The Impact of Recovery Specifications on Stochastic Duration and Convexity: A Reduced-Form Analysis

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Abstract

Reduced-form credit risk models have become the standard for pricing defaultable securities, yet the choice of recovery assumption—Recovery of Market Value (RMV) versus Recovery of Face Value (RFV)—is often driven by mathematical tractability rather than empirical realism. This paper extends the affine term structure framework to derive closed-form expressions for stochastic duration and convexity under the RFV specification, contrasting them with the widely used RMV baseline. We quantify the "model risk" inherent in this choice, demonstrating that the RMV assumption systematically overestimates interest rate sensitivity. Our numerical analysis reveals that RMV duration exceeds RFV duration by approximately 6% when default intensity is positively correlated with interest rates. Furthermore, RMV significantly overstates convexity, implying that standard models overestimate the natural gamma protection of corporate bonds. These findings suggest that relying on RMV-based hedging ratios leads to systematic over-hedging of interest rate risk for distressed debt portfolios.

1 Introduction

The valuation of defaultable bonds requires a joint specification of the risk-free term structure, the default intensity, and the recovery mechanism. Since the seminal contributions of [3] and [2], reduced-form models have dominated the academic and practitioner landscape. Among these, the Recovery of Market Value (RMV) assumption has achieved ubiquity, primarily because it allows the defaultable bond to be priced as if it were a risk-free bond discounted at a risk-adjusted rate $R(t) = r(t) + L\lambda(t)$. While mathematically convenient, this assumption implies that recovery is a fraction of the pre-default market value—a circularity that is often at odds with legal bankruptcy proceedings where claims are typically based on the Face Value (Par) of the debt.

This paper challenges the convenience of the RMV assumption by rigorously analyzing the Recovery of Face Value (RFV) specification, where the recovery payment is a fixed fraction of the bond's principal. While the distinction may appear technical, we demonstrate that it has profound implications for the risk management of credit portfolios. The RMV assumption induces a "double sensitivity" to interest rates: as rates rise, the bond price falls, and consequently, the recovery value (being a fraction of that price) falls as well. In contrast, the RFV assumption provides a "buffer effect," as the recovery cash flow remains fixed regardless of the pre-default market value.

Building on the sensitivity analysis of [1], which characterized the stochastic duration of RMV bonds, this paper addresses a critical gap in the literature by explicitly deriving the sensitivity dynamics under RFV. We extend the affine term structure framework to incorporate the RFV mechanism, deriving semi-analytical solutions for both stochastic duration and convexity.

Our contributions are threefold. First, we provide a tractable implementation of the RFV pricing kernel within a Vasicek framework, utilizing the forward measure to handle the independence between the default time and the recovery payoff. Second, we present a comparative numerical analysis demonstrating that the RMV assumption consistently overestimates both duration and convexity. Specifically, we find that the hedging error—defined as the relative difference between RFV and RMV duration—approaches -6% in environments where default intensity is positively correlated with interest rates. Finally, we show that the convexity bias is substantial, suggesting that standard RMV models exaggerate the convexity protection (gamma) embedded in corporate bonds.

The remainder of this paper is organized as follows. Section 2 outlines the theoretical framework and the mathematical derivation of the pricing equations. Section 3 details the numerical implementation and the calibration of the Vasicek parameters. Section 4 presents the results of the sensitivity analysis, quantifying the duration and convexity gaps. Section 5 concludes with implications for portfolio hedging.

2 Theoretical Framework

In this section, we derive the pricing and sensitivity dynamics of a defaultable zero-coupon bond under the Recovery of Face Value (RFV) assumption. We assume a filtered probability space $(\Omega, \mathcal{F}, (\mathcal{F}_t)_{t\geq 0}, \mathbb{P})$ supporting a short rate process $r(t)$ and a default intensity process $\lambda(t)$.

