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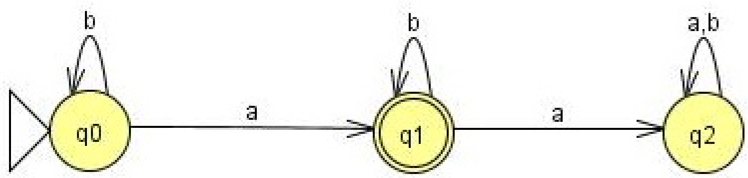
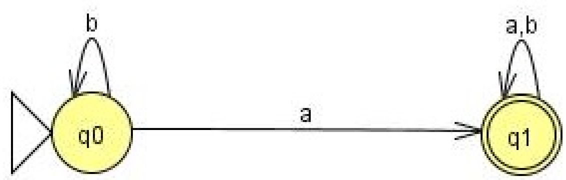
CS 421, SP15
21 January 2015
Lab #1

2. Construct a DFA with $\Sigma = \{a, b\}$ that accepts all the sets consisting of:

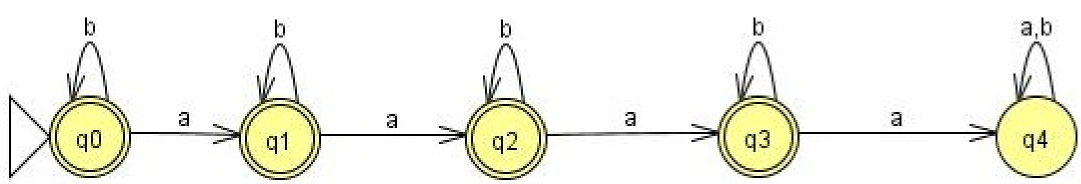
2a. All strings with exactly one a.



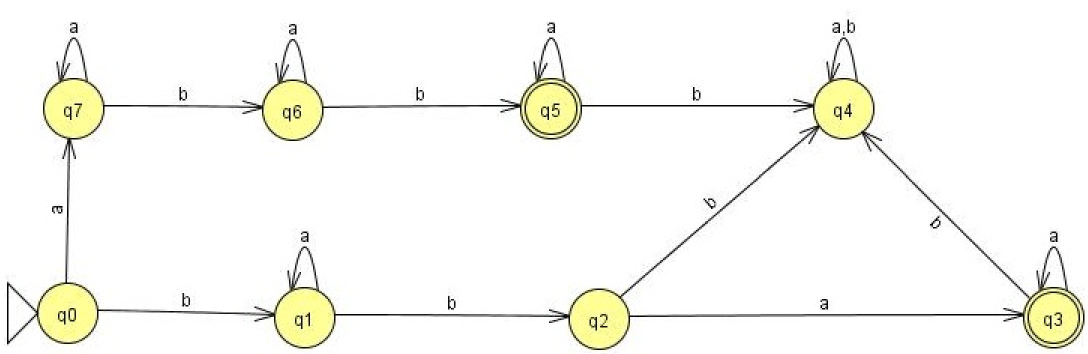
2b. All strings with at least one a.



