \hat{\beta}_2 = -s$ for $\hat{\beta}_1, \hat{\beta}_2 \le 0$.