VAnishing DOmino pRoblem - VaDoR

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Contents

1 Problem description

Vanishing domino problem. Full definition is available in the document provided by laboratories supervisor. From now on, the problem will be referred to as VaDoR.

2 Description of the input/output formats

2.1 Input

2.1.1 Text file

Described in the documentation provided by the laboratories supervisor.

2.1.2 Our own format

XML file.

```
<domino_board width="5" height="4">
    <piece x="0" y="0" orientation="vertical" value1="2" value2="1" />
    <piece x="0" y="2" orientation="vertical" value1="3" value2="2" />
    <piece x="1" y="0" orientation="horizontal" value1="1" value2="0" />
    <piece x="1" y="1" orientation="vertical" value1="3" value2="0" />
    <piece x="1" y="3" orientation="horizontal" value1="0" value2="0" />
    <piece x="2" y="1" orientation="vertical" value1="1" value2="1" />
    <piece x="3" y="0" orientation="horizontal" value1="3" value2="3" />
    <piece x="3" y="1" orientation="vertical" value1="0" value2="2" />
    <piece x="3" y="3" orientation="horizontal" value1="2" value2="2" />
    <piece x="4" y="1" orientation="vertical" value1="4" value2="4" />
</domino_board>
```

Description of the `piece` tag:

- x - coordinate, from zero, increasing from the right to the left side of the board
- y - coordinate, from zero, increasing from the top to the bottom side of the board
- orientation:
  - *horizontal* - the piece starts at (x, y) and ends at $(x + 1, y)$
  - *vertical* - the piece starts at (x, y) and ends at $(x, y + 1)$
- value1 - value of the beginning of the piece i.e. value at (x, y)
- value2 - value of the end of the piece i.e. location depends on the orientation

If a tag `removed_pieces` is present inside the `domino_board` tag, it is ignored. If an attribute `order` is attached to the `piece` tag, it is also ignored. That way the output file can be used as an input, and pieces can be rearanged and put back on the board using a text editor, without worrying about problems with parsing.

2.2 Output

Output is given either as output in command line, or as a graphical representation of a final state of the board in GUI. Output format depends on how user launched the application.

3 Description of the accurate algorithm

3.1 Class variables

Main class of the program is called domino_problem. Rough list of fields of the class:

```
# size_t is an integral type
# elements_t is a list of domino pieces
# board_t is a two-dimensional array of references to the halves of domino pieces
   size_t width
   size_t height
   # collection of pieces
   elements_t elements
   # collection of halves of pieces that come from 'elements' field
   board_t board
   # pieces currently on the board
   elements_t on_board
   # possible to remove in the next turn
   elements_t possible
   # not longer on board, removed in the previous turns
   elements_t removed
   # algorithm does not know anything about these pieces
   elements_t unresolved
   # possible to remove if other pieces are placed right
   elements_t checked
   # impossible to remove due to size of the board and/or dependencies on other invalid pieces
   elements_t invalid
```

There is also a derived class, called domino_problem_solver, which contains several other variables, which store data about every state analyzed by the algorithm.

```
# state_list_t is a list of states (domino_problem-s)
# dag_t is a directed acyclic graph
# hash_t is a radix-based hashing table for integral types
   # contains unexplored states
   state_list_t unexplored
   # contains current tree of states (some explored, some not) connected
   #  according to the rules given below
   dag_t tree
   # contains explored states
   hash_t hashed
   # reference to currently best known state (state with max. number of removed pieces)
   domino_problem best_state
```

3.2 Algorithm

Below is an outline of the algorithm for deterministic RAM:

```
   # 1. read input data
   width = from input
   height = from input
   elements = all pieces present on board # this list does not change with time
   # board is generated using 3 above variables
   board = generate_board(elements, width, height)
   # 'elements' are copied into 'on_board' and 'unresolved'
   on_board = unresolved = elements
   # 'unresolved' are partitioned into 'checked' and 'invalid'
   resolve(unresolved, checked, invalid)
```

```
# current state is copied from initial data of the problem, and it is put into the tree,
#  becoming its root
domino_problem current_state = self
tree += current_state
unexplored += current_state
best_state = current_state
# all pieces that can be removed in current state are found,
#  by analyzing the 'checked' list
find_possible(current_state)

# 2. build the tree of states
while unexplored is not empty do
   domino_problem s = unexplored.last_element
   unexplored -= s

   state_list_t new_states
   for element e in s.possible
      # make a copy of 's', with the exception of list of possible pieces
      domino_problem pr = s
      # from copy of current domino\_problem, the corresponding piece is added to 'removed',
      # and the same piece is deleted from 'checked' and 'on\_board'
      pr.on_board -= e
      pr.checked -= e
      pr.removed += e
      find_possible(pr)
      new_states += pr
   end

   # sort new_states ascending, using length of list of 'possible' as criteria
   sort(new_states)

   # add all new states to 'tree', with exception of those that already exist
   # in 'tree', because removing first piece A and then B gives the same resulting state
   # as removing first piece B and then A. Calculating all possibilities with duplicates would
   # result in factorial complexity of the algorithm (n!).
   for domino_problem st in new_states
      if hashed does not contain st
         hashed += st
         unexplored += st
         # add the new state to 'tree', as a child of 's'
         add_child(tree, s, st)
      end
   end

   if best_state.on_board.length > s.on_board.length then
      best_state = s
      if best_state.on_board.length == invalid.length then
         break
   end

   # this optimization causes complexity to go beyond 2^n, but it is used only if
   # algorithm would have ended computation otherwise due to no memory
   if running out of memory then
      hashed.remove_all
   end
end
```

```

    # 3. return final result
    return best_state
```

