Modeling, Control, and Experimental Validation of a Ball and Plate System

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01

Introduction and Motivation

You can describe the topic of the section here

System advantages

Portable

Affordable

Unstable

Controllable

Safe

Clear response observer

Convenient

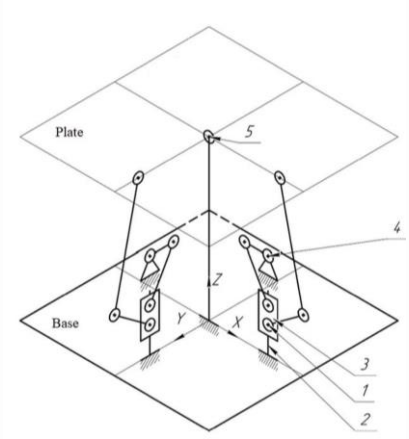
Scalable

02

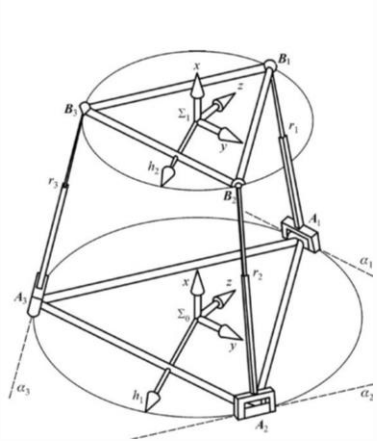
Plant Selection and Implementation

You can describe the topic of the section here

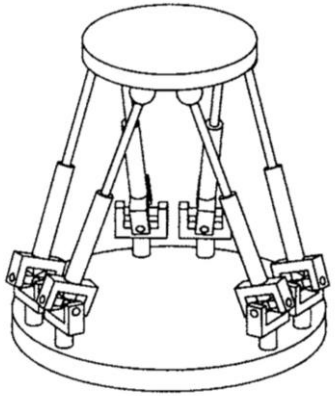
Plant selection based on degree of freedom



2 DOF



3 DOF



6 DOF

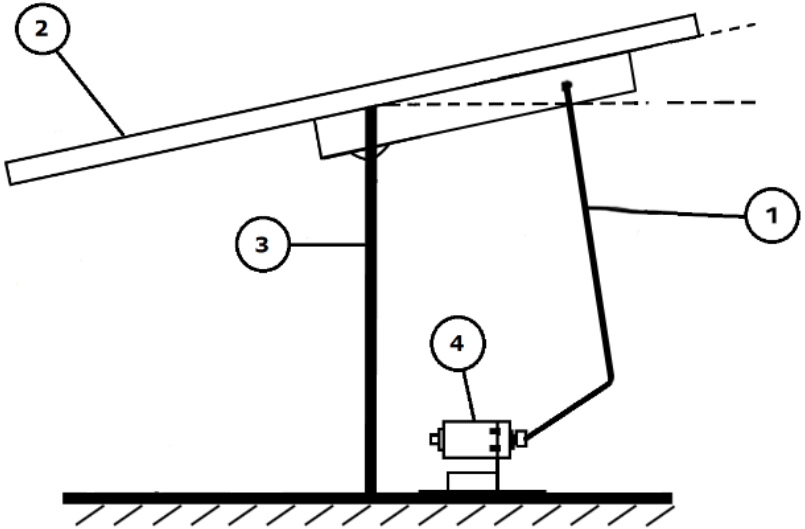
Mechanical design

1 Linkage Rods

A linkage part between the actuator and the plate

2 Plate

Transparent plastic flat platform in 200mm x 2mm dimension



3 Base and Universal Joint

Pivotal connection allowing the plate to move around two axis

4 Actuator holder

A simple aluminium structure that holds the actuator in place

Electrical parts selection

Electrical parts

- Sensor
  - Camera
  - Resistive touch screen
  - Infrared sensor
- Actuator
  - Servo motor
  - Stepper motor
  - DC motor
- Microcontroller
  - Arduino
  - PC via simulink
  - PCB circuitry

Electrical connection

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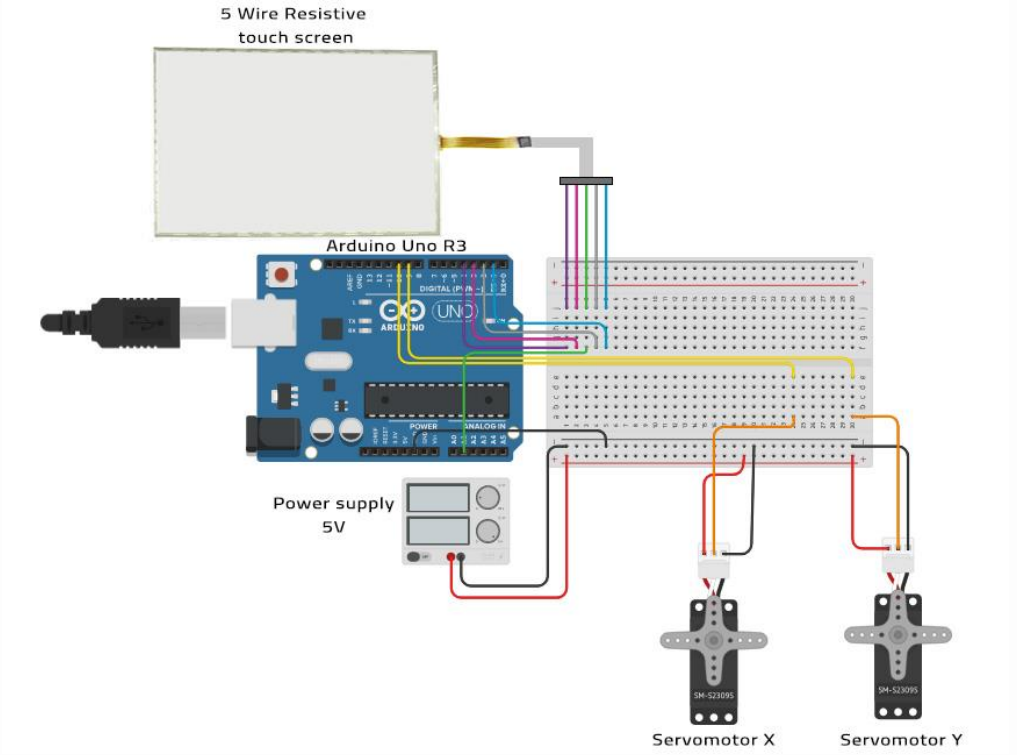
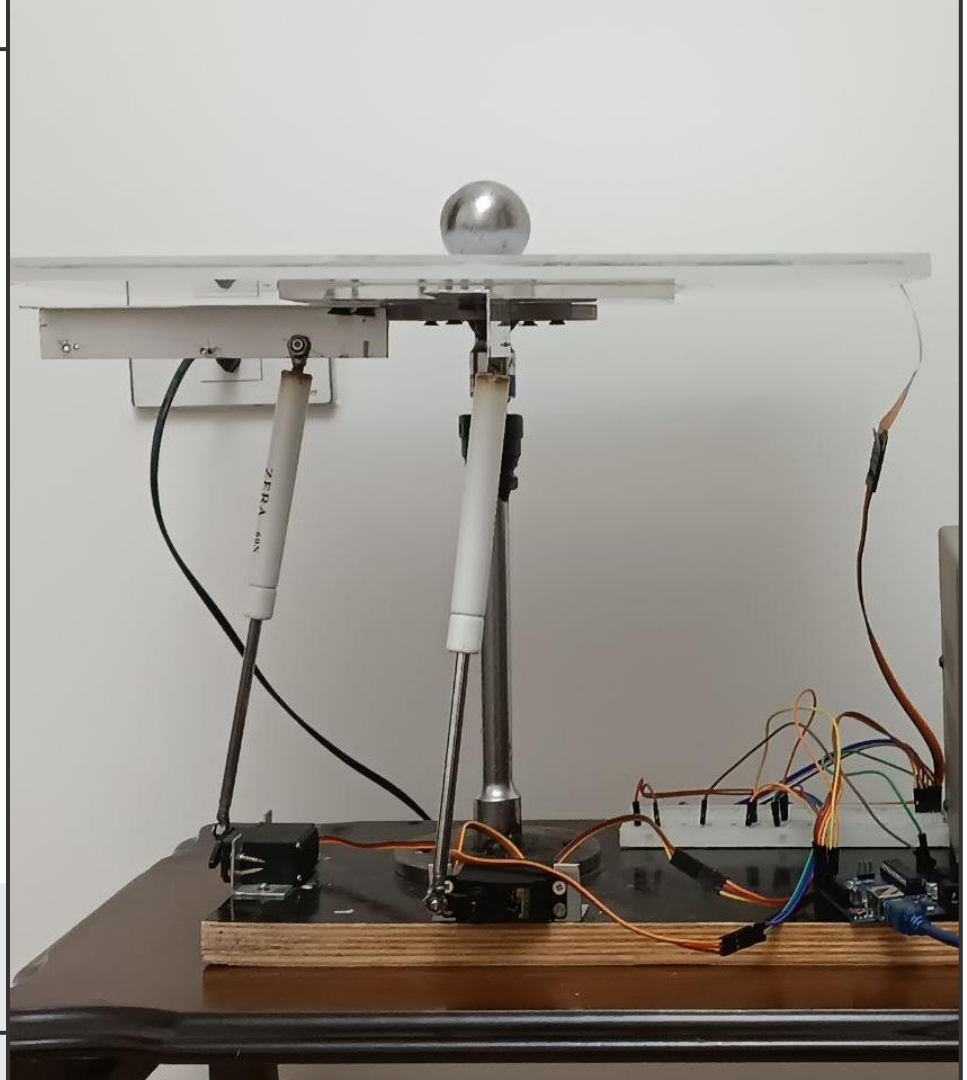


The final form of the BPS system

This image reveal the final appearance of the system with a steel ball resting on top of it



