The Sentiment Channel of Fiscal Policy[‡]

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Abstract

How does government spending stimulate the economy? We uncover a new transmission channel—the sentiment channel—by showing that government spending makes firms overoptimistic about their future demand, thereby stimulating investment and output. We assemble a novel dataset linking microdata on Italian firms' sales, sales forecasts, and public procurement contracts. Using a natural experiment that shifted public spending across municipalities, we find that an increase in government demand makes firms systematically overoptimistic about their future sales. This overoptimism is pervasive, as firms also raise their expectations about export sales. To interpret these findings, we develop a theory of expectations in which shocks to total sales make a firm overoptimistic about both its public and private sales. We embed this model of expectations in a heterogeneous-firm New Keynesian model disciplined with our causal estimates. Government spending boosts firms optimism which prompts them to invest, crowding-in investment in general equilibrium—thereby doubling the government spending multiplier. This amplification is state-dependent: our model predicts that the multiplier is a third smaller during financial crises than in recessions without financial distress.

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1 Introduction

How does government spending stimulate economic activity in a recession? There is a long-standing view, common in the press and in policy discussions, that fiscal policy operates not only through direct demand effects, but also by boosting business optimism and hence investment. This idea dates back to Keynes, who, at the height of the Great Depression, famously argued that government expenditures can move beliefs and sentiment just as much as fundamentals: "*Large schemes of work being undertaken [by the government] would give an immediate fillip to the industry of the country*" (Keynes and Henderson, 1929).

Yet, incorporating this channel in standard macroeconomic models has proven challenging. As Mankiw (2009) and Cochrane (2009) noted after the Global Financial Crisis, the lack of direct empirical evidence on how government spending affects private sector beliefs has made it difficult to discipline models that feature such mechanisms. The result has been that, as Mankiw puts it, that "*until we figure it out, it is best to be suspicious of any policy whose benefits are supposed to work through the amorphous channel of confidence*". Hence, the literature has largely resorted to models with rational expectations to understand the effects of fiscal stimulus. In this context, it is not possible to define a notion of optimism or assess the quantitative relevance of such a channel for fiscal policy.

This paper makes progress by formalizing empirically, theoretically and quantitatively what we call the sentiment channel of fiscal policy. Using firm-level data, we show that receiving a public procurement contract boosts firms' optimism about their future sales. Incorporating this sentiment channel in a heterogeneous-firm New Keynesian model has important policy implications. The firm-level optimism induced by an aggregate government spending shock overturns the crowding-out of investment predicted by standard models and leads to larger and empirically plausible multipliers.

First, to provide causal evidence for the sentiment channel, we assemble a novel dataset linking firms' sales forecasts with their realizations, and match it to the universe of public procurement contracts in Italy. Leveraging a natural experiment that induced plausibly exogenous variation in municipal spending, we find that positive government demand shocks make firms systematically overoptimistic: their sales forecasts rise by more than their actual sales. This optimism is not limited to public revenues and spills over to other domains like private (export) sales.

To rationalize these findings, we develop a new theory of expectation formation in which a positive sales shock generates systematic optimism, distorting beliefs across domains of demand. This bias is particularly relevant for investment decisions, which hinge on expectations of future demand. In this context, financial frictions play a central role, as they affect the ability of firms to make intertemporal choices. We therefore incorporate our model of expectation formation into a standard q-theory of investment with financial frictions, and show analytically that financial constraints are a key mechanism shaping the transmission of optimism to investment: investment is increasing in optimism about future demand, but the effect is dampened for financially constrained firms.

Finally, we embed this behavioral theory of investment in a heterogeneous-firm New Keynesian model to quantify the sentiment channel of fiscal policy and run policy counterfactuals. We calibrate the behavioral bias to our empirical estimates and find that the sentiment channel is quantitatively large: the optimism induced by government spending leads to a crowding-in of firm investment, doubling the government multiplier relative to a benchmark with no sentiment, from 0.49 to 1. Therefore our channel allows to reach empirically plausible multipliers and offers an alternative mechanism to Heterogeneous Agents New Keynesian models (Auclert et al., 2024).

