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Esercitazione

DISCo

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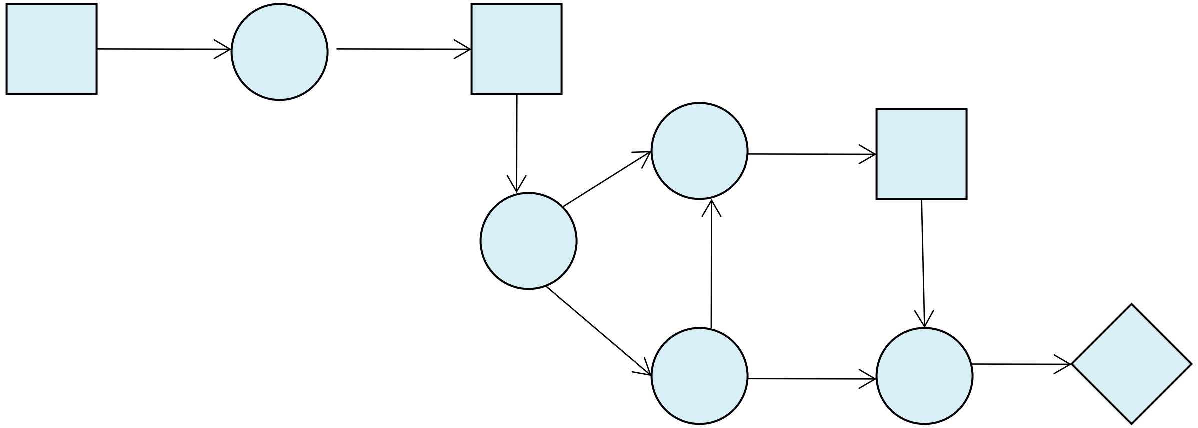
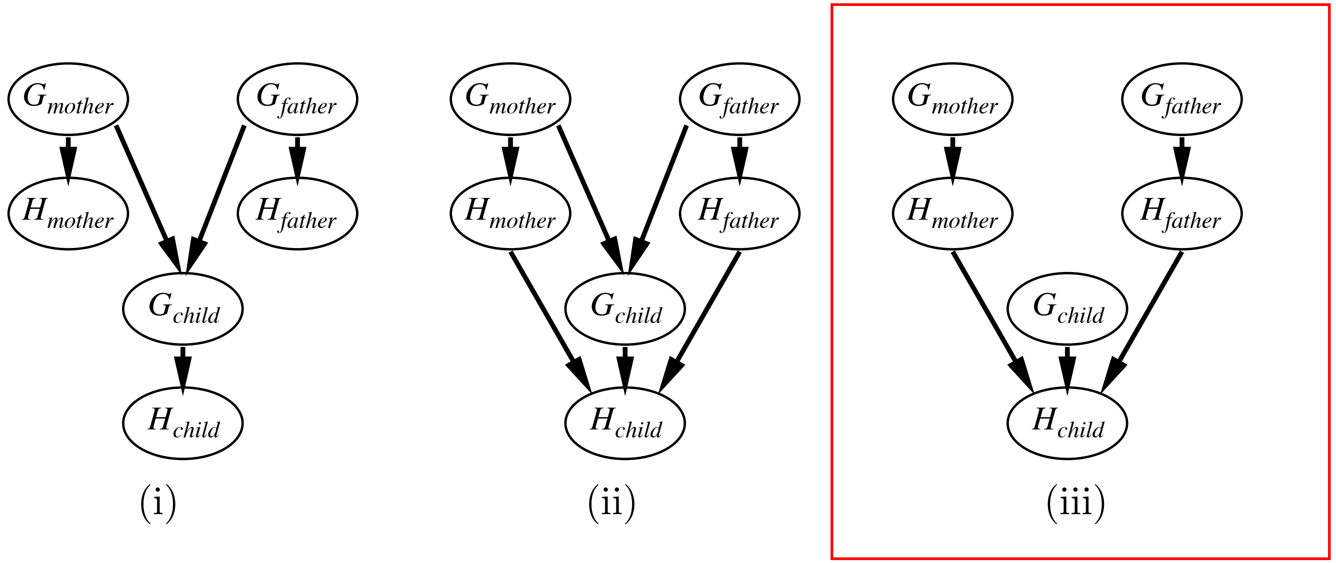


Esercizio 1

- Sia Hx una variabile casuale che denota la manualità di un individuo x, la quale può assumere valore L=Left e R=Right. Un'ipotesi comune è che l'essere destrorsi (R) o mancini (L) sia ereditato da un semplice meccanismo: c'è un gene Gx, che assume valori L or R, e con probabilità s l'individuo assume la stessa manualità del gene. Inoltre, il gene stesso ha uguale probabilità di essere ereditato da un genitore, ma con probabilità m può ereditare una manualità opposta a quella dei genitori.