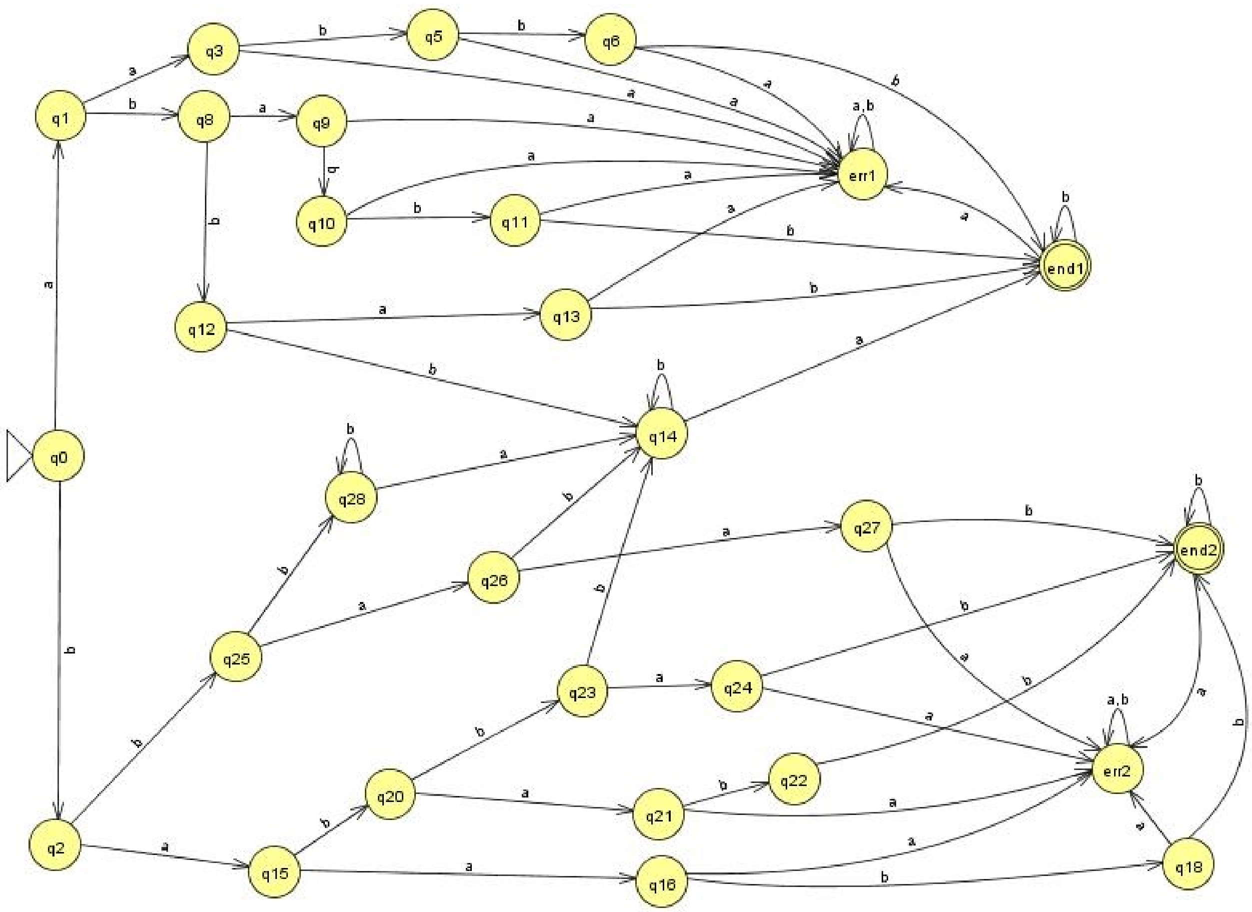
2c. All strings with no more than three as ($0 \leq n \leq 3$).



2d. All strings with at least one a and exactly two bs.

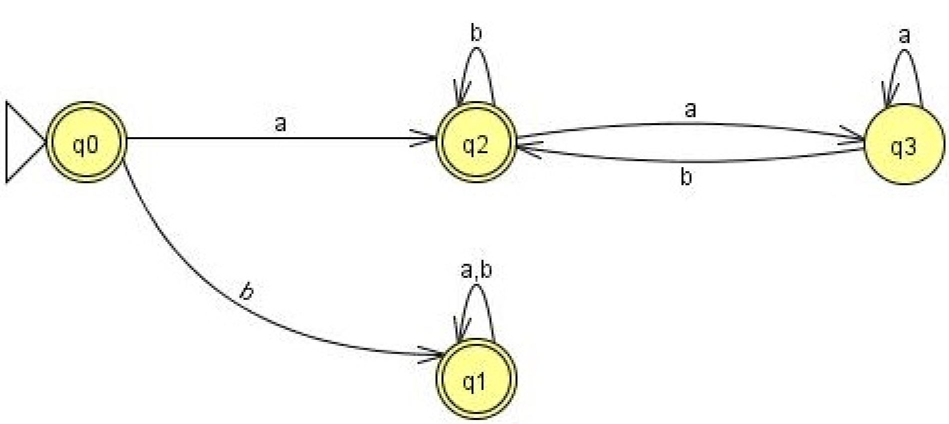


2e. All strings with exactly two as and more than two bs.



