

7.9 Exercises: Conceptual

1.) (a) Since $(x-\gamma)_+^3 = 0$ when $x \leq \gamma$, then for $x \leq \gamma$, $f(x)$

simplifies to:

$$f(x) = \beta_0 + \beta_1 \cdot x + \beta_2 \cdot x^2 + \beta_3 \cdot x^3 ; x \leq \gamma$$

Thus, for $f(x) = f_1(x)$, we simply set:

$$\begin{cases} a_1 = \beta_0 \\ b_1 = \beta_1 \\ c_1 = \beta_2 \\ d_1 = \beta_3 \end{cases}$$

(b) Now for $x > \gamma$,

~~$$f(x) = \beta_0 + \beta_1 \cdot x + \beta_2 \cdot x^2 + \beta_3 \cdot x^3 + \beta_4 (x-\gamma)^3$$~~

$$(x-\gamma)(x-\gamma) = x^2 - 2x\gamma + \gamma^2$$

$$(x^2 - 2x\gamma + \gamma^2)(x-\gamma) =$$

$$x^3 - 2x^2\gamma + x^2\gamma^2 - \gamma x^2 + 2x\gamma^2 - \gamma^3$$

$$f(x) = \beta_0 + \beta_1 \cdot x + \beta_2 \cdot x^2 + \beta_3 \cdot x^3 + \beta_4 (x^3 - 3\gamma x^2 + 3\gamma^2 x - \gamma^3)$$

$$f(x) = \underline{\beta_0 + \beta_1 x + \beta_2 x^2 + \beta_3 x^3} + \underline{\beta_4 x^3} - \underline{\beta_4 \cdot 3\gamma x^2} + \underline{\beta_4 3\gamma^2 x} - \underline{\beta_4 \gamma^3}$$

$$f(x) = (\beta_0 - \beta_4 \gamma^3) + (\beta_1 + \beta_4 \cdot 3\gamma^2) x + (\beta_2 - 3\beta_4 \gamma) x^2 + (\beta_3 + \beta_4) x^3$$

Now since $f_2(x) = a_2 + b_2 x + c_2 x^2 + d_2 x^3$,

simply set:

$$a_2 = (\beta_0 - \beta_4 \cdot \gamma^3) ; \quad c_2 = (\beta_2 - 3\beta_4 \gamma)$$

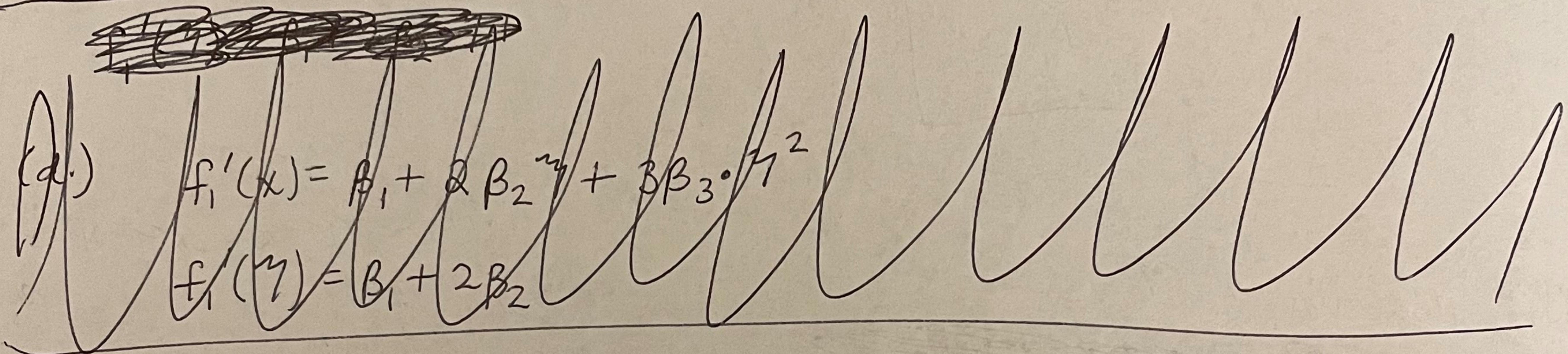
$$b_2 = (\beta_1 + 3\beta_4 \gamma^2) ; \quad d_2 = (\beta_3 + \beta_4)$$