This amplification mechanism is state-dependent, owing to the central role of financial frictions highlighted in our theory. In financial crises, when credit constraints bind and sentiment plays a smaller role, the fiscal multiplier is lower than during recessions where financing conditions are less impaired. These findings suggest that incorporating empirically disciplined models of expectations in standard theories has important consequences for policy analysis.

Empirics Estimating a sentiment channel of fiscal policy is challenging, as it requires data on the expectations and realizations of firms' outcomes, together with exogenous variation in government spending at the firm level. Our first contribution consists in assembling a novel dataset combining the quasi-universe of public procurement contracts in Italy with a large survey of firms' expectations and with administrative data on income statements.[1] This allows us to track firm-specific procurement revenues and link them to both forward-looking beliefs and realized outcomes.

Our second empirical contribution is a new source of identification: we exploit a 2015 reform to Italy's municipal fiscal rules that modified the formula determining municipal surplus targets. Before the reform, targets were based on average municipal expenses over 2009-2011. The reform revised the base years to 2010-2012, effectively loosening targets for municipalities that had high expenditures in 2009 relative to 2012. Since the details of the reform were not publicly debated prior to its announcement, the change in base years was plausibly unanticipated by both municipalities and firms, generating exogenous variation

[1] Public procurements are sizable, amounting to about 7% of GDP and accounting for 36% of government consumption.

in surplus goals across municipalities. We employ our contract-level procurement data to construct a measure of firms' exposure to different municipalities. We then rely on a shift-share event study combining firm exposures with the municipal variation in fiscal targets.

This enables us to provide the first causal estimate of how government spending shocks affect the firms' expectations. To accomplish this, we use a natural experiment to study how firms' beliefs respond to public sales shocks, thus complementing existing work that has primarily relied on unconditional correlations (Barrero, 2022, Ma et al., 2024). These methods pool together different sources of shocks, and are hence silent about the underlying drivers of behavioral biases. This matters, as recent evidence shows that expectations respond differently to different types of shocks (Born et al., 2023). Our findings thus help reconcile the mixed evidence on over- and under-reaction in the literature.[2]

A rise in government procurement spending makes firms overly optimistic. A procurement shock that increases firm revenues by 10% leads sales forecasts to exceed realizations by 9% in the subsequent year (while forecast errors are zero on average). This optimism extends beyond public revenues, as firms also raise their expectations about private sales, which we proxy with export sales. Together, these findings provide new causal evidence for a sentiment channel of fiscal policy, in which government spending affects economic activity through belief distortions.

A theory of cross-domain extrapolation Our empirical results show not only that firms overreact to procurement shocks, but also that they extrapolate these shocks into domains that should be unaffected, such as export sales. Standard models of overreaction, like diagnostic expectations, can account for the first finding but not for the latter. Models of imperfect information, on the other hand, are at odds with the fact that firms can precisely observe their private and public revenues. In order to rationalize our empirical findings we therefore build a model of expectation formation in which recent news about total sales shapes beliefs about its private and public sub-components. Our mechanism is a form of cross-domain extrapolation that combines two central ideas in the behavioral literature: *representativeness* and *salience* (Tversky and Kahneman, 1983, Bordalo et al., 2023). *Representativeness* implies that managers overweight the probability of events that have become more likely in light of recent news. *Salience* governs which variable captures their attention when applying this heuristic. In particular, we posit that total sales are salient, meaning that they stand out in the managers' minds more than its public and private sub-components.

[2]An alternative strategy in the literature is to rely on survey experiments (Coibion et al., 2022). To our knowledge, this approach has not been used to study the response of firms' expectations to government spending.

In this setting, any positive surprise to total sales—whether driven by public or by private revenues—cues memories of both high private *and* public sales. *Representativeness* then leads managers to overweight these states when forming expectations, making them overly optimistic about both components. The effect is symmetric for negative news. In this way, shocks to one domain (e.g. public sales) spill over into beliefs about the other domain (e.g. private sales).

Building on Maxted (2023) continuous time formulation of diagnostic expectations, we prove that this model of expectation formation can be conveniently represented in recursive form with a variable, *sentiment*, driving latent optimism. This variable summarizes recent sales news and is defined as an exponentially weighted memory of past sales shocks. Thus, good sales news originating from private and/or public demand endogenously raises sentiment, tilting beliefs about future public, private, and total sales upward, consistent with our empirical results.