03

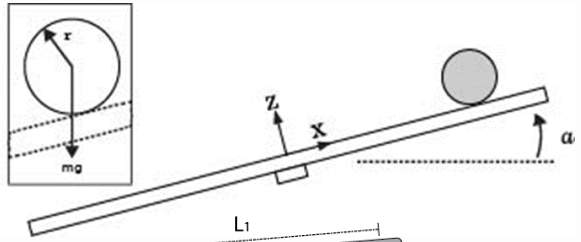
Mathematical Modeling

In this section the mathimatical of servo and BPS and linearization efforts will be disscussed

The system is divided into:

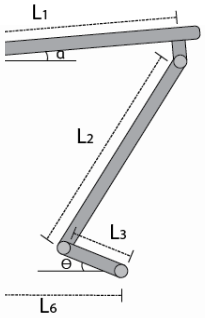
Plate and Ball

In which alpha is the input and ball position is the output



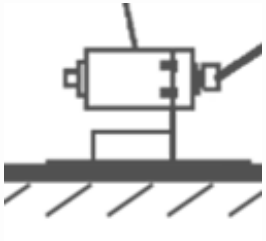
Linkage rod

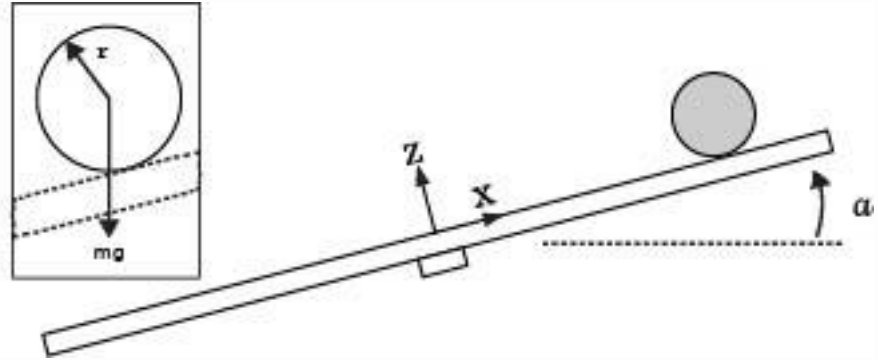
Gain that converts theta into alpha



Servomotor

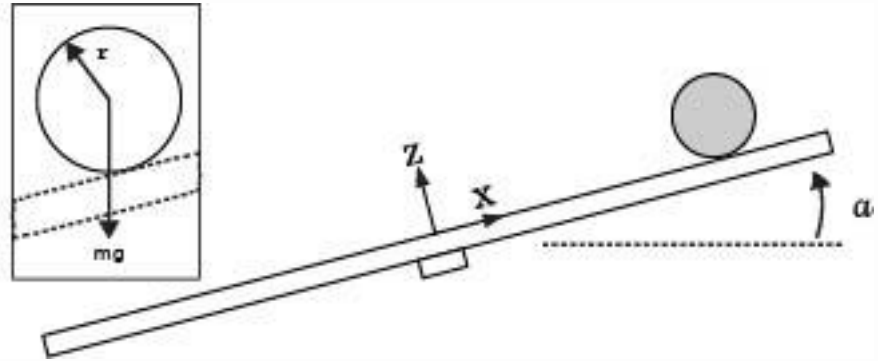
Which is the servo internal system





Assumptions

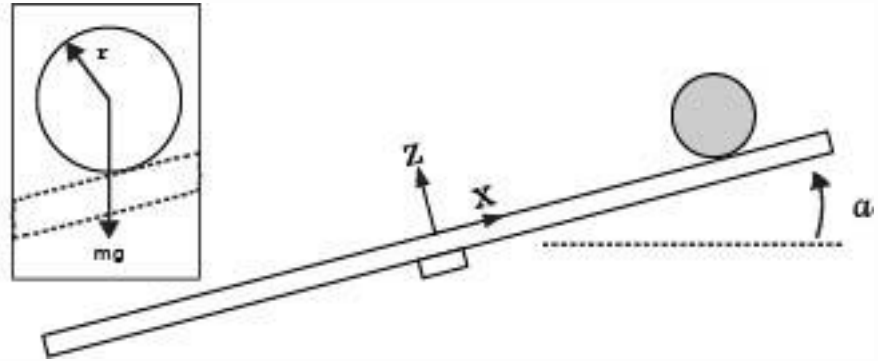
- All friction forces are neglected
- the ball is symetric and rounded
- The plate has infinite surface



Euler Lagrange Formula

$$\frac{d}{dt}\frac{dL}{d\dot{q}} - \frac{dL}{dq} = Q$$

$$\mathrm{L}(\mathrm{q}_i, \dot{q}_i, t) = T(\dot{q}_i, t) - V(q_i, t)$$



Kinetic and potential energy

$$T = T_b + T_p$$

$$T_b = \frac{1}{2} m_b (\dot{x}^2 + \dot{y}^2) + \frac{1}{2} \frac{J_b}{r^2} (\dot{x}^2 + \dot{y}^2)$$

$$= \frac{1}{2} \left(m_b + \frac{J_b}{r_b^2} \right) (\dot{x}^2 + \dot{y}^2)$$

$$T_p = \frac{1}{2} (J_b + J_p) \left(\dot{\alpha}^2 + \dot{\beta}^2 \right) + \frac{1}{2} m_b \left(x\dot{\alpha} + y\dot{\beta} \right)^2$$

$$V = m_b g h = m_b g (x \sin \alpha + y \sin \beta)$$

Equation of motion

Differential equations

$$0 = \left(m_b + \frac{J_b}{r^2}\right)\ddot{x} - m_b\left(x\dot{\alpha}^2 + y\dot{\alpha}\dot{\beta}\right) + m_b g \sin\alpha$$

$$0 = \left(m_b + \frac{J_b}{r^2}\right)\ddot{y} - m_b\left(x\dot{\beta}^2 + x\dot{\alpha}\dot{\beta}\right) + m_b g \sin\beta$$

$$\tau_x = (J_b + J_p + m_b x^2)\ddot{\alpha} + 2m_b x\dot{x}\dot{\alpha} + m_b xy\ddot{\beta} + m_b\dot{x}y\dot{\beta} + m_b x\dot{y}\dot{\beta} - m_b g x\cos\alpha$$

$$\tau_y = (J_b + J_p + m_b y^2)\ddot{\beta} + 2m_b y\dot{y}\dot{\beta} + m_b xy\ddot{\alpha} + m_b\dot{x}y\dot{\alpha} + m_b x\dot{y}\dot{\alpha} - m_b g y\cos\beta$$

In state space

$$\begin{bmatrix}\dot{x_1}\\ \dot{x_2}\\ \dot{x_3}\\ \dot{x_4}\\ \dot{x_5}\\ \dot{x_6}\\ \dot{x_7}\\ \dot{x_8}\end{bmatrix} = \begin{bmatrix} x_2 \\ \frac{m_b}{m_b+\frac{J_b}{r^2}}(x_1x_4^2 + x_4x_5x_8 - g\sin x_3) \\ x_4 \\ 0 \\ x_6 \\ \frac{m_b}{m_b+\frac{J_b}{r^2}}(x_5x_8^2 + x_1x_4x_8 - g\sin x_7) \\ x_8 \\ 0 \end{bmatrix} + \begin{bmatrix}0&0\\0&0\\0&0\\1&0\\0&0\\0&0\\0&0\\0&1\end{bmatrix}\begin{bmatrix}u_x\\u_y\end{bmatrix}$$

$$X = (x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8) = (x, \dot{x}, \alpha, \dot{\alpha}, y, \dot{y}, \beta, \dot{\beta})$$

Simplification and linearization

Simplification assumption

- The inclination angles assumed to be small < 15 therefore sine(α) = α
- The motor doesn’t lose any steps and load doesn’t affect their performance
- The ball is solid spherical object

Linearized model

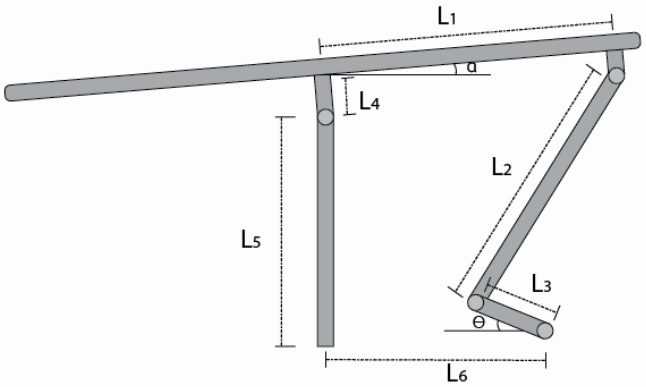
$$G(s) = \frac{x(s)}{\alpha(s)} = -\frac{5}{7}\frac{g}{s^2}$$

$$G(s) = \frac{y(s)}{\beta(s)} = -\frac{5}{7}\frac{g}{s^2}$$

$$\begin{bmatrix} \dot{x_1} \\ \dot{x_2} \end{bmatrix} = \begin{bmatrix} x_2 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{5}{7}(-g) \end{bmatrix} [\alpha]$$

$$\begin{bmatrix} \dot{x_5} \\ \dot{x_6} \end{bmatrix} = \begin{bmatrix} x_6 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{5}{7}(-g) \end{bmatrix} [\beta]$$

Linkge gain



Mapping the angles and using linear polynomial regression:

Kx = 0.0445 Ky = 0.0377

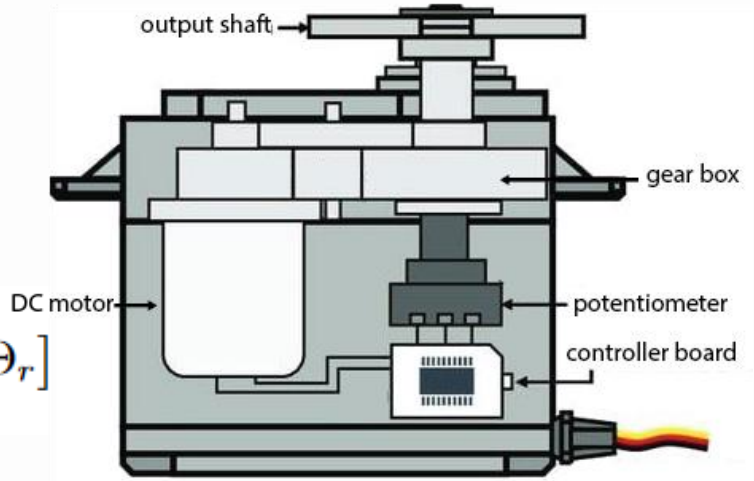
Mapping table

Servo motor angle	Inclination angle alpha	Inclination angle beta
0	-7.9	7.3
5	-7.8	7.0
10	-7.6	6.8
15	-7.2	6.6
.....		
85	0.8	-1.7
90	1.3	-2.3
95	2.0	-2.9
…….		
145	9.0	-9.0
150	9.6	-9.3
180	10.5	-10.2