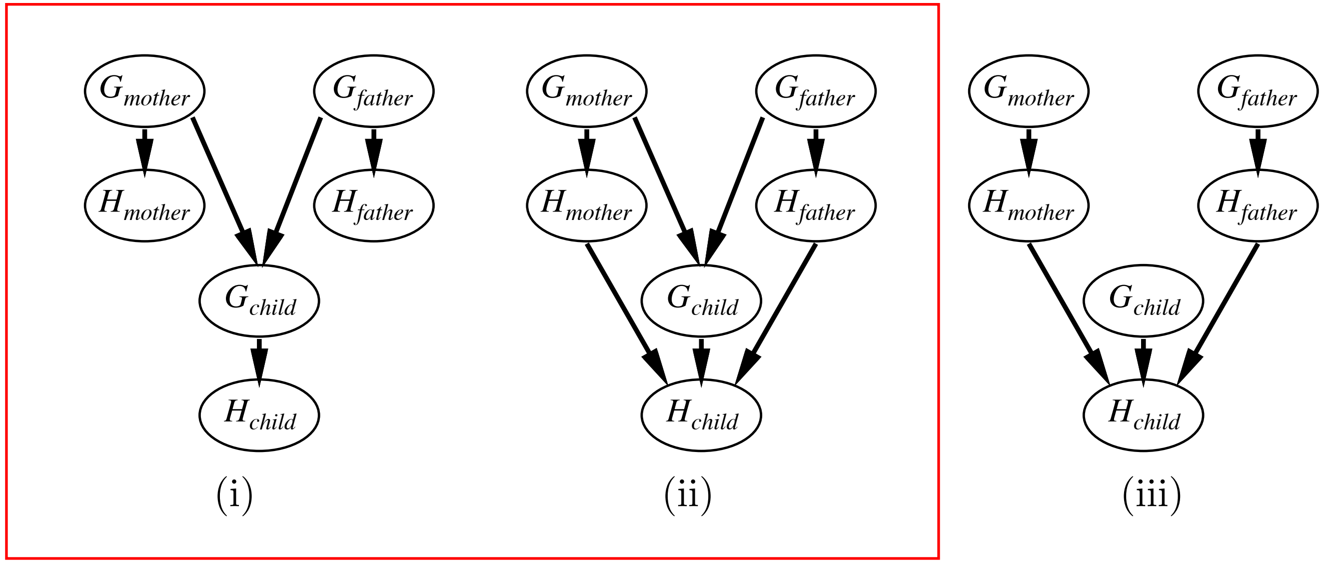
Esercizio 1

1. Quali delle tre reti asserisce che $P(G_{father}, G_{mother}, G_{child}) = P(G_{father})P(G_{mother})P(G_{child})$?



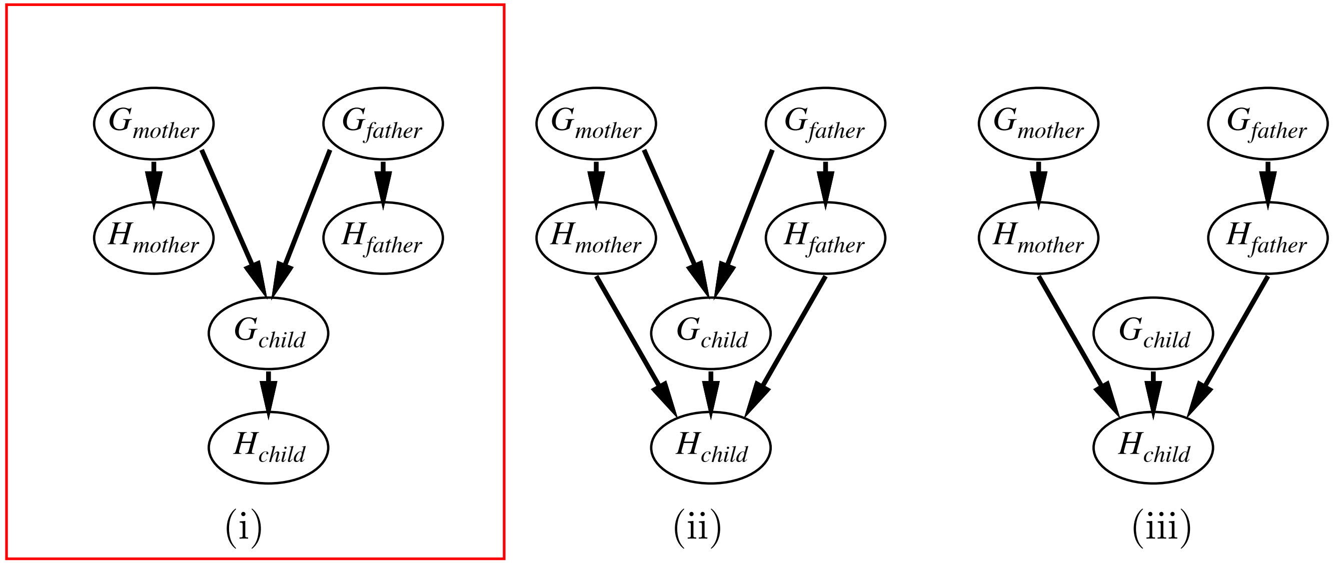
Esercizio 1

2. Quali delle tre reti bayesiane afferma le condizioni di indipendenza coerenti con l’ipotesi descritta nel testo del problema?



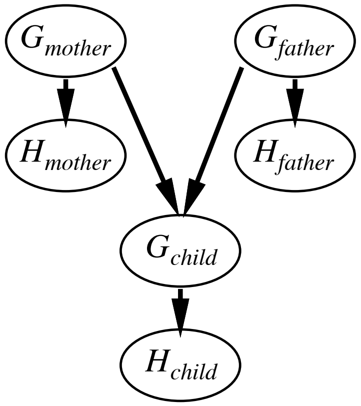
Esercizio 1

3. Quale delle tre reti descrive al meglio l'ipotesi descritta nel problema?



Esercizio 1

4. Scrivere una CPT per G_{child} relative alla rete (i) o (ii), coerente con le specifiche del problema.

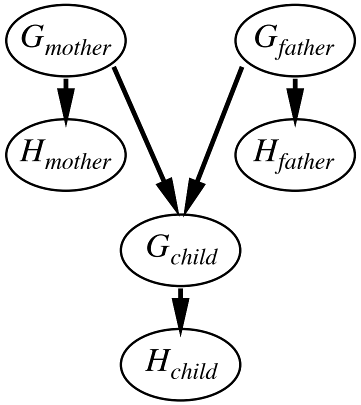


(i)

G_{mother}	G_{father}	$P(G_{child} \mid G_{mother}, G_{father})$	
		L	**R**
L	**L**		
L	**R**		
R	**L**		
R	**R**		

Esercizio 1