2.1 Pricing Equation

Under the RFV specification, the bond holder receives the face value (normalized to 1) at maturity T if no default occurs, or a fixed recovery fraction $w = (1 - L)$ of the face value at the instant of default τ if $\tau \leq T$. The price at time t is given by:

$$P^{RFV}(t,T) = \mathbb{E}_t^{\mathbb{Q}}\left[e^{-\int_t^T r(u)du}\mathbf{1}_{\{\tau>T\}} + \int_t^T e^{-\int_t^s r(u)du} w\mathbf{1}_{\{\tau\in ds\}}\right] \tag{1}$$

Using the standard reduced-form pricing result [4], we can express this in terms of the default intensity $\lambda(t)$:

$$P^{RFV}(t,T) = \underbrace{\mathbb{E}_t^{\mathbb{Q}}\left[e^{-\int_t^T (r(u)+\lambda(u))du}\right]}_{V_{surv}(t,T)} + \underbrace{w\int_t^T \mathbb{E}_t^{\mathbb{Q}}\left[\lambda(s)e^{-\int_t^s (r(u)+\lambda(u))du}\right] ds}_{V_{rec}(t,T)} \tag{2}$$

Here, V_{surv} represents the value of the survival claim, and V_{rec} represents the present value of the recovery payments.

2.2 Affine Dynamics

We assume the default intensity is an affine function of the short rate:

$$\lambda(t) = \Lambda_0 + \Lambda_1 r(t) \tag{3}$$

Consequently, the "risk-adjusted" or "effective" discount rate for the survival component becomes:

$$k(t) \equiv r(t) + \lambda(t) = \Lambda_0 + (1 + \Lambda_1)r(t) \tag{4}$$

Assuming $r(t)$ follows a Vasicek process $dr(t) = \kappa(\theta - r(t))dt + \sigma dW(t)$, the survival value V_{surv} admits a closed-form affine solution:

$$V_{surv}(t,T) = e^{A^*(\tau)-B^*(\tau)r(t)} \tag{5}$$

where $B^*(\tau) = \frac{1+\Lambda_1}{\kappa}(1 - e^{-\kappa\tau})$ and $A^*(\tau)$ is determined by the standard Riccati equations adjusted for the effective rate $k(t)$.

2.3 Measure Change and Recovery Value

The recovery component V_{rec} involves the expectation of the product of the discount factor and the intensity. To evaluate the integrand $\mathbb{E}_t^{\mathbb{Q}}[\lambda(s)e^{-\int_t^s k(u)du}]$, it is convenient to switch to the s-forward measure $\mathbb{Q}^s$ associated with the numeraire $N(t) = e^{\int_0^t k(u)du}$. However, since $k(u)$ is stochastic, we proceed by identifying the expectation as the price of a security paying $\lambda(s)$ at time s, discounted by k.

Let $D(t,s) = \mathbb{E}_t^{\mathbb{Q}}[e^{-\int_t^s k(u)du}] = V_{surv}(t,s)$. We can rewrite the integrand as:

$$\mathbb{E}_t^{\mathbb{Q}}\left[\lambda(s)e^{-\int_t^s k(u)du}\right] = D(t,s)\cdot\mathbb{E}_t^{\mathbb{Q}^s}[\lambda(s)] \tag{6}$$

where $\mathbb{Q}^s$ is the measure defined by the Radon-Nikodym derivative related to the bond price $V_{surv}(t,s)$. Under the affine specification, the expectation of $\lambda(s)$ under $\mathbb{Q}^s$ is:

$$\mathbb{E}_t^{\mathbb{Q}^s}[\lambda(s)] = \Lambda_0 + \Lambda_1\mathbb{E}_t^{\mathbb{Q}^s}[r(s)] \tag{7}$$

The expectation of the short rate under the forward measure includes a convexity adjustment (Girsanov shift) relative to the physical measure expectation:

$$\mathbb{E}_t^{\mathbb{Q}^s}[r(s)] = \text{Mean}^{\mathbb{P}}(r(s)) - \text{Shift}(t,s) \tag{8}$$

where the shift term arises from the correlation between the interest rate and the bond price volatility:

$$\text{Shift}(t,s) = \int_0^{s-t} e^{-\kappa(s-t-u)}\sigma^2 B^*(u)du \tag{9}$$

2.4 Analytical Derivation of Stochastic Duration

The stochastic duration is defined as the semi-elasticity of the price with respect to the short rate $D(t,T) = -\frac{1}{P}\frac{\partial P}{\partial r(t)}$. Applying Leibniz's rule to the pricing equation, we obtain:

$$\frac{\partial P^{RFV}}{\partial r} = \frac{\partial V_{surv}}{\partial r} + w\int_t^T \left(\frac{\partial V_{surv}(t,s)}{\partial r}\mathbb{E}_t^{\mathbb{Q}^s}[\lambda(s)] + V_{surv}(t,s)\frac{\partial \mathbb{E}_t^{\mathbb{Q}^s}[\lambda(s)]}{\partial r}\right) ds \tag{10}$$