Servomotor

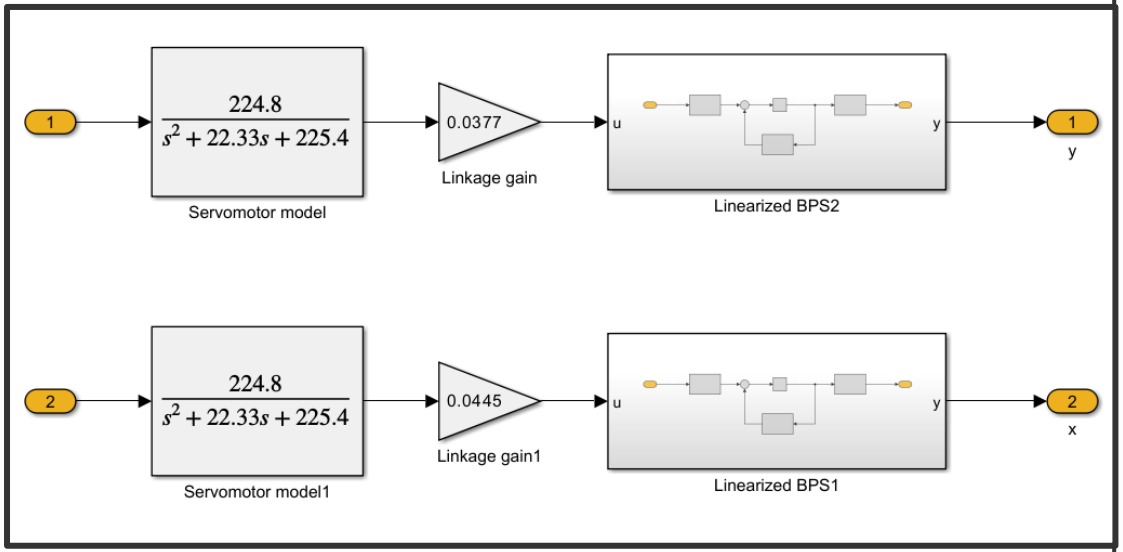
$$\frac{\Theta_l(s)}{\Theta_r(s)} = \frac{224.8}{s^2+22.33s+225.4}$$

$$\begin{bmatrix} \dot{\Theta}_l \\ \ddot{\Theta}_l \end{bmatrix} = \begin{bmatrix} 1\dot{\Theta}_l \\ 225.4\Theta_l + 22.33\dot{\Theta}_l \end{bmatrix} + \begin{bmatrix} 0 \\ 224.8 \end{bmatrix} [\Theta_r]$$



Overall model

Taking in account the linearized plant model and the actuator model with the linkage gain



$$\frac{x}{\Theta_r} = \frac{70.1}{s^4 + 22.33s^3 + 225.4s^2}$$

$$\begin{bmatrix} \dot{x_1} \\ \ddot{x_2} \\ \dot{x_3} \\ \ddot{x_4} \end{bmatrix} = \begin{bmatrix} x_1 \\ \frac{-5}{7} g x_3 \\ x_4 \\ -225.4x_x - 22.33x_4 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 70.1 \end{bmatrix} [\Theta_r]$$

04

Control Design and Challenges

You can describe the topic of the section here

Control Requirements

- **the plate to incline** only in 15 deg in both coordinates.
- **Settling time**: less than 5 seconds
- **Rise time**: less than 1 seconds
- **Overshoot:** less than 20%
- **Steady state error:** less than 0.1%