5. Supponiamo che $P(G_{father} = L) = P(G_{mother} = L) = x$. Si derivi per la rete (i) or (ii), l'espressione per $P(G_{child} = L)$ in termini di m ed x, condizionado solo rispetto ai nodi genitori.



(i)

$$P(G_{child} = L) = \sum_{G_{father}} \sum_{G_{mother}} P(G_{father}) * P(G_{mother}) * P(G_{child} | G_{father}, G_{mother})$$

$$= P(G_{father} = L) * P(G_{mother} = L) * P(G_{child} = L | G_{father} = L, G_{mother} = L) +$$

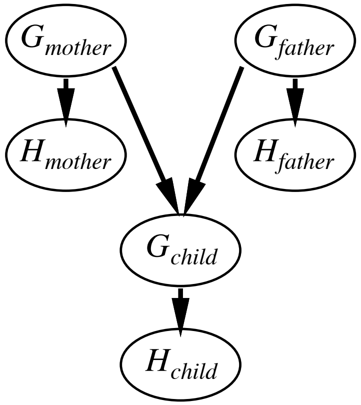
$$+ P(G_{father} = R) * P(G_{mother} = R) * P(G_{child} = L | G_{father} = R, G_{mother} = R) +$$

$$+ P(G_{father} = L) * P(G_{mother} = R) * P(G_{child} = L | G_{father} = L, G_{mother} = R) +$$

$$+ P(G_{father} = R) * P(G_{mother} = L) * P(G_{child} = L | G_{father} = R, G_{mother} = L)$$

Esercizio 1

5. Supponiamo che P(G_{father} =L) = P(G_{mother} =L) = x. Si derivi per la rete (i) or (ii), l'espressione per P(G_{child} = L) in termini di m ed x, condizionado solo rispetto ai nodi genitori.