3.3 Estimation of complexity

3.3.1 Number of states

Let us assume that we have a set of n domino pieces that are arranged on a board. Then, let us assume that we are taking away pieces from the board, one at a time. If we consider each state after removing a piece, how many different states do we have? For a given state, for each piece, we have a choice: the piece either is on the board, or it is not. In total we have 2 possible scenarios for one piece, and n pieces, which gives us exactly 2^n different states.

3.3.2 Relations between the states

We can construct a following directed acyclic graph (DAG) G:

1. each vertex V represents a state (configuration of pieces on the board)
2. each directed edge E represents the relation between states. Edges are positioned in such way, that start point is at state V_1, and such that the end point of the edge is connected to the state V_2 that can be obtained directly from state V_1 by removing one domino piece from the board.

3.3.3 Proof that VaDoR is NP

Below, outline of a polynomial-time algorithm for a non-deterministic RAM:

```
input: b # list of elements on board

var r # list of removed elements, initially empty

for each element e in b
    # magical 'if' that knows when putting current element to the list
    #  is the best possible option - such 'if-better' is allowed for non. det. RAMs
    if-better
        add e to r
    else
        next

return r
```

3.3.4 Final estimation of complexity

Having a DAG G, the domino problem is equivalent to finding the longest path of DAG. Unfortunately, graph has to be constructed at runtime, and that takes time.

Since, for each state, the calculation of related states will be performed (and there are 2^n states in total) I estimate the complexity of accurate algorithm as exponential. Due to this maximum number of states, I also informally classify this algorithm as NP-complete, however, it is only my intuition.

After the DAG is constructed, by finding its longest path, an optimal solution is obtained. But, because the DAG is not given as input, but rather constructed during algorithm's operation, finding longest path can be done while constructing the DAG, therefore no extra time is needed for it.

My estimations indicate that calculation of possible sub-states for a given single state takes polynomial time. Therefore, the total complexity of the accurate algorithm should be roughly 2^n In practice, however, I have noticed that it is not such, because problems encountered in practice tend to have far less possible choices, due to strong inter-piece dependencies.

4 Description of the approximate algorithms

4.1 Approximation by Mateusz Bysiek

4.1.1 Class variables

The same as in case of accurate algorithm.

4.1.2 Algorithm

Development of approximations of NP problems is usually a consequence of deep understanding of the problem, and extensive experience in solving it using some kind of accurate algorithm.

During my experiments, I have discovered that depth-first construction/search of DAG is far more efficient than breadth-first, when it comes to actual time needed to find the best result. It is so because we need to find any best result, not all of them. In case of breadth-first,

I have put all above optimizations into the accurate algorithm, and I have created approximate one by simply limiting number of possible options to be explored. I have limited number of explored options to n^2, where n is number of elements initially on board. In short, after line 60, I added this:

```
if hashed.element_count > square(elements.length)
  break
```

4.1.3 Optimizations

I have also noticed that when I use depth-first search in such way as described in the accurate algorithm's pseudo-code, the tree is searched from 'right' to 'left', and when algorithm is analyzing any state x, that means that all states to the right of it are already explored.

I have used this to develop optimization which removes obsolete explored branches, i.e. those branches that do not contain currently known best state. This greatly reduced memory usage.

I have also noticed that list of pieces currently on board can be easily converted to integral type (if number of pieces initially on board is less than maximum number of bits stored in that type). Keeping all lists of pieces (with exception of the 'reference' list used to convert them back) as integers also greatly decreased memory usage, and allowed creation of a very cheap (in terms of time and memory) hash table (used as the field 'hashed'), which has constant access/storage/search time equal to $\log(k)$ where k is number of pieces initially on board.

The above (and some other, minor) optimizations caused the algorithm to be able to solve problems with at most 64 not-invalid pieces. This means that if length of 'checked' exceeds 64, the program will crash, when either accurate algorithm, or approximate algorithm created by me (M. Bysiek) is used.

4.1.4 Complexity

Due to hard limit of checking at most n^2 states, and assuming that checking each state takes polynomial time, such approximate algorithm is also polynomial.