Under the Vasicek specification, we can solve for these partial derivatives explicitly. First, the sensitivity of the survival component is determined by the affine coefficient B^*:

$$\frac{\partial V_{surv}(t,s)}{\partial r} = -B^*(s-t)V_{surv}(t,s) \tag{11}$$

Second, the sensitivity of the expected default intensity depends on the mean reversion of the short rate. Recall that $\mathbb{E}^{\mathbb{Q}^s}[\lambda(s)] = \Lambda_0 + \Lambda_1(\text{Mean}^{\mathbb{P}} - \text{Shift})$. The shift term is deterministic and does not depend on $r(t)$. Thus:

$$\frac{\partial \mathbb{E}_t^{\mathbb{Q}^s}[\lambda(s)]}{\partial r} = \Lambda_1 \frac{\partial}{\partial r}\left(r(t)e^{-\kappa(s-t)} + \theta(1-e^{-\kappa(s-t)})\right) = \Lambda_1 e^{-\kappa(s-t)} \tag{12}$$

Substituting these back yields the final analytical expression for the sensitivity of the recovery component:

$$\frac{\partial V_{rec}}{\partial r} = w\int_t^T V_{surv}(t,s)\left[\underbrace{-B^*(s-t)\mathbb{E}_t^{\mathbb{Q}^s}[\lambda(s)]}_{\text{Discount Effect}} + \underbrace{\Lambda_1 e^{-\kappa(s-t)}}_{\text{Payoff Effect}}\right] ds \tag{13}$$

This explicit formula highlights why RMV fails: RMV assumes the recovery value sensitivity is driven solely by the discount effect (via the adjusted rate), ignoring the distinct decay rate ($e^{-\kappa\tau}$) of the default intensity expectation.

3 Numerical Results

We perform a comparative analysis of the stochastic duration and convexity under RMV and RFV specifications. We calibrate the Vasicek model using the parameters from [1] (Table 5.1): $r_0 = 0.04$, $\kappa = 0.15$, $\sigma = 0.01$, $\theta = 0.0522$, $L = 0.4$, and $\Lambda_0 = 0.025$. We analyze a 10-year bond ($T = 10$) and vary the sensitivity of the default intensity to interest rates, Λ_1, from -0.10 to 0.10.

3.1 Duration Gap Analysis

Figure 1 displays the stochastic duration of the bond under both recovery assumptions.

We observe that D_{RMV} is consistently higher than D_{RFV} across the entire range of Λ_1.

- **Baseline ($\Lambda_1 = 0$):** Even with zero correlation between default and rates, there is a significant gap. The RMV duration is approximately 5.18 years, while the RFV duration is 4.90 years.
- **Relative Error:** The relative error, defined as $(D_{RFV} - D_{RMV})/D_{RFV}$, is approximately -6% when Λ_1 is positive. This implies that using an RMV model to hedge an RFV instrument results in a hedge ratio that is 6% too large.

The RMV assumption implies that as interest rates rise (and bond prices fall), the recovery value—being a fraction of the market price—also falls. This creates a "double sensitivity" to rates. In the RFV model, the recovery is fixed at $(1 - L)$ of par, providing a buffer that dampens the price sensitivity.