(i)

$$= \overbrace{P(G_{father} = L) * P(G_{mother} = L)}^{x^2} * \overbrace{P(G_{child} = L | G_{father} = L, G_{mother} = L)}^{(1-m)} +$$

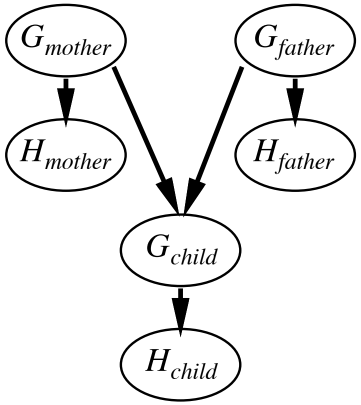
$$+ \overbrace{P(G_{father} = R) * P(G_{mother} = R)}^{(1-x^2)} * \overbrace{P(G_{child} = L | G_{father} = R, G_{mother} = R)}^{m} +$$

$$+ \overbrace{P(G_{father} = L) * P(G_{mother} = R)}^{(x)*(1-x)} * \overbrace{P(G_{child} = L | G_{father} = L, G_{mother} = R)}^{0.5} +$$

$$+ \overbrace{P(G_{father} = R) * P(G_{mother} = L)}^{(1-x)*(x)} * \overbrace{P(G_{child} = L | G_{father} = R, G_{mother} = L)}^{0.5}$$

Esercizio 1

5. Supponiamo che P(G_{father} =L) = P(G_{mother} =L) = x. Si derivi per la rete (i) or (ii), l'espressione per P(G_{child} = L) in termini di m ed x, condizionado solo rispetto ai nodi genitori.



(i)

$$P(G_{child} = L) = \sum_{G_{father}} \sum_{G_{mother}} P(G_{father}) * P(G_{mother}) * P(G_{child}|G_{father}, G_{mother})$$

$$= (x^2 - mx^2) + (m + mx^2 - 2mx) + 0.5x - 0.5x^2 + 0.5x - 0.5x^2 =$$

$$= x^2 - mx^2 + m + mx^2 - 2mx + 0.5x - 0.5x^2 + 0.5x - 0.5x^2 =$$

$$= m + x - 2mx$$

Esercizio 2

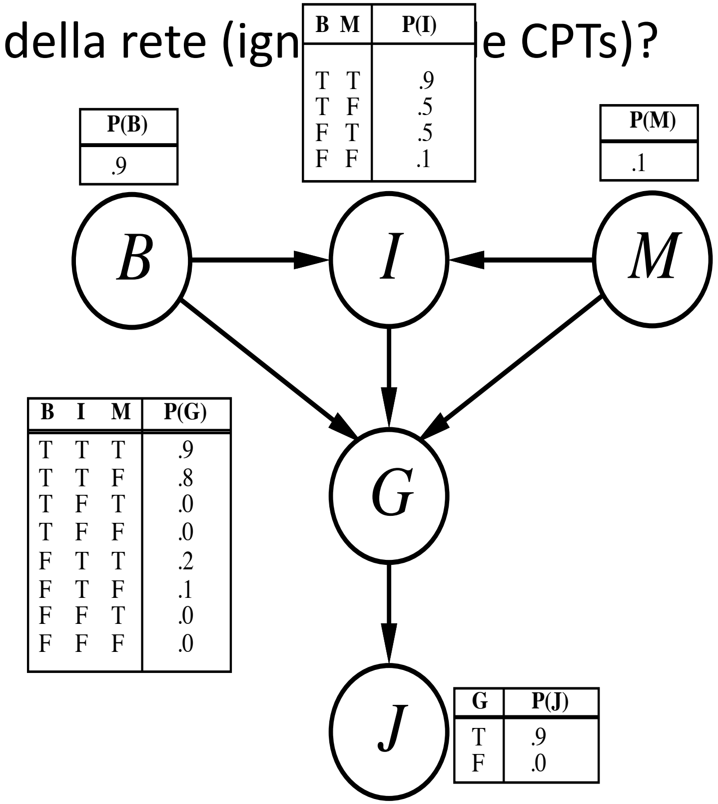
- Si consideri la seguente Rete Bayesiana.