A q-theory of investment with sentiment Expectations matter for real activity only insofar as they affect firms' choices. Among these, investment is the natural decision for which beliefs are important, as it is inherently forward looking. At the same time, firm investment is constrained by financing capacity, making financial frictions central to how sentiment translates into real activity. To study how sentiment shapes firms' investment choices, we therefore embed our theory of expectation formation in a canonical q-theory (Hayashi, 1982, Abel and Eberly, 1994) with financial frictions (Fazzari et al., 1988). This setting yields an analytical mapping between firms' potentially biased expectations about future demand and investment decisions.

We analytically decompose the optimal investment choice in the frictionless, rational expectations benchmark augmented by two wedges: one reflecting financial frictions and another capturing belief distortions about future profitability driven by sentiment.

A key prediction of our theory is that the relationship between investment and sentiment is weaker for financially constrained firms, as financing constraints limit firms' ability to make intertemporal decisions. Because sentiment enters our framework only through forecast errors about future profits, we can test this prediction directly in our survey data. In line with the theory, financial constraints dampen the sensitivity of investment to expectations in the data.

General equilibrium model Finally, to assess the macroeconomic relevance of the sentiment channel, we embed our behavioral q-theory of investment in a general equilibrium heterogeneous-firm New Keynesian model calibrated to our microdata and reduced-form estimates. We cast the economy as a mean field game featuring a "rationality wedge", reflecting the gap between firms' perceived and actual economic dynamics. Our methodology is portable and can be applied to a wide range of heterogeneous-agent models exhibiting deviations from rational expectations, see for example Bellifemine et al. (2025). This allows us to solve for the general equilibrium response of the economy to a government spending shock. Consistent with our empirical setting, we model government spending shocks as correlated shocks to individual firms' public procurement.

We calibrate the expectation formation process to our reduced-form empirical estimates and find that the sentiment channel is quantitatively important. The optimism induced by government spending more than offsets the negative impact of higher interest rates on investment, overturning the standard crowding-out result: investment is crowded-in in general equilibrium. This is consistent with the empirical literature that finds a crowding-in effect of government expenses on private investment (Edelberg et al., 1999, Auerbach and Gorodnichenko, 2012). This effect amplifies the total output response by almost twofold, raising the cumulative multiplier to 1 from a baseline of 0.49.

The strength of this channel depends on the state of the economy. During financial crises, when constraints bind more tightly, the sensitivity of firm investment to expectations is muted, thus weakening the sentiment channel. In particular, our model predicts that the government spending multiplier was 34% smaller during the Global Financial Crisis than during Covid, a difference driven entirely by the sentiment channel.

Related literature We contribute to three main strands of literature. First, we add to the growing body of work, both empirical and theoretical, studying the macroeconomic consequences of departures from rational expectations. While most of this literature has focused on expectations of aggregate variables (Coibion and Gorodnichenko, 2012, Broer and Kohlhas, 2024, Auclert et al., 2020, Flynn and Sastry, 2024a,b), we study how firms' expectations about their *idiosyncratic* future outcomes respond to shocks. In doing so, we contribute to the debate on overreaction versus underreaction: existing evidence often points to underreaction in response to aggregate news but overreaction to idiosyncratic shocks (Born et al., 2023). Our results suggest that government spending shocks are perceived as salient, firm-specific demand shocks, giving rise to overreaction. Our contribution to this literature is twofold. First, we build a linked dataset combining firms' expectations about future sales, the realization of these sales, and sales to the public sector. We leverage a natural experiment to provide the first causal estimates on how government spending shapes firms' expectations. In doing so, we complement prior work that has mostly relied on survey experiments or unconditional correlations to study how expectations respond to shocks. Among these, Ma et al. (2024) use our same expectations survey to document *unconditional* deviations from rational expectations and study their implication for misallocation. Relative to them, we study the *conditional* response of firms' expectations to government demand shocks and trace its implications for aggregate fluctuations, rather than misallocation. These empirical estimates allow us to discipline and quantify our channel, thus adding to Angeletos and Lian (2021), who develop a theory where household expectations depart from full-information and introduce the notion of a "confidence multiplier".[3] Second, we develop a new theory of expectation formation featuring cross-domain extrapolation. Our theory is a generalization of diagnostic expectations (Bordalo et al., 2018, Maxted, 2023) to a multivariate setting. We cast it in a recursive form that can be embedded easily in heterogeneous agent general equilibrium models and disciplined to micro-evidence on expectations, in the spirit of Moll (2024). In this sense our paper is close to Bordalo et al. (2025) who incorporate diagnostic expectations in a heterogeneous-firm model to study boom-bust cycles. Relative to their analysis, we propose deviations from rational expectations as a channel through which fiscal policy affects real macroeconomic aggregates and bring new causal evidence on how government spending shocks affect firms' expectations.