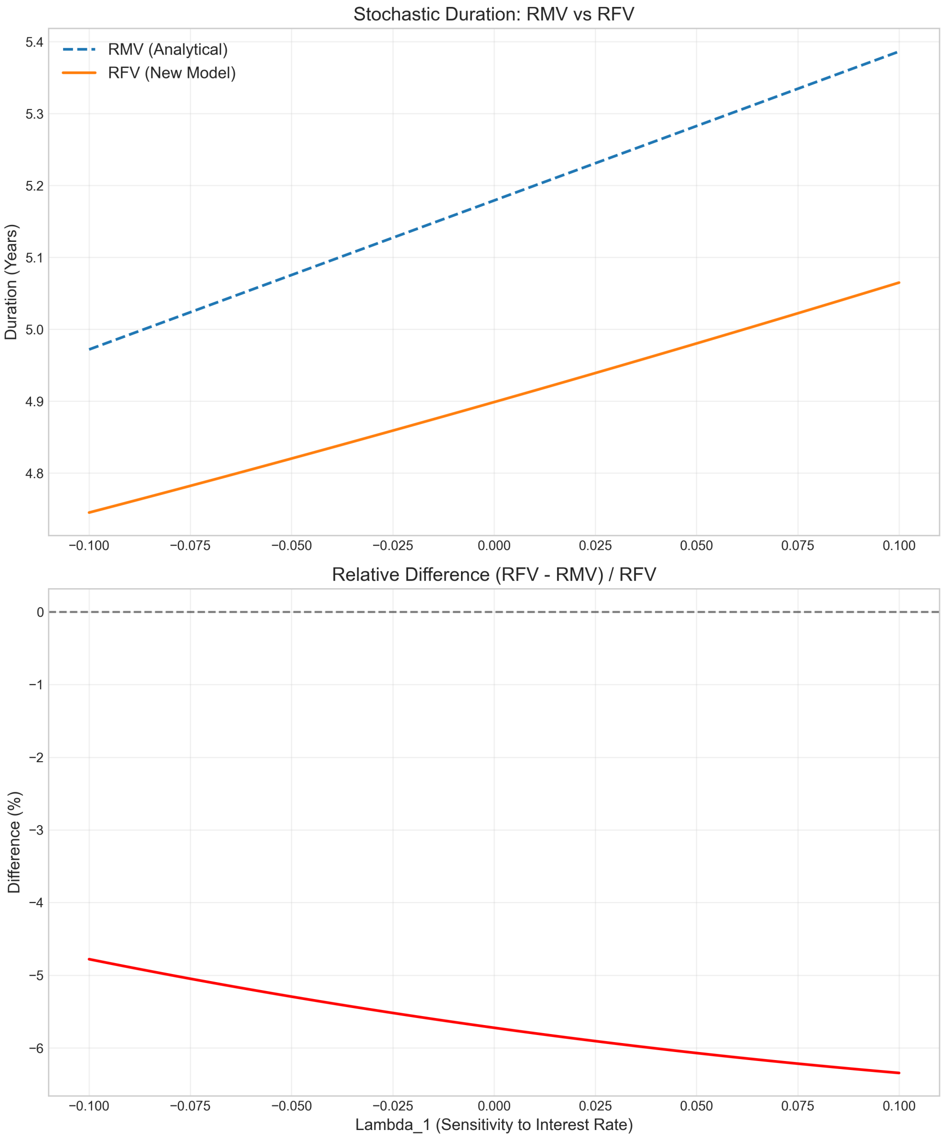

Figure 1: Stochastic Duration Comparison: RMV vs. RFV. The top panel shows absolute duration levels; the bottom panel shows the relative difference.

3.2 Convexity Analysis

Figure 2 illustrates the convexity profiles.

Consistent with the duration results, C_{RMV} significantly exceeds C_{RFV}. The gap is approximately 2.0 units. This result indicates that standard RMV models overestimate the "convexity-on-convexity" effect. By assuming the recovery value is sensitive to the asset price (which is itself convex in yields), the RMV model compounds the convexity. The RFV model, with its fixed recovery cash flow, removes one layer of this convexity, resulting in a lower gamma profile.

4 Conclusion

This paper has generalized the reduced-form credit risk framework to incorporate the Recovery of Face Value (RFV) assumption, providing a rigorous comparison with the standard Recovery of Market Value (RMV) baseline. By deriving semi-analytical solutions for stochastic duration and convexity under RFV, we have quantified the "model risk" associated with the choice of recovery specification.