Designing steps

01

Analyzing

Controllability check, Rootlocus analysis

02

Simulation

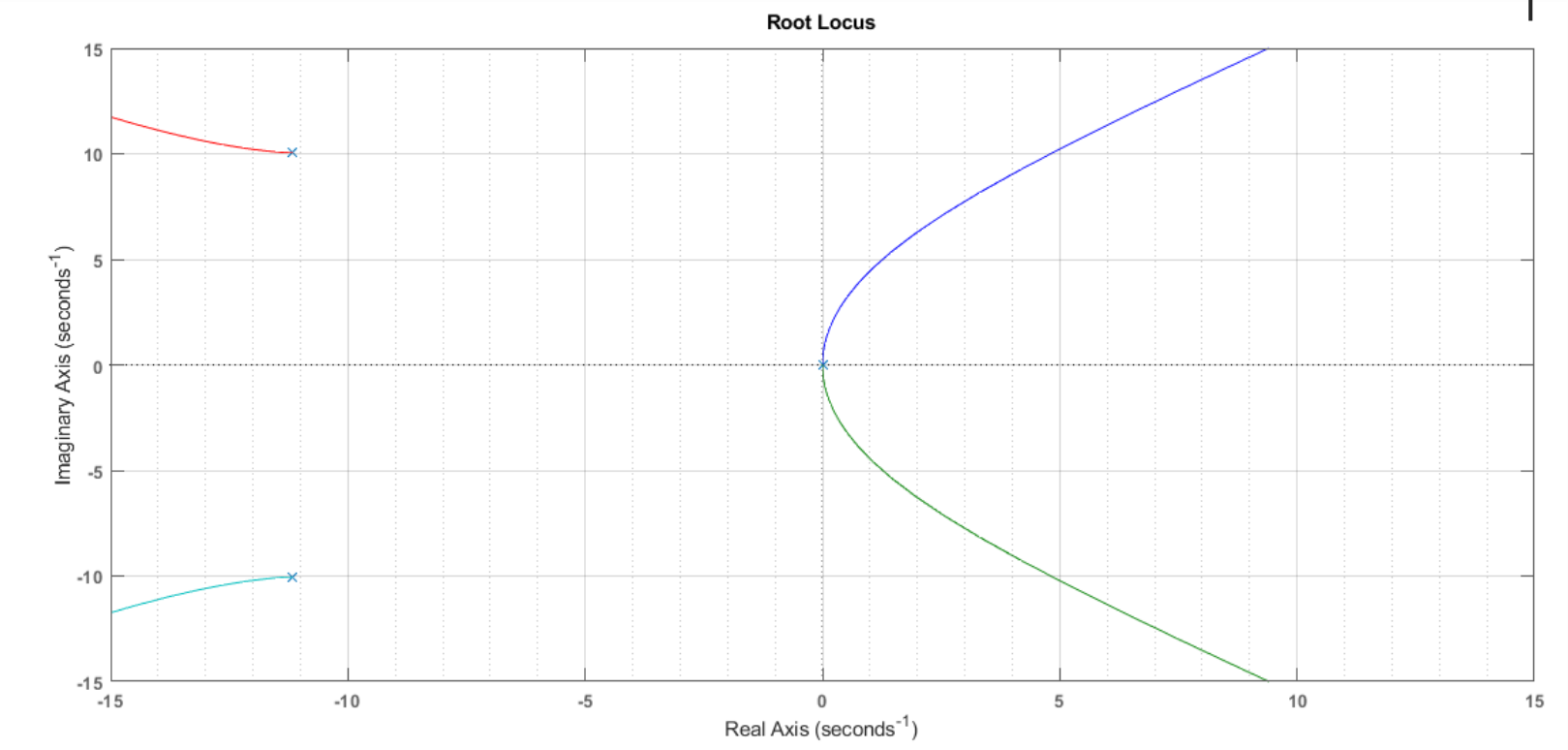
Simulink modeling and tuning

03

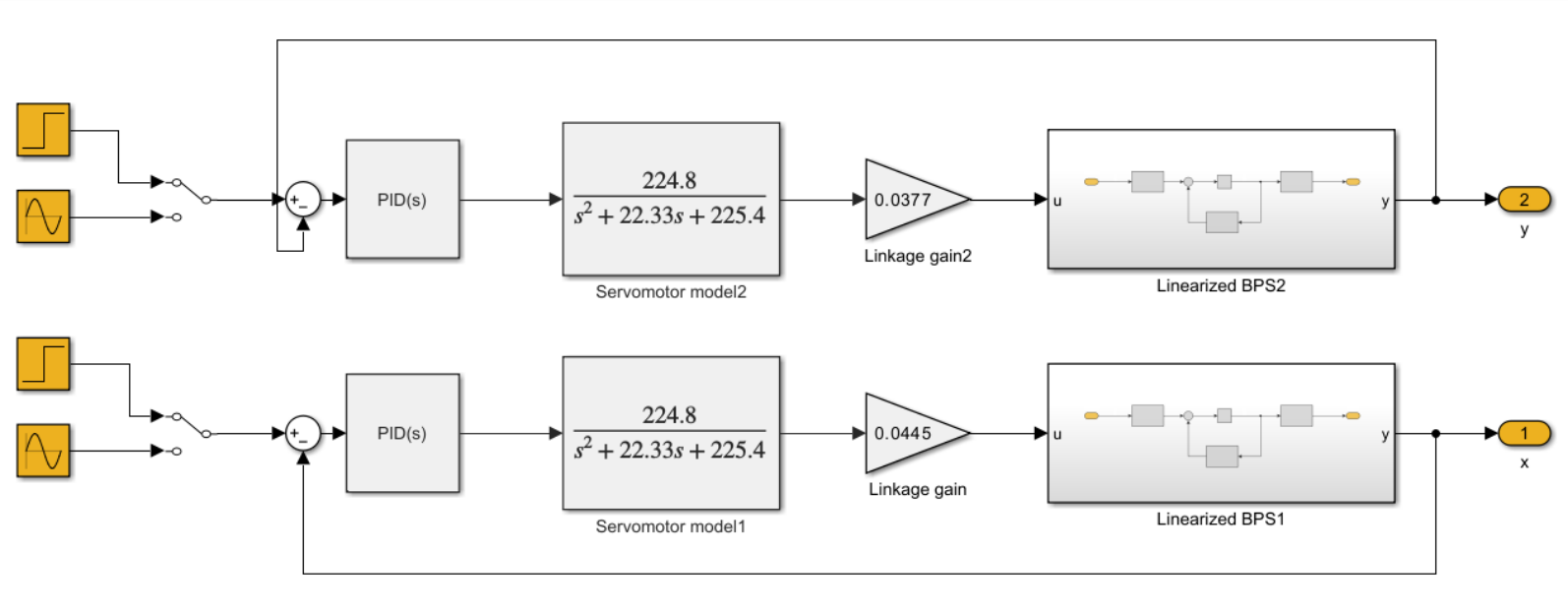
Experimental

Implementation and testing via simulink and arduino

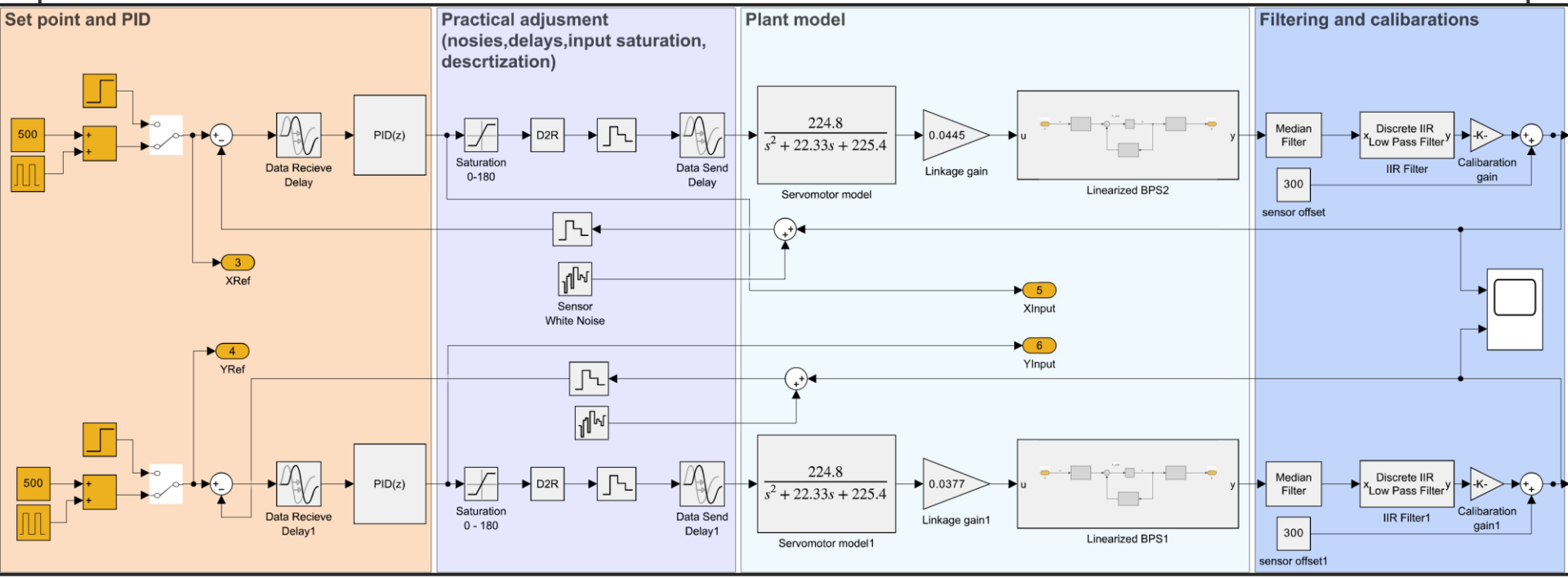
Root locus with no controller



Continuous time Simulink model for PID controller



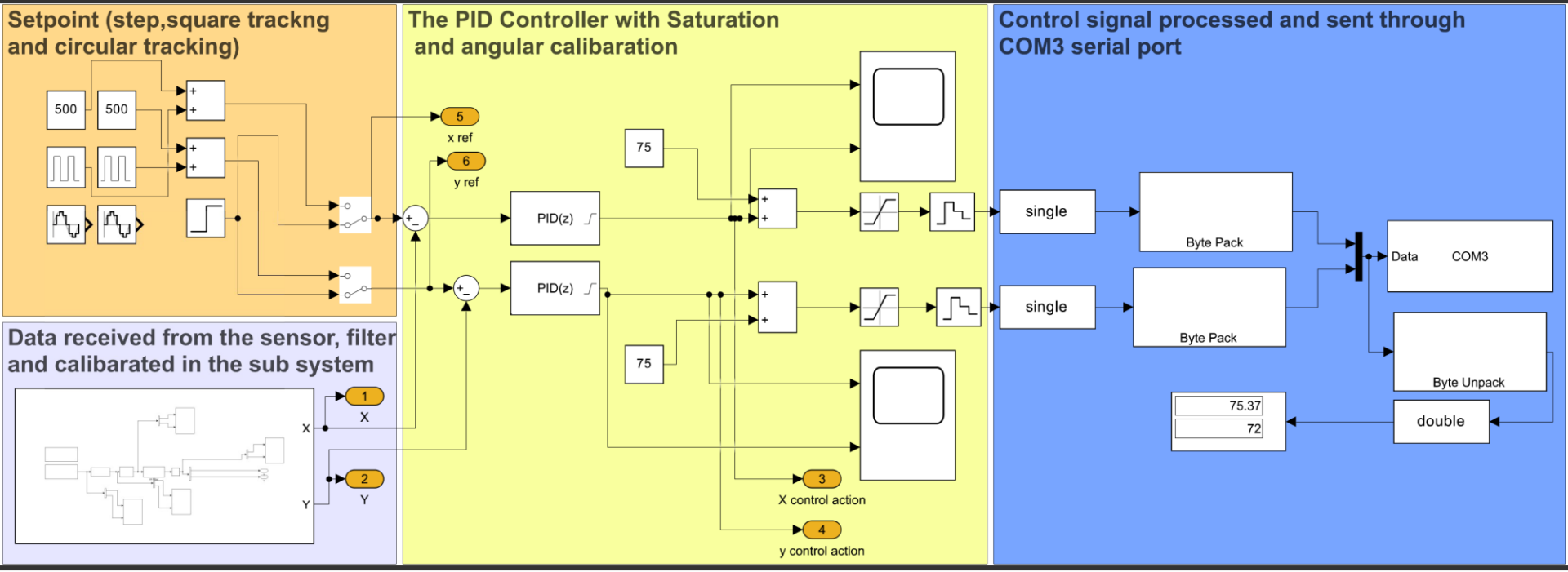
Discrete simulink model with practical adjustment PID



Tuned PID parameters

	Prpotional gain	Integral gain	Derivative gain
PID X 1	0.125	0.00184	0.098
PID Y 1	0.127	0.00084	0.088
PID X 2	0.165	0.00085	0.11
PID Y 2	0.167	0.00084	0.105
PID X 3	0.105	0.00084	0.07
PID Y 3	0.107	0.00184	0.069

Simulink model with practical adjustment PID

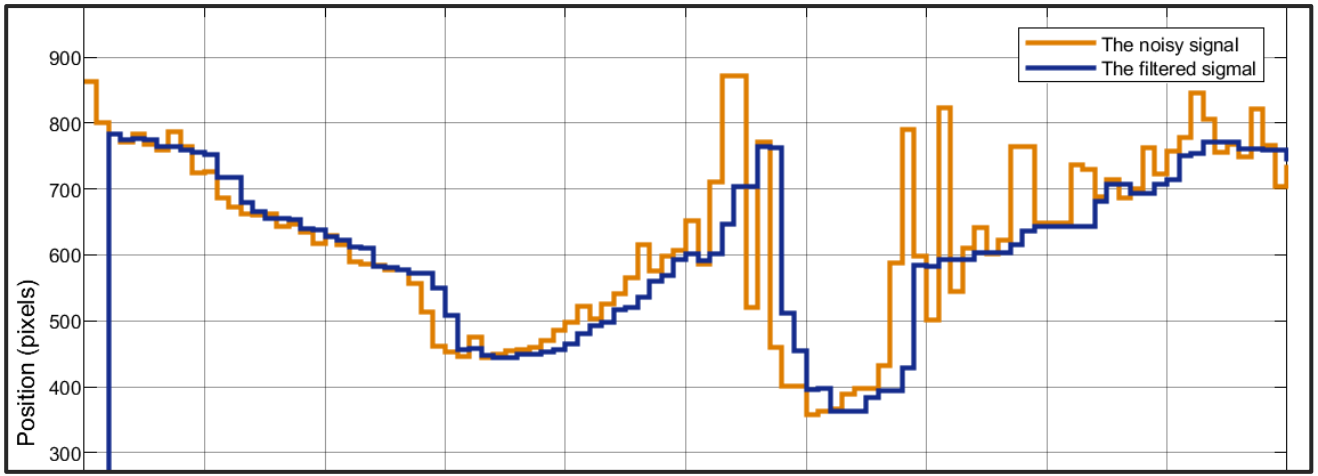


Challenges and limitation

Sensor noises and filtering delay

To eliminate signal noises 3 filters was applied:

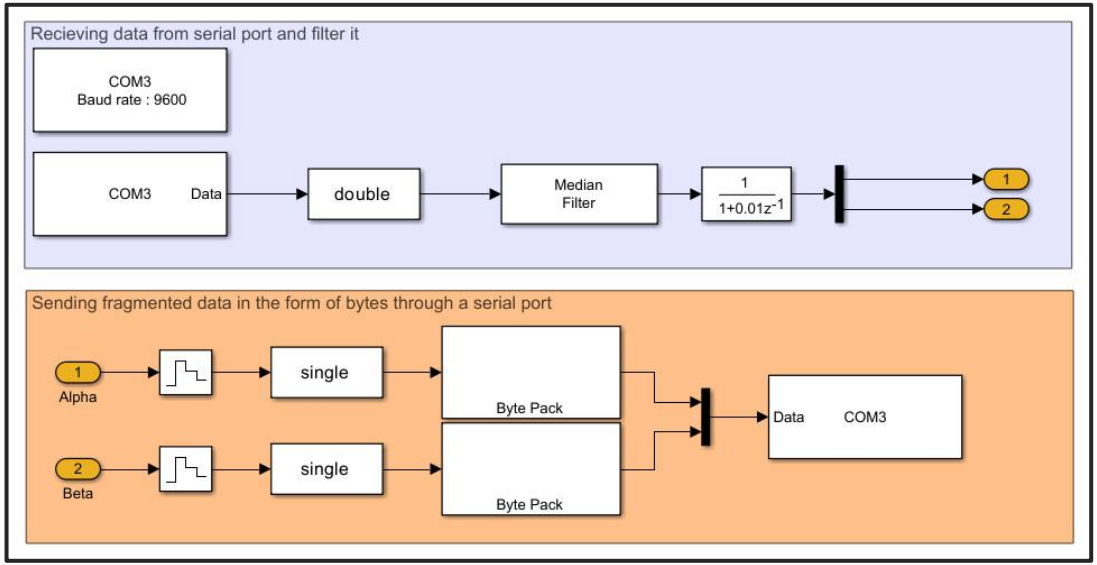
- Arduino code detect when a touch happen
- Athematic median filter with 3 windows size
- Low pass filter



Challenges and limitation

Connecting Arduino with Matlab

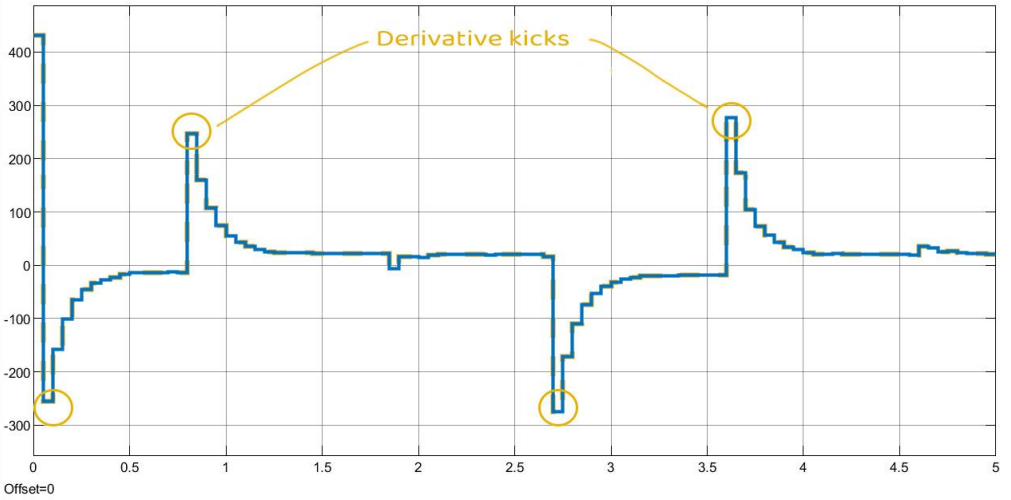
To connect arduino with matlab serial communication was used, this may cause a transpotation delay



Challenges and limitation

Derivative kicks and noises degradation

The suddenly changes in setpoint or the input make an undesired phenomenon called Derivative kicks



05

Validation and Results

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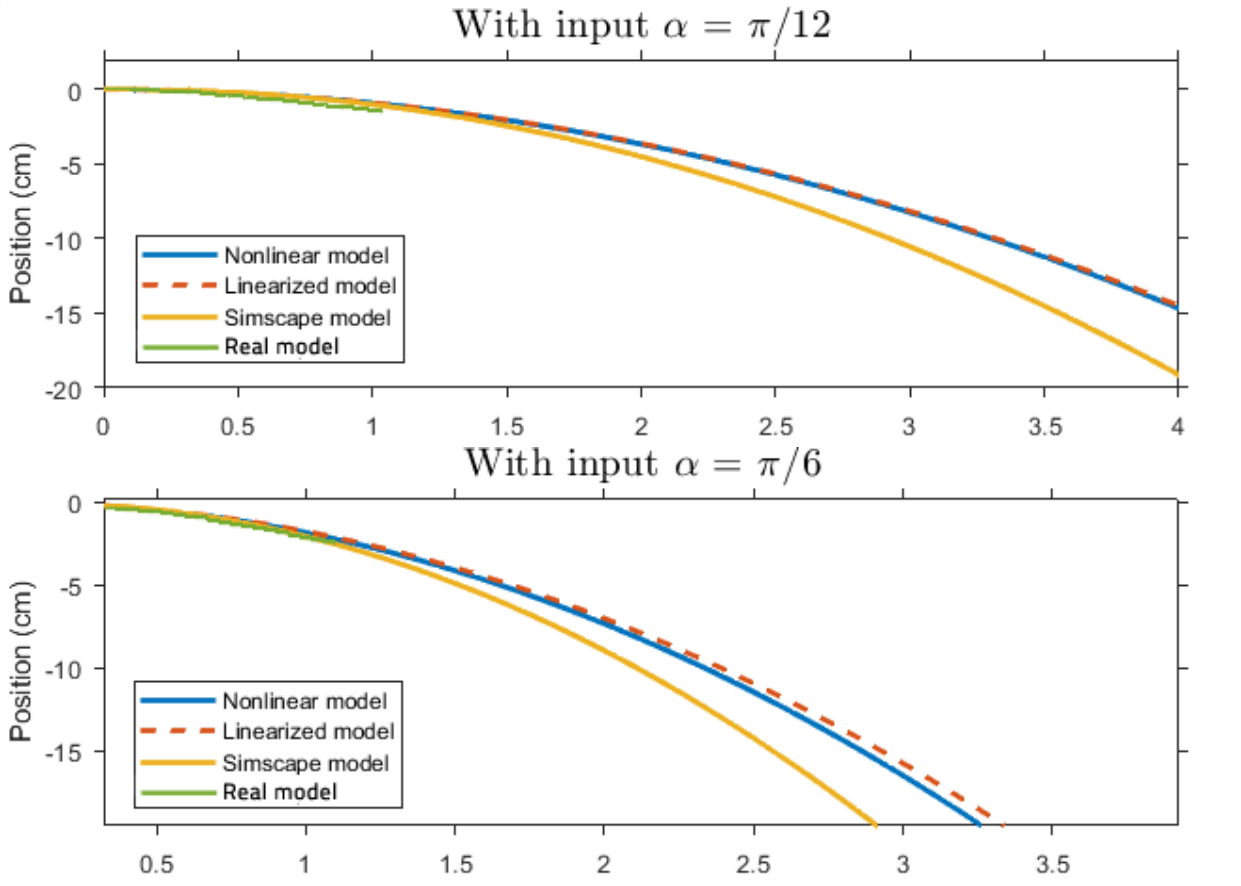
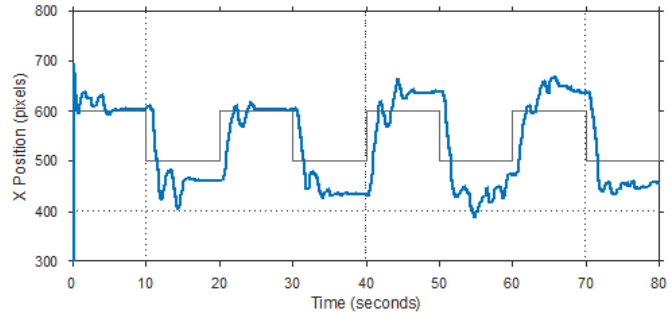
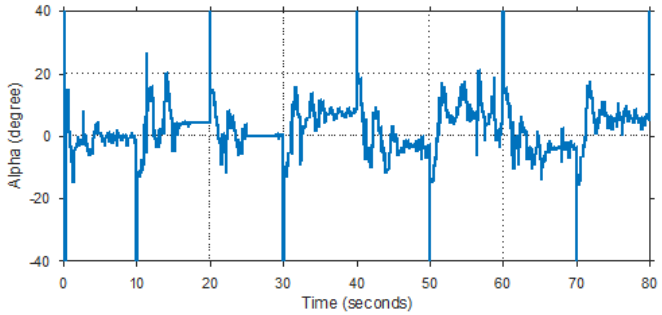
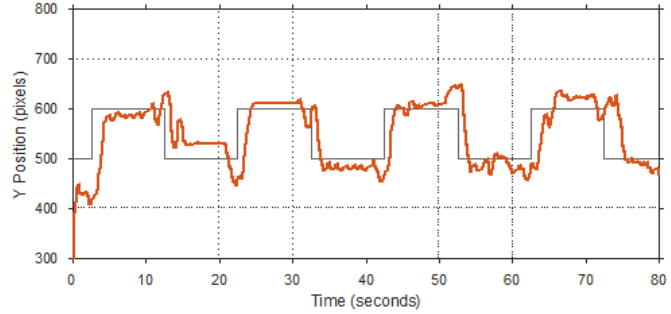
Validation of modeling and linearization