1. Quali delle seguenti affermazioni è asserita dalla struttura della rete (ign e CPTs)?

➢ P(B, I, M) = P(B)P(I)P(M) falso

➢ P(J|G) = P(J|G, I) vero

➢ P(M|G, B, I) = P(M|G, B, I, J) vero

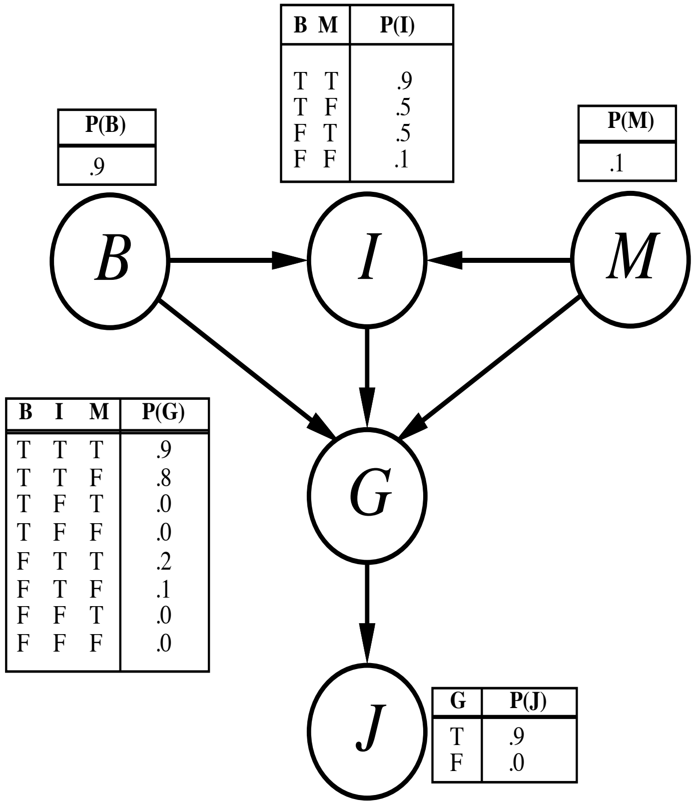


Esercizio 2

2. Calcolare il valore di P (b, i, ¬m, g, j).

$$P\ (b, i, \neg m, g, j) = P(b) * P(\neg m) * P(i|b, \neg m) * P(g|b, i, \neg m) * P(j|g) =$$