Our analysis demonstrates that the RMV assumption is not a neutral mathematical convenience but a structural choice that systematically biases risk sensitivities upward. We found that RMV models

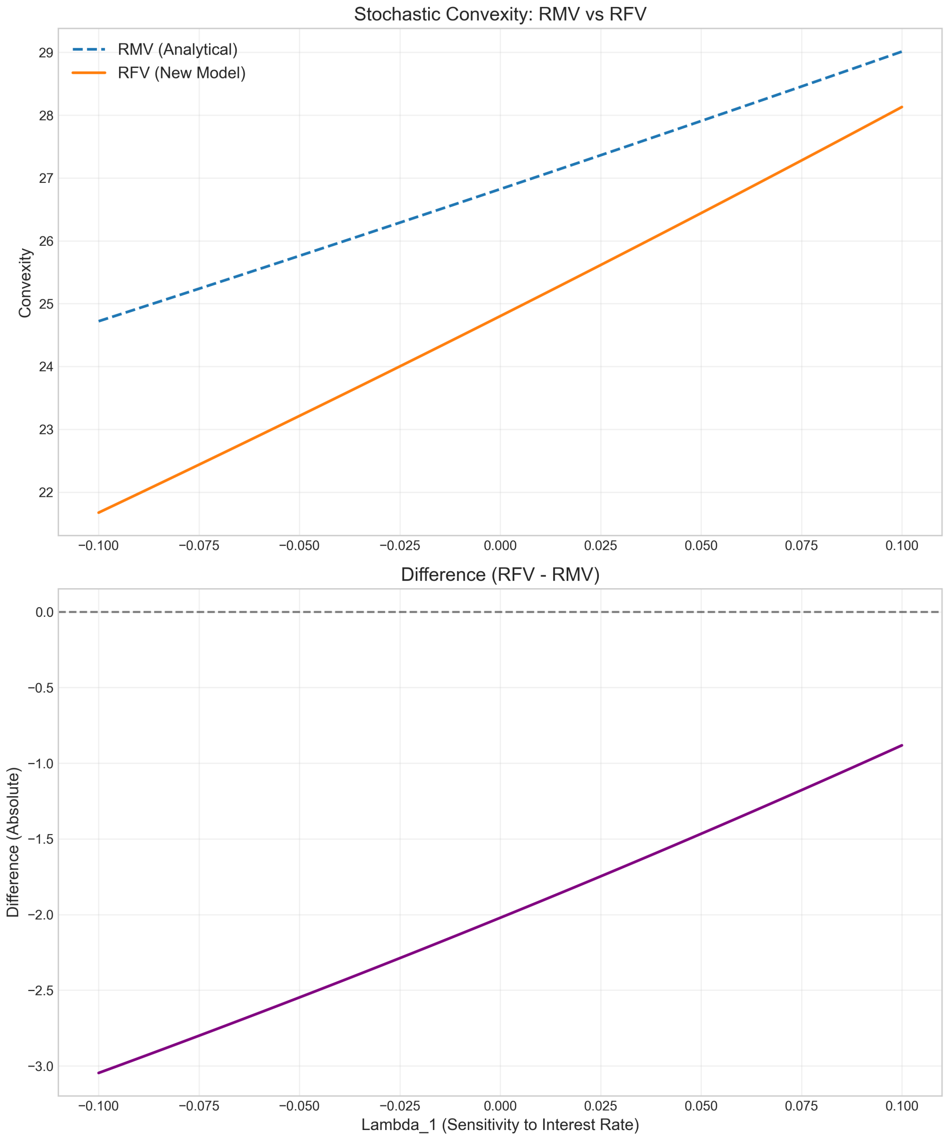

Figure 2: Stochastic Convexity Comparison: RMV vs. RFV. The RMV model consistently overestimates the convexity.

overestimate duration by approximately 6% and significantly overstate convexity. This bias stems from the implicit assumption in RMV that recovery values decline when interest rates rise (via the bond price channel), a mechanic that is absent in the fixed-recovery RFV framework.

For practitioners managing portfolios of distressed debt (where $\Lambda_1 > 0$ and default risk is material), relying on standard RMV-based risk systems will lead to systematic over-hedging. Specifically, hedging interest rate risk based on RMV duration would result in selling more treasury futures or buying more gamma protection than is necessary to offset the true economic risk of the position. As such, we advocate for the use of RFV-consistent pricing kernels when managing credit-sensitive assets where legal recovery is tied to par value.

References

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