This graph was generated from a tree, and only a minimal effort was made to simplify it. No doubt there is a simpler graph.

3. Show that if we change Figure 2.6, making q_3 a nonfinal state and making q_0, q_1, q_2 final states, the resulting DFA accepts $\overline{L}$. Modified graph:



We want to show that the DFA accepting L accepts $\overline{L}$ when certain states are changed. After making the changes, we have the above transition graph. By definition, $\overline{L} = \Sigma^* - L$, which means $\overline{L} = \Sigma^* - \{awa : w \in \{a, b\}^*\}$. (Note that since Σ^* contains λ, we must also consider the empty string.) This means that $\overline{L}$ essentially describes a language that contains strings that neither begin with a nor (necessarily) end in a. In other words, $\overline{L}$ removes all sentences explicitly of the form awa, which are described by L.

Looking at the aforementioned transition graph, we can easily see that the case involving a string of zero length (i.e., λ) is trivial, as the string is automatically accepted at q_0. We also notice that the only other final states are q_1 and q_2. If we input a string v beginning with b, then we have $\delta(q_0, b) \to q_1$. Since q_1 is a trap state, this DFA will accept any input string from $\overline{L}$ beginning with b (and ending in *either* a or b). Alternatively, if v begins with a, then we have $\delta(q_0, a) \to q_2$. If at any time we encounter another a (we can disregard b while in q_2), then we become trapped in q_3. Hence, if v begins with a, then the DFA will reject v if it ends in a. If we encounter b while in q_3, then we have $\delta(q_3, b) \to q_2$, which will trap all bs until another a is encountered. If another a is encountered, the DFA will return to q_3, which is a trap state. This will lead to the rejection of v. Thus, we have that this DFA accepts $\overline{L}$, which implies the rejection of L.

#QED

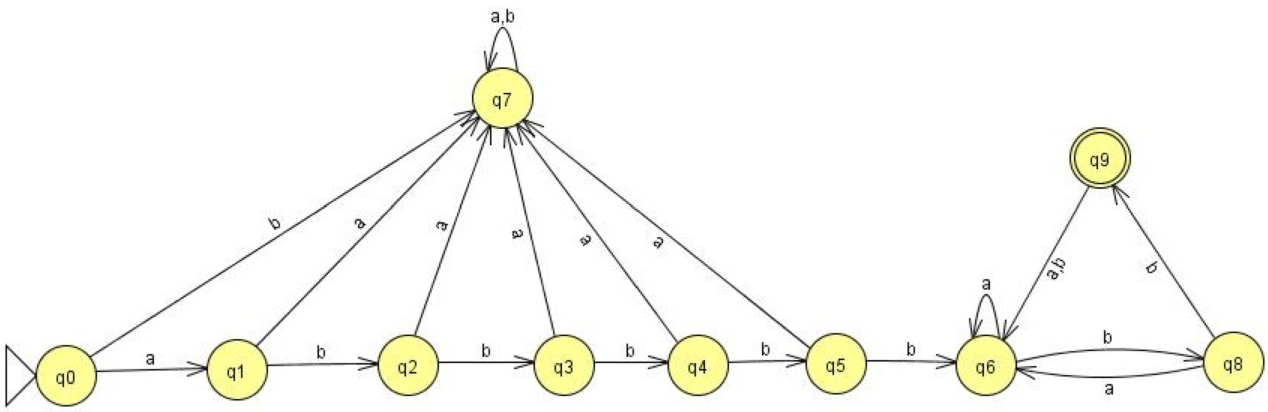
4. Generalize the observation in the previous exercise. Specifically, show that if $M = (Q, \Sigma, \delta, q_0, F)$ and $\widehat{M} = (Q, \Sigma, \delta, q_0, Q - F)$ are two DFAs, then $\overline{L(M)} = L(\widehat{M})$.