$$= 0.9 * 0.9 * 0.5 * 0.8 * 0.9 = 0.291$$



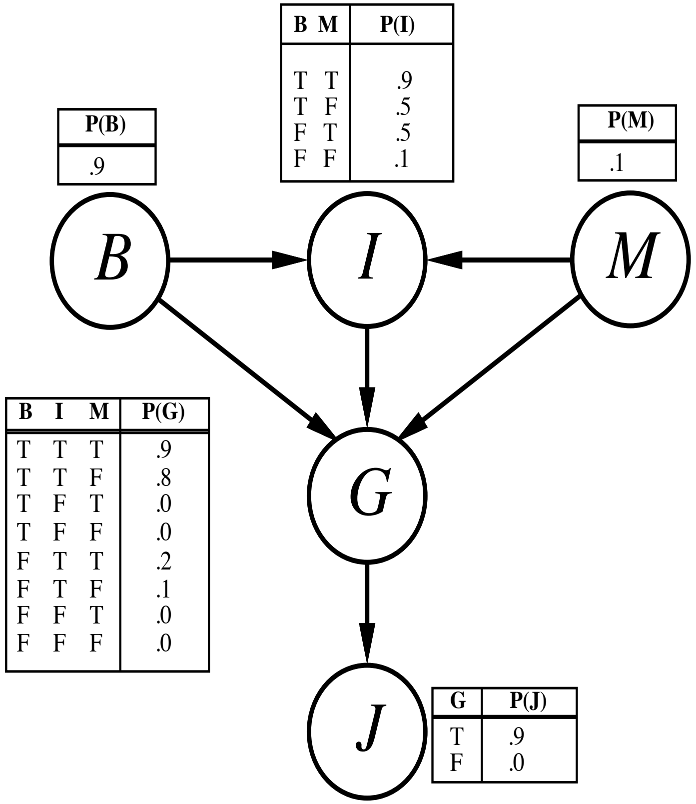
Esercizio 2

3. Calcolare P(J|b,i,m)

$$P(J|b,i,m) = \alpha \sum_{G} P(J,b,i,m,G)$$

$$P(j|b,i,m)$$

$$P(\neg j|b,i,m)$$



P(B)
.9

B	M	P(I)
T	T	.9
T	F	.5
F	T	.5
F	F	.1

P(M)
.1

B	I	M	P(G)
T	T	T	.9
T	T	F	.8
T	F	T	.0
T	F	F	.0
F	T	T	.2
F	T	F	.1
F	F	T	.0
F	F	F	.0

G	P(J)
T	.9
F	.0

Esercizio 2

3. Calcolare P(J|b,i,m)

$$P(j|b,i,m) = \alpha \sum_{G} P(j,b,i,m,G) =$$

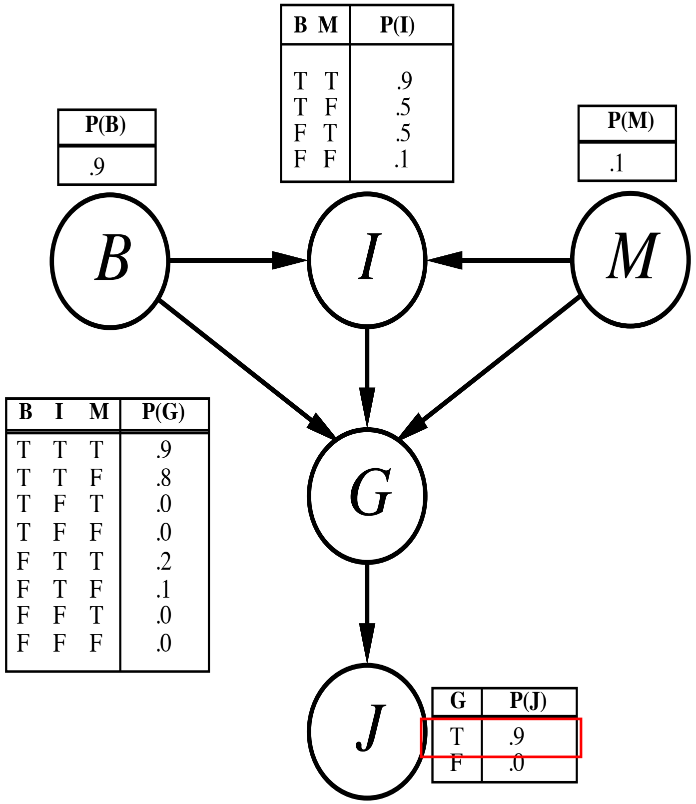
$$= \alpha \sum_{G} P(j|G) * P(G|b,i,m) * P(b) * P(m) * P(i|b,m)$$