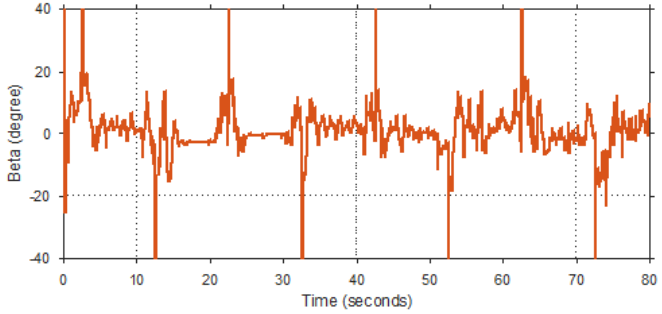


Experimental response and control action

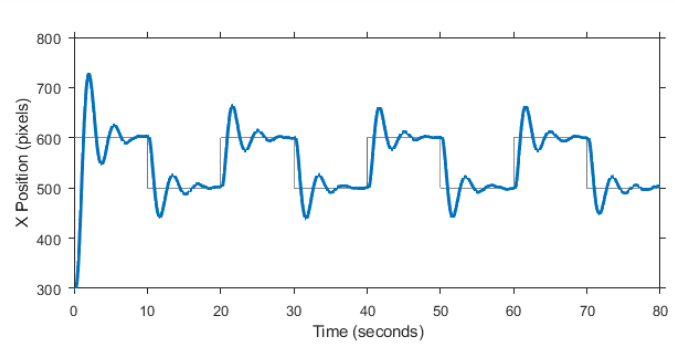


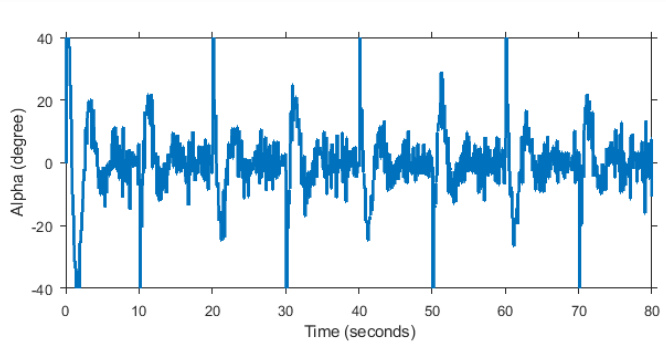


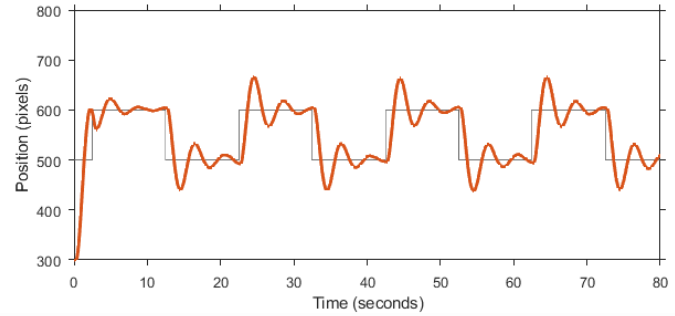


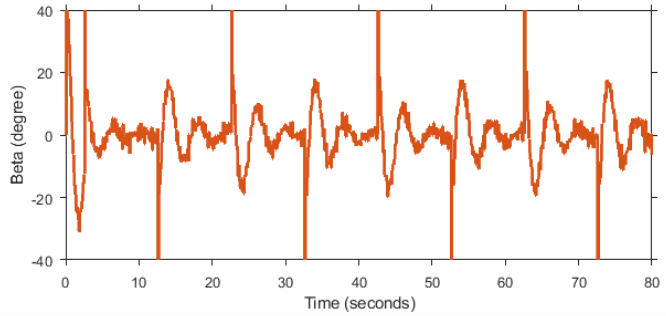


Simulation response and control action









06

Conclusions and Recommendations

You can describe the topic of the section here

Conclusions

- Mercury is
- Jupiter is a gas giant, the biggest planet in the Solar System and the fourth-brightest object in the night sky
- Neptune is the farthest planet from the Sun. It's also the fourth-largest planet by diameter in the Solar System
- Venus has a beautiful name and is the second planet from the Sun. It’s terribly hot—even hotter than Mercury
- Saturn is a gas giant and has several rings. This planet is composed mostly of hydrogen and helium
- Earth is the third planet from the Sun and the only one that harbors life in the Solar System

Thanks!

Do you have any questions?

Purpose statement

What about Mercury?

Mercury is the closest planet to the Sun and the smallest one in the Solar System. The planet’s name has nothing to do with the liquid metal

What about Venus?

Venus has a beautiful name and is the second planet from the Sun. It’s terribly hot and its atmosphere is extremely poisonous

What about Jupiter?

Jupiter is a gas giant and the biggest planet in the Solar System. It's the fourth-brightest object in the sky. It was named after a Roman god

“This is a quote. Words full of wisdom that someone important said and can make the reader get inspired”

—Someone Famous

98,300,000

Big numbers catch your audience’s attention

9h 55m 23s

Jupiter's rotation period

333,000

The Sun’s mass compared to Earth’s

386,000 km

Distance between Earth and the Moon

A picture is worth a thousand words

A picture is worth a thousand words

Thanks!

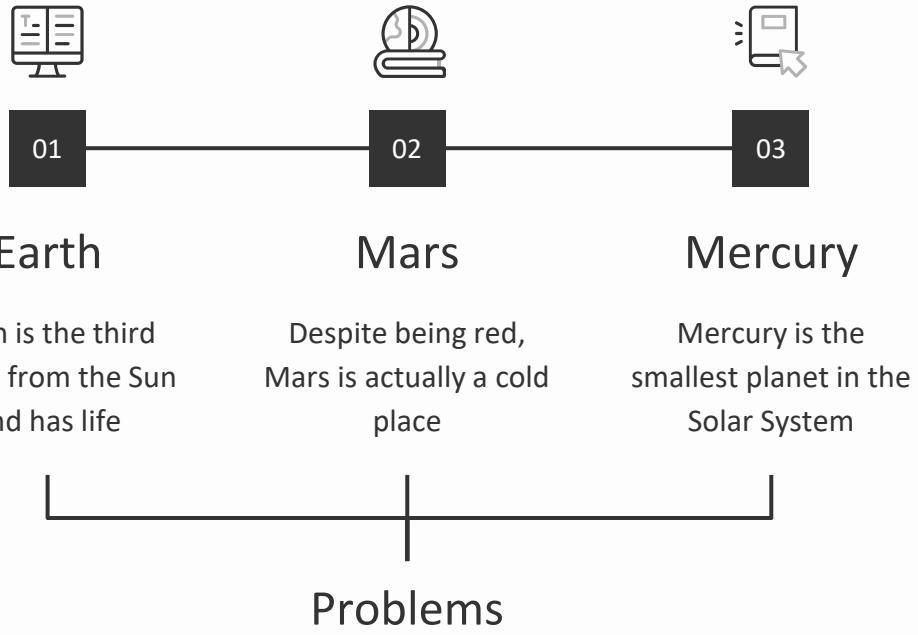
Do you have any questions?

youremail@freepik.com
+34 654 321 432
yourwebsite.com

Current situation & problems statement

Current situation

Jupiter is a gas giant and the biggest planet in the Solar System. It's the fourth-brightest object in the night sky. It was named after the Roman god of the skies and lightning



Hypotheses

Hypothesis 1

Mercury is the closest planet to the Sun.The planet’s name has nothing to do with the liquid metal

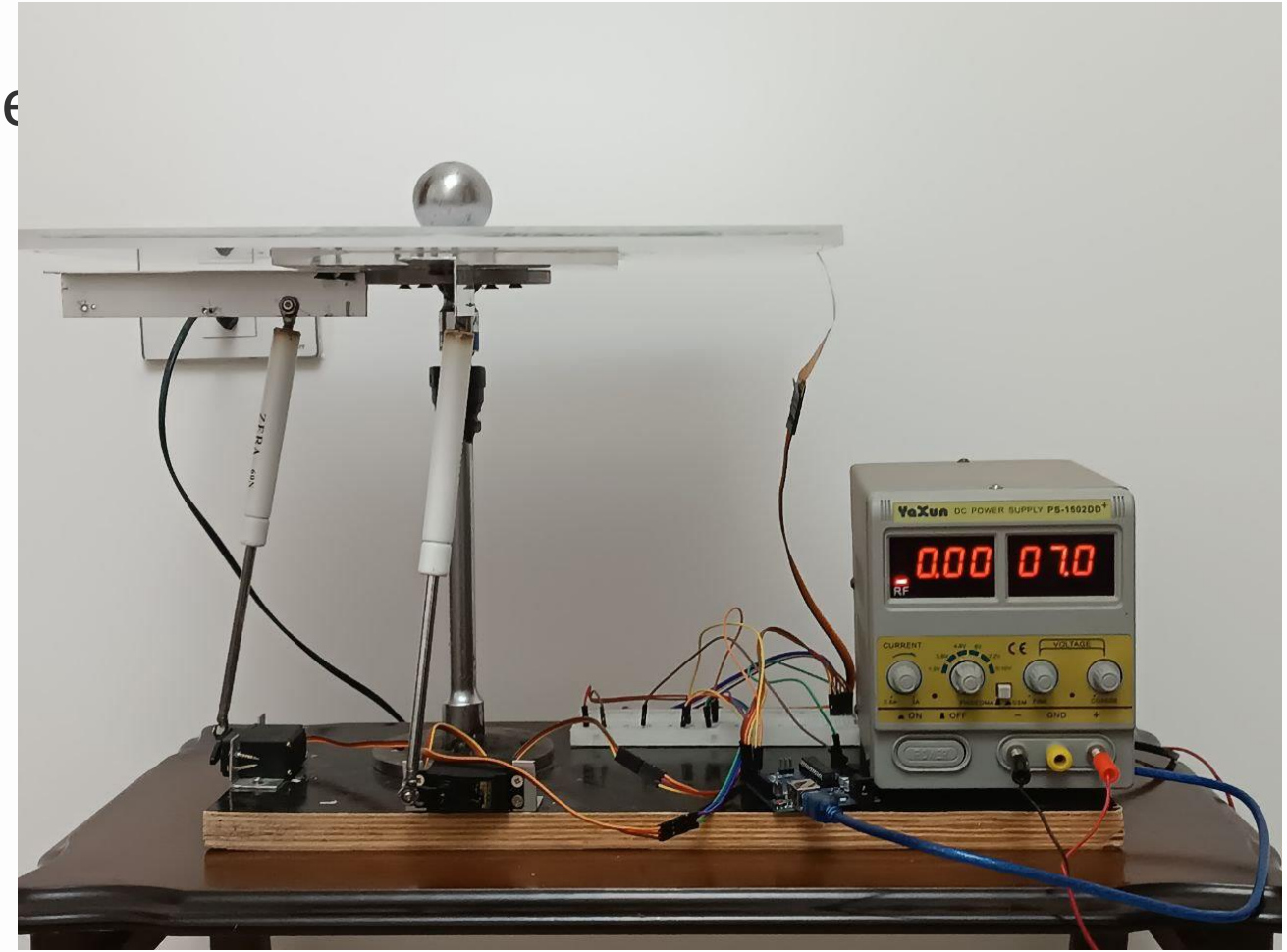
Hypothesis 2

Venus has a beautiful name and is the second planet from the Sun. It’s hot and its atmosphere is poisonous