Second, we add to a large body of work studying the transmission of fiscal policy to aggregate economic activity. Empirically, our contribution is to study how firms' sales expectations respond to government spending, thus complementing an existing literature mostly focusing on the response of real outcomes.[4,5] In doing so, our empirical approach is close in spirit to recent work using granular procurement data to study government spending (di Giovanni et al., 2023, Cox et al., 2024).[6] We advance this literature by combining procurement data with survey data on firms' expectations. Our theoretical framework relates to a growing literature that uses heterogeneous-agent models to study fiscal transmission.[7] Relative to these papers, we shift the focus from households to firms and propose a new transmission channel whereby fiscal policy stimulates investment via biases to firms' expectations. Hence, our channel relates to the seminal contribution of Ramey and Shapiro (1998), who study the response of business investment to fiscal shocks.

[3]We also differ from Angeletos and Lian (2021) in that (i) we focus on firms rather than households, and (ii) highlight the importance of financial frictions in shaping the role of expectations for aggregate dynamics.

[4]See Hall (2009), Ramey (2011a, 2019) for detailed reviews of the empirical literature on the effects of government spending.

[5]Bachmann and Sims (2012) use a structural VAR to study how indices of consumer sentiment respond to government spending.

[6]Other papers that rely on procurement data to study government spending include Coviello et al. (2021), Hebous and Zimmermann (2021).

[7]See, among others, Galí et al. (2007), Oh and Reis (2012), McKay and Reis (2016), Auclert et al. (2024), Andre et al. (2025).

Finally, we relate to a long-standing literature on firm investment. Our framework builds on the classic q-theory of investment (Tobin, 1969, Hayashi, 1982, Abel and Eberly, 1994) augmented with financial frictions, in the spirit of Fazzari et al. (1988) and Ottonello and Winberry (2020). We extend these models by allowing firms to hold expectations about future demand that depart from rationality. In doing so, our work connects to recent heterogeneous-firm models that emphasize the role of idiosyncratic shocks for investment dynamics (Khan and Thomas, 2008, Winberry, 2021, Koby and Wolf, 2020).

Outline The rest of the paper is organized as follows. Section 2 describes the data and presents our empirical findings. Section 3 develops our theory of cross-domain extrapolation and its recursive formulation. In Section 4 we embed this theory in a model of firm investment and analyze the role of sentiment for investment. Section 5 describes our calibration strategy, quantifies the sentiment channel of fiscal policy, and examines its state-dependence. Section 6 concludes.

2 Empirical Evidence

We begin by studying how government spending affects firms' expectations. To this end, we assemble a novel dataset that links the quasi-universe of Italian public procurement contracts to survey data on firms' expectations and administrative balance sheet and income statement records. Our identification strategy exploits a reform to municipal budget rules that generates plausibly exogenous variation in public spending across Italian municipalities. We leverage this variation to estimate the causal effects of government spending on firms' sales forecast errors.

2.1 Identifying a sentiment channel of fiscal policy

To motivate our empirical strategy, we first describe the ideal specification for testing a sentiment channel of fiscal policy: a regression of firms' forecast errors on shocks to their public sales:

$$\text{Forecast Error}_{it} = \alpha + \beta \text{ Public Sales Shock}_{it} + u_{it}. \tag{1}$$

Forecast errors are a natural outcome variable, as they provide a direct way to test for deviations from rational expectations (RE). Under the null hypothesis of rationality, forecast errors should be orthogonal to any variable in the forecaster's information set; otherwise, it could be possible to improve the forecast with available information. Under the assumption that firms observe their public sales shock and understand the stochastic process for public sales, a finding of $\beta \neq 0$ in (1) therefore constitutes a rejection of the null.

