Michael Beaver
CS 421, SP15
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Lab #4

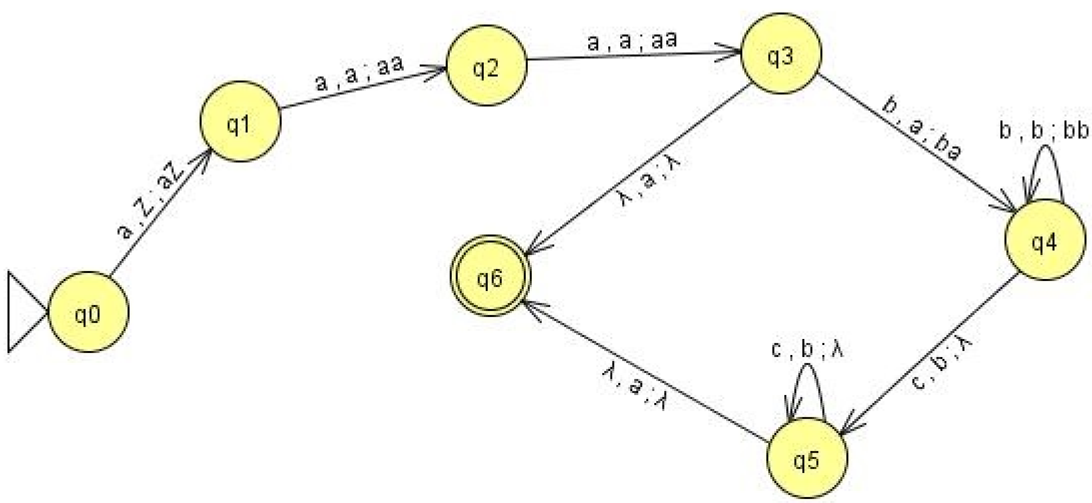
§5.1 7. Find context-free grammars for the following languages (with $n \geq 0$, $m \geq 0$).

§5.1 7c. $L = \{a^n b^m : n \neq 2m\}$
Note $m = 0 \Rightarrow n \neq 0$, so $\lambda \notin L$.

$S \to aCb \ | \ a \ | \ b$
$C \to A \ | \ B$
$A \to aA \ | \ a \ | \ \lambda$
$B \to aB \ | \ b \ | \ \lambda$

§7.1 4. Construct NPDAs that accept the following languages on $\Sigma = \{a, b, c\}$.

§7.1 4e $L = \{a^3 b^n c^n : n \geq 0\}$



§8.1 2. Show that the language $L = \{a^n : n \text{ is a prime number}\}$ is not context-free.

Assume L is context-free and therefore pumpable with the Pumping Lemma for CFLs. Suppose we have a string $w = a^q$, where q is a prime number such that $q \geq m$. We have that $w \in L$ and $|w| \geq m$. The string w can then be subdivided in the following fashion: $w = a^r a^s a^t a^u a^{q-r-s-t-u}$. We see that $s + u \geq 1$ and $s + t + u \leq m$ satisfy our criteria for the Pumping Lemma for CFLs. We can pump w to generate strings of the following form:

$$w_i = a^r a^{si} a^t a^{ui} a^{q-r-s-t-u}$$

$$\Rightarrow w_i = a^{si+ui-s-u+q}$$

$$\Rightarrow w_i = a^{(s+u)(i-1)+q}.$$

Now consider $i = q + 1$: $w_{q+1} = a^{(s+u)(q+1-1)+q} = a^{(s+u+1)q}$. Notice that $w_{q+1} \notin L$ because $(s + u + 1)q$ is not a prime number. Therefore, L is not pumpable and is not context-free.

#QED

§8.2 17. Show that the language $L = \{a^n b^n : n \geq 0, n \text{ is not a multiple of } 5\}$ is context-free.

Let $L_1 = \{a^n b^n : n \geq 0\}$ and $L_2 = \{a^n b^n : n \text{ is a multiple of } 5\}$. We know that L_1 is not a regular language. We should be able to construct a finite automaton to accept L_2, so it should be a regular language. We can construct NFAs for $L_{2a} = \{a^n : n \text{ is a multiple of } 5\}$ and $L_{2b} = \{b^n : n \text{ is a multiple of } 5\}$. From here we can concatenate $L_2 = L_{2a} L_{2b}$ since regular languages are closed under concatenation. It is easy to see that $L = L_1 \cap \overline{L_2}$. Since L_2 is regular, $\overline{L_2}$ is also regular. Thus, we have the conditions for regular intersection, which is closed for CFLs. It follows then that L is context-free.

#QED