4.2 Approximation by Stanisław Peryt

4.2.1 Input Data

collection of domino pieces from input file

4.2.2 Description and Class Variables

This algorithm is based on the idea of Greedy algorithms. I assume some additional classification which is done only once for each element. For classification purposes each dominoPiece should contain some additional fields:

```
boolean isIndependent; //can be removed even from empty board, so it does not have to
   // be removed immediately
boolean canBeRemoved; //initially false. Changes to true, when fulfills removal conditions
list<waitingFor>; // list of pieces for which it is waiting;
list<waitForMe>; // list of pieces waiting for it;
```

I decided to add this classification, because in greedy strategy, we could remove pieces immediately when they fulfill removal requirements. But it may happen, that there are some pieces, for which pieces we already removed are necessary to fulfill removal requirements. When we have this classification based on scopes of domino pieces, we can wait with removing, and eliminate at least some of those situations described above, providing better local optimal choices.

For this algorithm we also need: board – contains all domino pieces that have remained on the board roots – contains possible starting pieces, i.e. containing zero neighbors – contains domino pieces, which are neighbors of deleted piece, i.e. pieces alongside empty fields

4.2.3 Algorithm

```
1. Create board from input
2. Find possible roots and assign them to neighbors
3. for each dominoPiece in board assign scopes and waiting property
   (a) determine scopes based on numbers given on dominoPiece
   (b) determine dominoPieces on scopes and assign them to the waitForMe list
   (c) tell those pieces to assign this dominoPiece on their waitingFor list
4. stop=false;
5. while(!stop){
     • for each dominoPiece in neighbors{
         (a) if dominoPiece can be removed {
               i. if (domino Piece isIndependent) and (list of pieces it is waiting for is not empty) {
                  wait with removal;
                  }
              ii. else remove it(from neighbors and board) and tell waiting pieces, if there were any waiting for this
                  piece, that it does not need waiting any more. (i.e. delete piece from waiting list)
             }
       }
       determine new neighbors into temp (each removed piece can produce at most 6 new neighbors)
     • if temp equals neighbors{
         – check if (something can be removed): remove first;
         – else stop=true;
       }
     • else neighbors=temp;
   }
6. calculate score
```

4.2.4 Estimation of complexity

Partial Complexities:
n-complexity of classification
n^2-complexity of part at “while(!stop) loop”

In order to associate attributes for classification we need to iterate once through whole collection which is of size n. Then in the while loop we are trying to remove pieces. In worst case we delete only one piece in one iteration which gives us at most n runs of while and inside while loop, we iterate through neighbors collection which is always smaller than n, but it is n dependent. Therefore for each n, algorithm executes number of operations proportional to n, which gives us quadratic time complexity.

Final worst case complexity : $n^2 + n$

4.2.5 Changes after tests

Classification feature was completely removed from the algorithm due to the fact that its complexity exceeded estimations.

4.3 Approximation by Radosław Łojek

4.3.1 Introduction

The solution I have designed is the most intuitive and straight forward algorithm. It checks all possible elements along its way. If a domino piece satisfies necessary conditions of being removed, it is thrown away from a board of elements. The operation is repeated for all consecutive elements until nothing can't be removed, or the domino board is empty.

4.3.2 Data structure

The data structure I have decided to use is a two dimensional array of objects, where object is a single domino field. Each object consists of fields such as: its value, location X and Y, boolean saying if a piece is placed horizontally/vertically, boolean saying if it is successor or predecessor field (value above or on the left side of a domino piece is successor, otherwise predecessor) and of course pointer to the other half of the piece. If a single cell in a matrix is empty, meaning that no piece is there, it is marked as a null pointer.

```
class dominoField {
   int value;
   int X;
   int Y;
   boolean isSuccessor;
   boolean isVertical;
   dominoField *twinField;
}
```

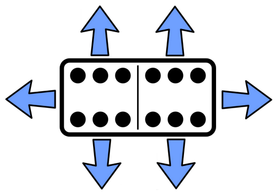
4.3.3 Algorithm Pseudo-Code

The algorithm takes as a parameters two dimensional array of elements and its vertical and horizontal size. Returns elements that were removed during the calculations. Below algorithm is simple, since it does not involve more complicated concepts such as Artificial Intelligence or complex graph algorithms.

```
List<dominoPiece> solveDominoProblem(dominoField **board, int dimX, int dimY){
   List<dominoPiece> result_list;
   int iterPiecesRemoved = 1;

   while iterPiecesRemoved != 0 {
      iterPiecesRemoved = 0;
      for i = 0 to dimX {
         for j = 0 to dimY {
            if !isEmpty(board,i,j) and isElemRemovable(board, dimX, dimY, i, j){
               result_list.add( removePiece(board,i,j) );
               iterPiecesRemoved = iterPiecesRemoved + 1;
            }
         }
      }
   }
```

```
    return result_list;
}
```



I will let myself omit pseudo code of functions used above. Just briefly explain how isElemRemovable will work: depending on the location of a 'twinField' of a single piece, function counts the number of empty fields in each possible direction (as depicted on the right). Once the directions are counted, function checks if they meet requirements for the piece to be removed - if it does, returns true or false otherwise.

4.3.4 Time complexity

Time complexity of the considered algorithm is n^2, since we go in each iteration through all possible domino fields n times. Each iteration is repeated in worst case *1/2*n* times, because may occur situation in which only one domino element (two fields) disappears in one *while* loop iteration.

4.3.5 Example