$$= \alpha * \boxed{[P(j|g)} * P(g|b,i,m) * P(b) * P(m) * P(i|b,m) +$$

$$+ P(j|\neg g) * P(\neg g|b,i,m) * P(b) * P(m) * P(i|b,m)]$$

$$= \alpha * \boxed{[0.9} * 0.9 * 0.9 * 0.1 * 0.9 + 0 * 0.1 * 0.0 * 0.1 * 0.9]$$

$$= \alpha * [0.06561]$$



P(B)
.9

B	M	P(I)
T	T	.9
T	F	.5
F	T	.5
F	F	.1

P(M)
.1

B	I	M	P(G)
T	T	T	.9
T	T	F	.8
T	F	T	.0
T	F	F	.0
F	T	T	.2
F	T	F	.1
F	F	T	.0
F	F	F	.0

G	P(J)
T	.9
F	.0

Esercizio 2

3. Calcolare P(J|b,i,m)

$$P(\neg j|b,i,m) = \alpha \sum_G P(\neg j,b,i,m,G) =$$

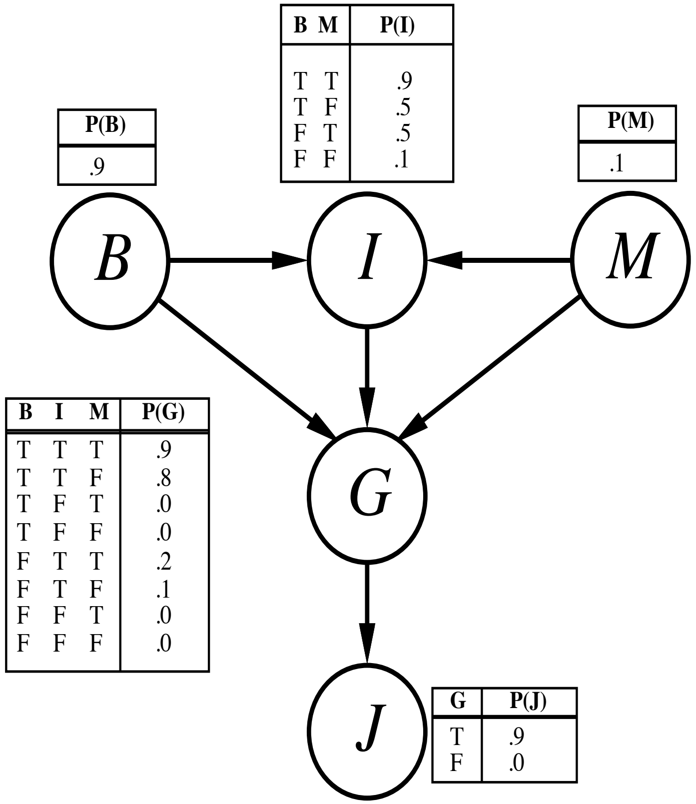
$$= \alpha \sum_G P(\neg j|G) * P(G|b,i,m) * P(b) * P(m) * P(i|b,m)$$

$$= \alpha * [P(\neg j|g) * P(g|b,i,m) * P(b) * P(m) * P(i|b,m) +$$

$$+ P(\neg j|\neg g) * P(\neg g|b,i,m) * P(b) * P(m) * P(i|b,m)]$$

$$= \alpha * [0.1 * 0.9 * 0.9 * 0.1 * 0.9 + 1 * 0.1 * 0.0 * 0.1 * 0.9]$$

$$= \alpha * [0.01539]$$



P(B)
.9

B	M	P(I)
T	T	.9
T	F	.5
F	T	.5
F	F	.1

P(M)
.1

B	I	M	P(G)
T	T	T	.9
T	T	F	.8
T	F	T	.0
T	F	F	.0
F	T	T	.2
F	T	F	.1
F	F	T	.0
F	F	F	.0

G	P(J)
T	.9
F	.0

Esercizio 2

3. Calcolare P(J|b,i,m)

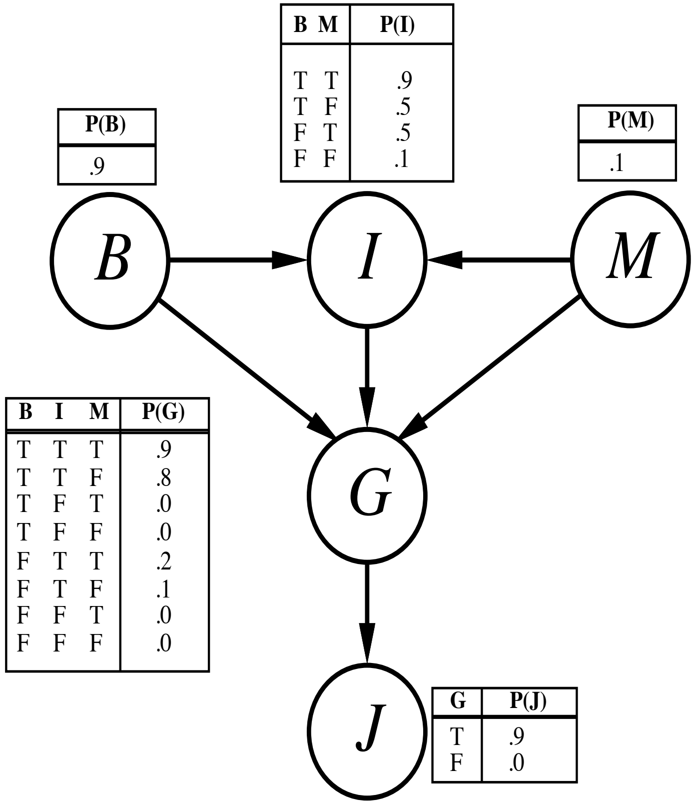
$$P(j|b,i,m) = \alpha * [0.06561]$$

$$P(\neg j|b,i,m) = \alpha * [0.01539]$$

$$P(J|b,i,m) = \alpha * < 0.06561; 0.01539 >$$

$$\alpha = \frac{1}{0.06561 + 0.01539} = 12{,}3456790123$$

$$P(J|b,i,m) = < 0.81; 0.19 >$$



Esercizio 3

- Si consideri la seguente Rete Bayesiana. Determine which of the following conditional independence statements follow from the structure of the Bayesian network:

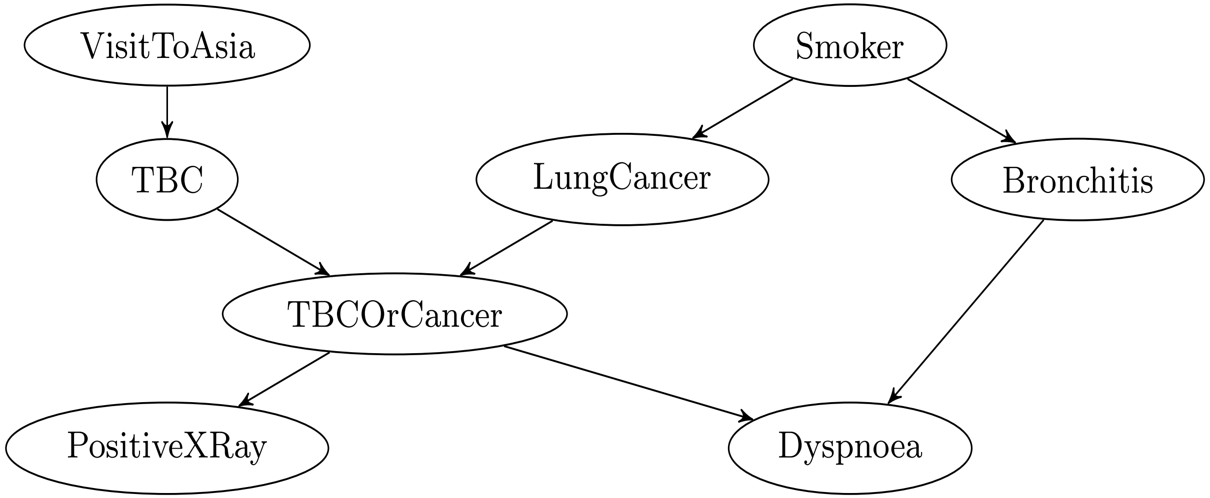
(i) Ind(TBC,VisitToAsia)

(ii) Ind(VisitToAsia,Smoker)

(iii) Ind(VisitToAsia,PositiveXRay|TBCOrCancer)

(iv) Ind(VisitToAsia,Dyspnoea|TBCOrCancer)

(v) Ind(TBC,Smoker|PositiveXRay)



Esercizio 3

[True/False]: Supponiamo che le variabili $X_1, X_2, \ldots, X_k$ non abbiano genitore in una determinata Rete Bayesiana che contiene n variabili in tutto, dove n>k. La Rete Bayesiana asserisce che $P(X_1,X_2,\ldots,X_k)=P(X_1)P(X_2)\cdots P(X_k)$.

vero

Esercizio 4

- Calcolare P(Dyspnoea|Smoker,¬TBC). The entries necessarie delle CPT sono le seguenti:
  - P (LungCancer|Smoker) = 0.1
  - P (LungCancer|¬Smoker) = 0.01
  - P (Bronchitis|Smoker) = 0.2
  - P (Bronchitis|¬Smoker) = 0.1
  - P (TBCOrCancer|TBC, LungCancer) = 1
  - P (TBCOrCancer|TBC, ¬LungCancer) = 1
  - P (TBCOrCancer|¬TBC, LungCancer) = 1
  - P(TBCOrCancer|¬TBC,¬LungCancer)= 0
  - P(Dyspnoea|TBCOrCancer,Bronchitis)=0.9
  - P(Dyspnoea|TBCOrCancer,¬Bronchitis)=0.7
  - P(Dyspnoea|¬TBCOrCancer, Bronchitis) = 0.6
  - P(Dyspnoea|¬TBCOrCancer, ¬Bronchitis) = 0.05