Hypothesis 3

Jupiter is a gas giant and the biggest planet in the Solar System. It was named after the Roman god

Revie


YaXun DC POWER SUPPLY PS-1502DD+
0.00
07.0
CURRENT
VOLTAGE
ON
OFF
GND

Reviewing concepts is a good idea

Mars

Mars is actually a very cold place

Venus

Venus has extremely high temperatures

Neptune

Neptune is the farthest planet from the Sun

Mercury

Mercury is the closest planet to the Sun

Saturn

Saturn is a gas giant with several rings

Jupiter

Jupiter is the biggest planet of them all

Literature review

- AUTHOR. (YEAR). *Title of the publication*. Publisher
  - Mercury is the closest planet to the Sun and the smallest one
- AUTHOR. (YEAR). *Title of the publication*. Publisher
  - Mars is full of iron oxide dust, which gives the planet its reddish cast
- AUTHOR. (YEAR). *Title of the publication*. Publisher
  - Jupiter is a gas giant and the biggest planet in the Solar System
- AUTHOR. (YEAR). *Title of the publication*. Publisher
  - Venus has a beautiful name and is the second planet from the Sun
- AUTHOR. (YEAR). *Title of the publication*. Publisher
  - Earth is the third planet from the Sun and harbors life

Theoretical framework

Theoretical framework		
Key terms	Relevant theories	Our framework
● Mercury is small ● Earth harbors life ● Jupiter is big	**Theory 1** Saturn is a gas giant and has rings. It’s composed mostly of hydrogen and helium **Theory 2** Neptune is the farthest planet from the Sun in Solar System and also an ice giant	 Venus has a beautiful name and is the second planet from the Sun. It’s very hot

Schedule

Task	Description	Date	J	F	M	A	M	J	Status
Task 1	Despite being red, Mars is cold	Jan 1 - Feb 15							Completed
Task 2	Earth is the planet with life	Feb 1 - Apr 30							In progress
Task 3	Venus has a beautiful name	Mar 15 - Apr 30							Delayed
Task 4	Neptune is far away from us	Apr 20 - May 15							Unstarted
Task 5	Jupiter is a huge gas giant	Jun 4 - Jun 30							Unstarted

Methodology

Mercury

- **Type of data** — Mercury is the closest planet to the Sun and the smallest one
- **Motives** — Venus has a beautiful name and is the second planet from the Sun
- **Data collection** — Mars is full of iron oxide dust, which gives the planet its reddish cast
- **Sampling** — Jupiter is a gas giant and the biggest planet in the Solar System

Analysis & development

Phase 01

- Mercury is the closest planet to the Sun and the smallest one in the Solar System—it’s a bit larger than the Moon
- Jupiter is a gas giant, the biggest planet in the Solar System and the fourth-brightest object in the night sky
- Neptune is the farthest planet from the Sun. It's also the fourth-largest planet by diameter in the Solar System

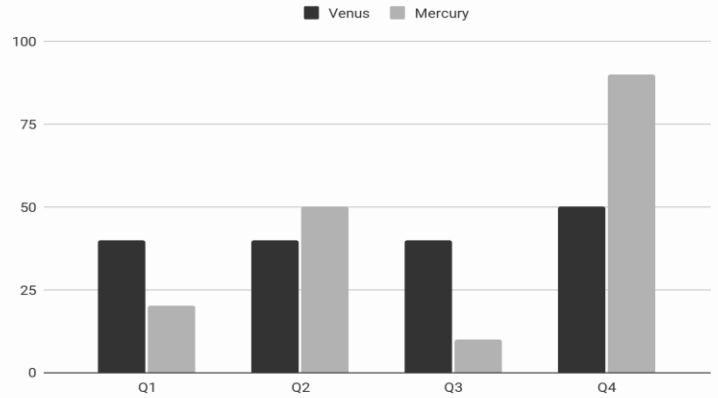
Phase 02

- Venus has a beautiful name and is the second planet from the Sun. It’s terribly hot—even hotter than Mercury
- Saturn is a gas giant and has several rings. This planet is composed mostly of hydrogen and helium
- Earth is the third planet from the Sun and the only one that harbors life in the Solar System

Analysis & development

Mercury is the closest planet to the Sun and the smallest one in the Solar System The planet's name has nothing to do with the liquid metal

- The Sun is the star at the center of the Solar System
- Jupiter is the biggest planet in the entire Solar System
- Saturn is composed mostly of hydrogen and helium



Venus

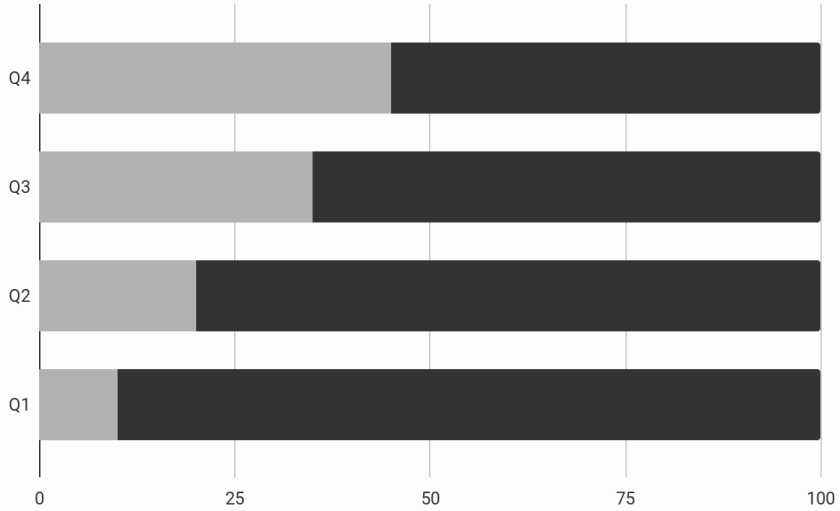
Venus has a beautiful name

Mercury

Mercury is quite a small planet

Follow the link in the graph to modify its data and then paste the new one here. **For more info, click here**

Analysis of the results



Mercury

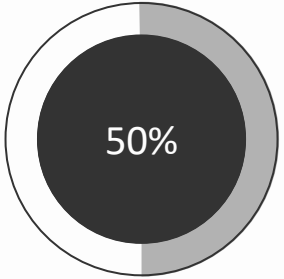
Mercury is quite a small planet

Venus

Venus has a beautiful name

Mars

Despite being red, Mars is a planet very cold



Follow the link in the graph to modify its data and then paste the new one here. **For more info, click here**

This is a map

Venus

Venus is the second planet from the Sun

Mercury

Mercury is the closest planet to the Sun

Mars

Despite being red, Mars is a cold place

Discussion

Mars & Earth

Despite being red, Mars is actually a cold place. It's full of iron oxide dust, which gives the planet its reddish cast. Earth is the third planet from the Sun and the only one that harbors life in the Solar System. This is where we all live:

- Ceres is located in the main asteroid belt
- The Moon is Earth’s natural satellite
- Neptune is very far away from us
- Pluto is now considered a dwarf planet

Discussion 1

Mercury is the closest planet to the Sun and the smallest one in the Solar System. It’s bit larger than the Moon

Discussion 2

Venus has a beautiful name and is the second planet from the Sun. Venus is a lot hotter than Mercury

Conclusions

Mars

Despite being red, Mars is actually a cold place. It’s full of iron oxide dust

Venus

Venus has a very beautiful name and is the second planet from the Sun

Mercury

Mercury is the closest planet to the Sun and the smallest one in the Solar System

Neptune

Neptune is a ig planet. It is the fourth-largest planet by diameter in the Solar System

Bibliographical references

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- Medium shot people at traveling agency
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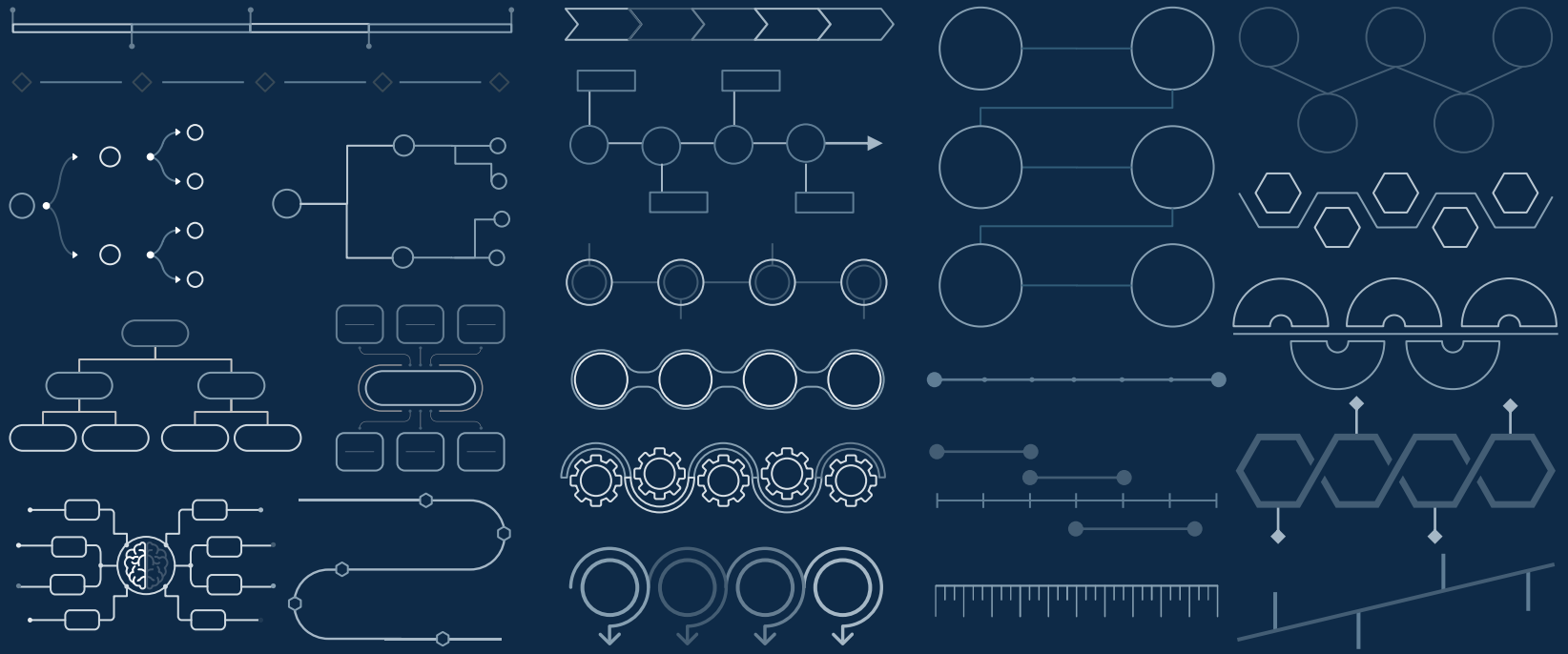