$$(c.) \quad f_1(\gamma) = \beta_0 + \beta_1 \cdot \gamma + \beta_2 \cdot \gamma^2 + \beta_3 \cdot \gamma^3$$

~~(d)~~

$$\begin{aligned} f_2(\gamma) &= (\beta_0 - \beta_4 \gamma^3) + (\beta_1 + 3\beta_4 \gamma^2) \cdot \gamma + (\beta_2 - 3\beta_4 \gamma) \cdot \gamma^2 + (\beta_3 + \beta_4) \cdot \gamma^3 \\ &= \beta_0 - \beta_4 \gamma^3 + \beta_1 \cdot \gamma + 3\beta_4 \cdot \gamma^3 + \beta_2 \cdot \gamma^2 - 3\beta_4 \cdot \gamma^3 + \beta_3 \cdot \gamma^3 + \beta_4 \cdot \gamma^3 \\ &= \beta_0 + \beta_1 \cdot \gamma + \beta_2 \cdot \gamma^2 + \beta_3 \cdot \gamma^3 \end{aligned}$$

Thus, $f_1(\gamma) = f_2(\gamma) \Rightarrow f(x)$ is continuous at γ . 



$$(d.) \quad f_1'(x) = \beta_1 + 2\beta_2 \cdot x + 3\beta_3 x^2$$

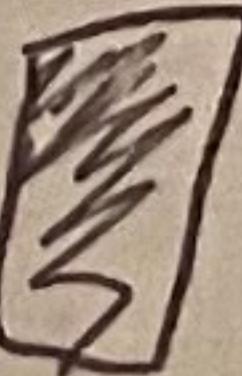
$$f_1'(\gamma) = \underline{\beta_1 + 2\beta_2 \cdot \gamma + 3\beta_3 \cdot \gamma^2}$$

$$f_2'(x) = \beta_1 + 3\beta_4 \gamma^2 + (2\beta_2 - 6\beta_4 \gamma) \cdot x + (3\beta_3 + 3\beta_4) \cdot x^2$$

~~$f_2'(x) = \beta_1 + 3\beta_4 \gamma^2 + 2\beta_2 x - 6\beta_4 \gamma x + 3\beta_3 x^2 + 3\beta_4 x^2$~~

$$f_2'(\gamma) = \beta_1 + 3\beta_4 \gamma^2 + 2\beta_2 \gamma - 6\beta_4 \gamma^2 + 3\beta_3 \gamma^2 + 3\beta_4 \gamma^2$$

$$f_2(\gamma) = \underline{\beta_1 + 2\beta_2 \gamma + 3\beta_3 \gamma^2}$$

Thus, $f_1'(\gamma) = f_2'(\gamma) \Rightarrow f'(x)$ is continuous at γ . 

(1) continued.

(e.) $f_1''(x) = 2\beta_2 + 3(2)\beta_3 \cdot x$

$f_1''(\gamma) = 2\beta_2 + 6\beta_3 \cdot \gamma$

$f_2''(x) = 2\beta_2 - 6\beta_4 \cdot \gamma + 6\beta_3 x + 6\beta_4 x$

$f_2''(\gamma) = 2\beta_2 - 6\beta_4 \gamma + 6\beta_3 \gamma + 6\beta_4 \gamma$

$f_2''(\gamma) = 2\beta_2 + 6\beta_3 \cdot \gamma$

Thus, $f_1''(\gamma) = f_2''(\gamma) \Rightarrow f''(x)$ is continuous at γ . 