We want to show that $\overline{L(M)} = L(\widehat{M})$. Suppose we have two DFAs $M = (Q, \Sigma, \delta, q_0, F)$ and $\widehat{M} = (Q, \Sigma, \delta, q_0, Q - F)$. Since we have a DFA M, there exists a language L accepted by M such that $L(M) = \{w \in \Sigma^* : \delta^*(q_0, w) \in F\}$. If we take the complement of L, then we have $\overline{L(M)} = \{w \in \Sigma^* : \delta^*(q_0, w) \notin F\}$. In terms of the equivalent transition graph G for M, this means we change the final states to nonfinal states and vice versa. In other words, we perform the set operation $F_1 = Q - F$, where Q and F are the finite sets of internal states and final states, respectively, of M. Thus, we have a new set of final states F_1 (consequently, note that $F_1 \subseteq Q$), which is identical to the finite set of final states of $\widehat{M}$. It is obvious to see, then, that $\widehat{M}$ accepts the language $L(\widehat{M}) = \{w \in \Sigma^* : \delta^*(q_0, w) \in F_1\}$, which is equivalent to $L(\widehat{M}) = \{w \in \Sigma^* : \delta^*(q_0, w) \notin F\}$. Thus, we see that $\overline{L(M)} = L(\widehat{M})$.

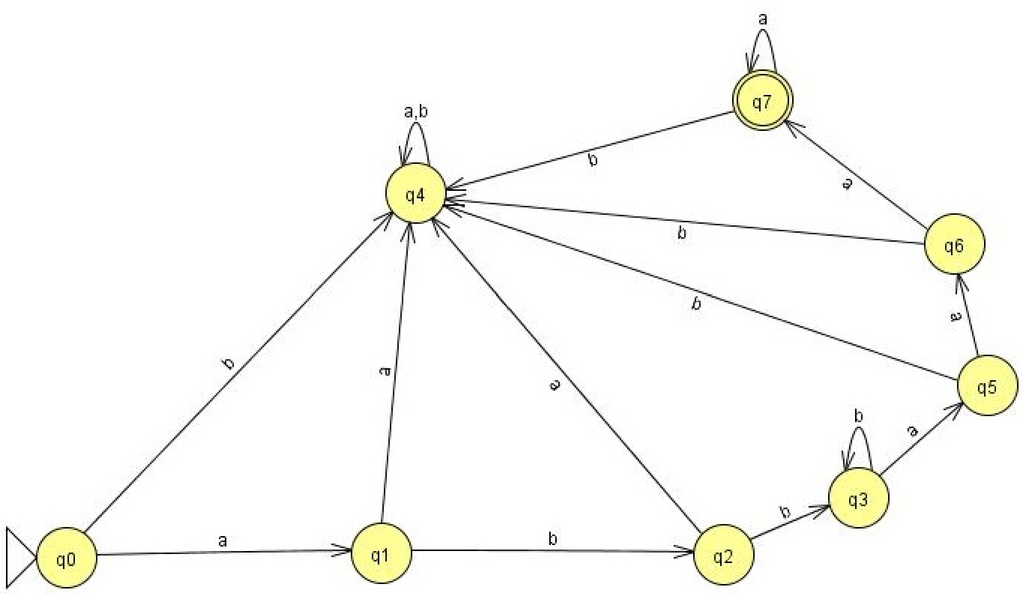
#QED

5. Give DFAs for the languages:

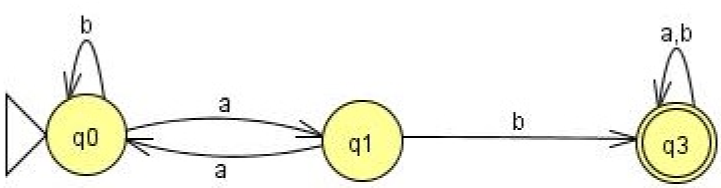
5a. $L = \{ab^5wb^2 : w \in \{a,b\}^*\}$.



5b. $L = \{ab^na^m : n \geq 2, m \geq 3\}$.



5c. $L = \{w_1abw_2 : w_1 \in \{a,b\}^*, w_2 \in \{a,b\}^*\}$.



5d. $L = \{ba^n : n \geq 1, n \neq 5\}$.