

A close-up portrait of a man with dark hair and a beard, smiling broadly. He is wearing a light-colored button-down shirt. The background is blurred, showing what appears to be a brick wall or a similar textured surface.

Michael Clausen

2. a. yes  
b. yes  
c. yes  
d. no  
e. yes  
f. yes

3. a. yes  
b. no  
c. yes  
d. yes  
e. no

$$4. a. P \rightarrow (Q \rightarrow R) \Rightarrow P \rightarrow (\neg Q \vee R)$$

$$\Rightarrow \neg P \vee (\neg Q \vee R) \Rightarrow \neg P \vee \neg Q \vee R$$

1.  $\neg P \vee \neg Q \vee R$

$$b. \neg(P \rightarrow R) \Rightarrow \neg(\neg P \vee R) \Rightarrow \neg\neg P \wedge \neg R$$

$$\Rightarrow P \wedge \neg R$$

1.  $P$   
2.  $\neg R$

$$c. P \rightarrow (R \wedge S) \Rightarrow \neg P \vee (R \wedge S) \Rightarrow (\neg P \vee R) \wedge$$

$$(\neg P \vee S)$$

1.  $\neg P \vee R$   
2.  $\neg P \vee S$

$$d. \neg(Q \vee R) \Rightarrow \neg Q \wedge \neg R$$

1.  $\neg Q$   
2.  $\neg R$

$$\begin{aligned}
 e. P \rightarrow (Q \vee (R \rightarrow S)) &\rightarrow P \rightarrow (Q \vee (\neg R \vee S)) \\
 \Rightarrow \neg P \vee (Q \vee (\neg R \vee S)) &\Rightarrow \neg P \vee Q \vee \neg R \vee S
 \end{aligned}$$

1.  $\neg P \vee Q \vee \neg R \vee S$

5. a.  $\neg P \vee Q \vee R, P \vee Q \vee R$

$\neg P \vee Q \vee R$

b.  $\neg P \vee Q \vee R, P \vee Q \vee S$

$\neg P \vee Q \vee R \vee S$

c.  $\neg P \vee Q \vee R, \neg R$

cannot be resolved

6. a.i.  $Q \rightarrow \neg P$   
 $Q \vee R$   
 $\neg R$   
 $\therefore \neg P$

Valid

Q	P	R	$Q \rightarrow \neg P$	$Q \vee R$	$\neg R$	$\neg P$
T	T	T	F	T	F	
T	T	F	F	T	T	
T	F	T	T	T	F	
T	F	F	T	T	F	T
F	T	T	T	T	F	
F	T	F	T	T	T	J
F	F	T	T	F	F	
F	F	F	T	F	T	

Q	P	$Q \rightarrow \neg P$
T	T	F
T	F	T
F	T	T
F	F	T

# Solve a propositional formula:

[help](#)
$$(Q \rightarrow \neg P) \wedge (Q \vee R) \wedge \neg R \wedge P$$

Solve using truth table: better showing console trace ?

Build a truth table ?

Generate a problem of type random 3-sat for 10 variables. Clear Browse... No file selected. ?

More problems: [satlib](#), [competitions](#)

## Result

Clause set is **false** for all possible assignments to variables.

b. i.

$$\frac{1. P \rightarrow (Q \rightarrow R) \\ 2. Q}{3. P \rightarrow R}$$

CNF

$$\frac{1. \neg P \vee \neg Q \vee R \\ 2. Q}{3. \neg P \vee R}$$

Negate conclusion

$$\frac{1. \neg \neg P \vee \neg Q \vee R \\ 2. Q \\ 3. P \quad \text{D} \\ 4. \neg R}{\neg(\neg P \vee R) \\ P \wedge \neg R}$$

Apply Resolution

$$\frac{1. \neg Q \vee R \quad [\text{unities } 1, 3] \\ 2. Q \\ 3. \neg R}{\perp}$$

$$\frac{1. R \\ 2. \neg R}{\perp} \quad [\text{unities } 1, 2]$$

Valid

# Solve a propositional formula:

[help](#)

$(P \rightarrow (Q \rightarrow R)) \& Q \& \neg(P \rightarrow R)$

[Solve](#)

using

truth table: better

showing

console trace

?

[Build](#)

a

truth table

?

Generate a problem

of type

random 3-sat

for

10

variables.

[Clear](#)[Browse...](#)

No file selected.

?

More problems: [satlib](#), [competitions](#)

## Result

Clause set is **false** for all possible assignments to variables.

c. i.

$$\begin{array}{l} 1. \neg P \vee Q \\ 2. P \rightarrow (R \wedge S) \\ 3. S \rightarrow Q \\ \hline 4. Q \vee R \end{array}$$

CNF

$$\begin{array}{l} 1. \neg P \vee Q \\ 2. (\neg P \vee R) \wedge (\neg P \vee S) \\ 3. \neg S \vee Q \\ \hline 4. Q \vee R \end{array} \quad \begin{array}{l} \neg P \vee (R \wedge S) \\ (\neg P \vee R) \wedge (\neg P \vee S) \end{array}$$

Negate conclusion

$$\begin{array}{l} 1. \neg P \vee Q \\ 2. (\neg P \vee R) \wedge (\neg P \vee S) \\ 3. \neg S \vee Q \\ \hline 4. \neg Q \wedge \neg R \end{array} \quad \begin{array}{l} \neg (Q \vee R) \\ \neg Q \wedge \neg R \end{array}$$

Resolution

$$\begin{array}{l} 1. \neg P \vee Q \\ 2. \neg P \vee R \\ 3. \neg P \vee S \\ 4. \neg S \vee Q \\ 5. \neg Q \\ 6. \neg R \end{array}$$

Invalid, cannot resolve any two sentences.

# Solve a propositional formula:

[help](#)
$$(\neg P \vee Q) \wedge (P \rightarrow (R \wedge S)) \wedge (S \rightarrow Q) \wedge \neg(Q \vee R)$$

Solve using truth table: better showing console trace ?

Build a truth table ?

Generate a problem of type random 3-sat for 10 variables. Clear Browse... No file selected. ?

More problems: [satlib](#), [competitions](#)

## Result

Clause set is **true** if we assign values to variables as:  $\neg P \neg Q \neg R \neg S$

8.a.

- 1. AVB
- 2. 7AVC
- 3. 7BVD
- 4. 7CVG
- 5. 7DVG
- 6. 7G

- 1. BVC [Res 1,2]
- 2. 7BVD
- 3. 7CVG
- 4. 7DVG
- 5. 7G

- 1. CVD [Res 1,2]
- 2. 7CVG
- 3. 7DVG
- 4. 7G

- 1. DVG [Res 1,2]
- 2. 7DVG
- 3. 7G

- 1. G [Res 1,2]
- 2. 7G

1

b.  $(2^n C 2) + n$  clauses

c. Per part b, the maximum number of clauses to be resolved given  $n$  propositions is  $(2^n C 2) + n$ .

Per question 7a, the length of the result clause when resolving two 2-CNF clauses is either one or two. As such, resolution for 2-CNF decreases the number of clauses by one until either a contradiction is reached or no more clauses can be resolved, which leaves at most two clauses remaining. This makes the worst case runtime  $O(2^n C 2 + n - 1)$  which is polynomial in  $n$ .

d. The argument in c does not apply for 3-CNF since the result clause after resolving two clauses of length three increases to four. Clauses longer than two increase in size after resolution. Therefore, the runtime for resolution could be much larger than the input size.