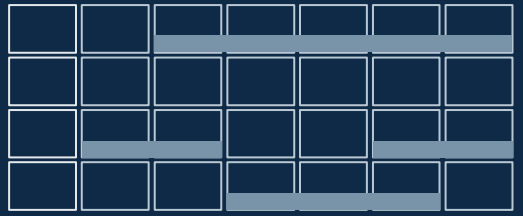

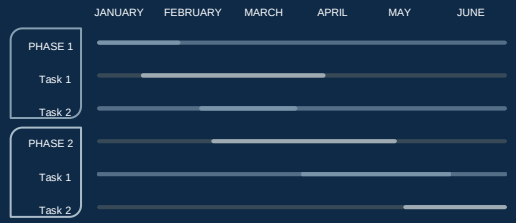
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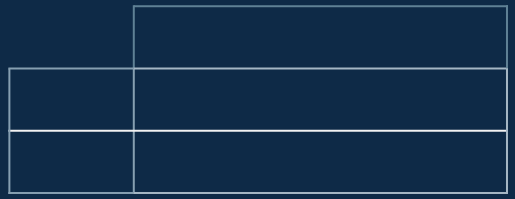

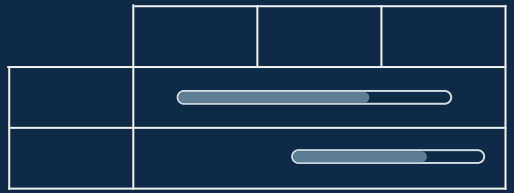

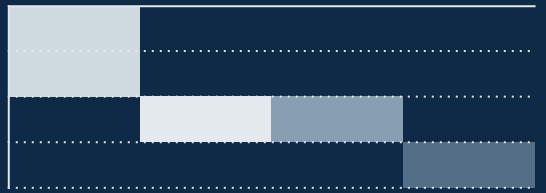

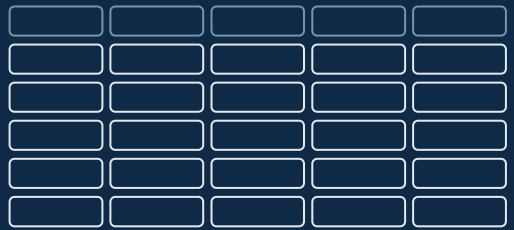

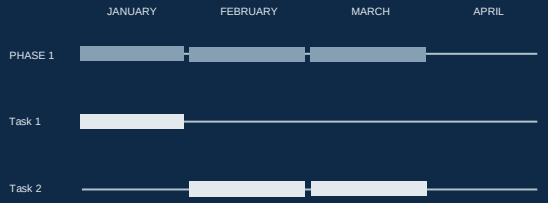
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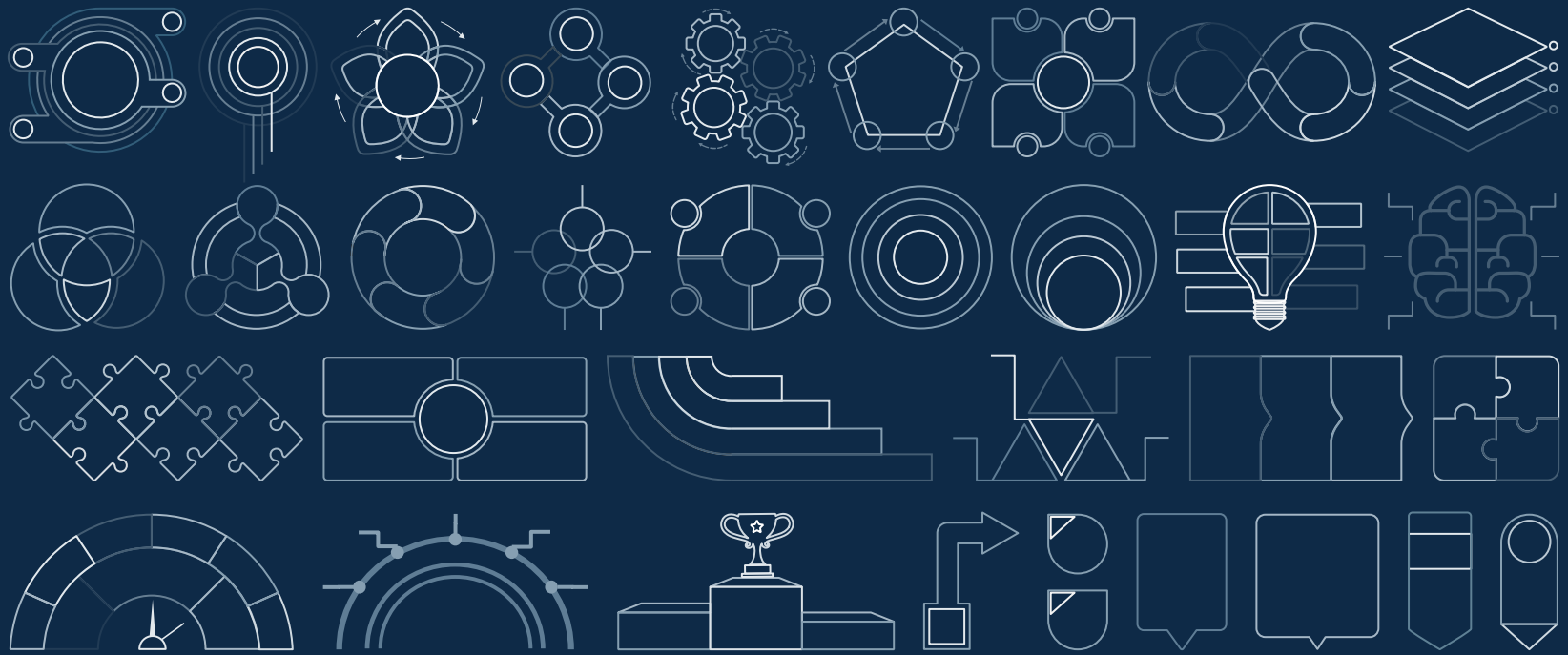

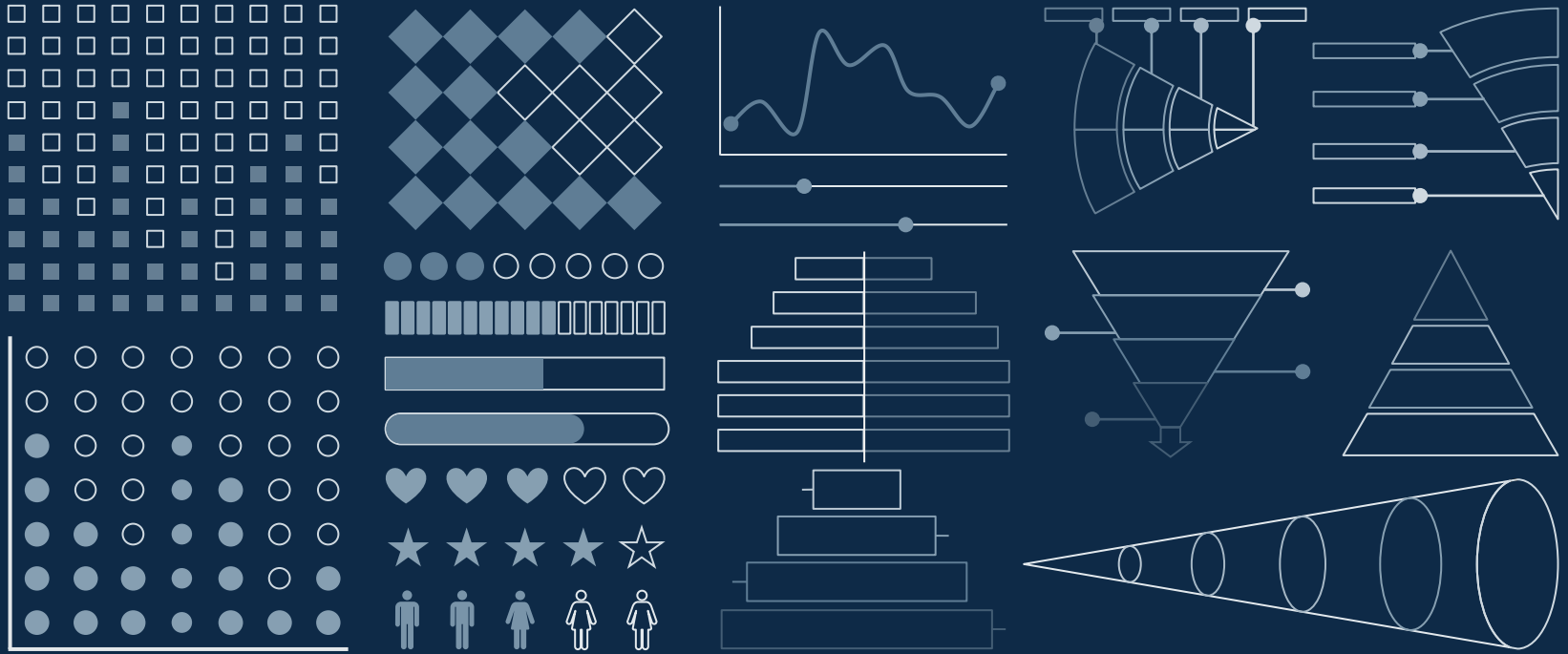

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