Importantly, testing for deviations from RE does not itself require the public sales shock to be exogenous. This point is illustrated by recent work documenting systematic biases in managerial expectations from *unconditional* forecast error correlations (Barrero, 2022, Ma et al., 2024). Such correlations, however, only establish the presence of biases; they are silent on what drives them. This distinction matters, as recent evidence shows that expectations respond differently to different types of shocks (Born et al., 2023).[8] By contrast, to establish that the deviations from RE are caused by fiscal policy—that is, to identify an expectation channel of fiscal policy—requires exogenous shocks to government spending.[9]

Taken together, these considerations underscore the challenge of identifying a sentiment channel of fiscal policy: such an endeavor requires firm-level data that jointly span (i) expectations, (ii) realizations, (iii) public sales, and (iv) exogenous fiscal shocks. We now describe how we assemble a dataset that meets these requirements.

2.2 Data sources

We construct our dataset by combining four data sources. First, we obtain firm-level sales forecasts from the Bank of Italy's Survey of Industrial and Service Firms (INVIND). Second, we link forecasts to realized outcomes from administrative balance sheet and income statement records from the Company Accounts Data System (CADS). Third, we merge this firm-level panel with contract-level procurement records from the Italian Anti-Corruption Authority (ANAC). This combined dataset allows us to track firms' expected and realized sales and investment, as well as their municipality-specific exposure to procurement revenues. Finally, to measure the public spending shock induced by the reform, we use fiscal accounts for all Italian municipalities from Orbis' proprietary database Aida PA. We now describe each source and the construction of the merged dataset in detail.

Survey of Industrial and Service Firms Our data on firm-level sales expectations come from the Bank of Italy's Survey of Industrial and Service Firms (INVIND), an annual sur-

[8] In addition, autocorrelations of forecast errors cannot disentangle underreaction from persistent firm heterogeneity as positive autocorrelation may simply reflect fixed optimistic or pessimistic types. Controlling for firm fixed effects mitigates this concern but introduces dynamic panel bias (Nickell, 1981), especially in short survey panels. To address this, Ma et al. (2024) rely on a dynamic panel GMM estimator (Arellano and Bover, 1995), restricting the analysis to a sample of firms with sufficiently long time series.

[9] A common approach in the literature to identify the drivers of behavioral biases is to use survey experiments. To our knowledge, this has not been applied in the context of government spending and firms. Moreover, survey experiments may suffer from limited external validity, as responses are elicited in low-stakes settings.

vey conducted between February and May of each year since 1972. From 1995 onward, respondents have reported numerical forecasts for end-of-year sales, thus providing us with nearly three decades of expectations data. Relative to existing surveys eliciting quantitative forecasts, a key advantage of INVIND is its large sample size and strong panel dimension. In recent years, about 5,000 firms participate in each wave and respondents are observed on average for 11 years. The survey targets firms with at least 20 employees headquartered in Italy operating in industrial and non-financial private service sectors. For the construction sector, the reference universe comprises firms with at least 10 employees. To ensure representativeness, the sample design is stratified by sector of economic activity, geographical area, and firm size. Response rates are high, averaging around 70 percent in recent years.

Company Accounts Data System To compare firms' expectations with realized outcomes, we merge our INVIND data with the Company Accounts Data System (CADS), a proprietary database maintained by Cerved Group S.p.A. CADS aggregates administrative balance-sheet and income-statement filings from all Italian limited liability companies. Italian law mandates that firms report these data annually to local Chambers of Commerce. From CADS, we extract firms' annual revenues, total assets, investment, and a proprietary credit score measure based on Altman's Z-score. We also retrieve a number of firm characteristics that will be useful for our analysis, including 2-digit NACE classification of economic sector, location, and year of incorporation.

Public procurement contracts To link firms' forecasts to their revenues from the public sector, we collect contract-level procurement data from the Italian anti-corruption authority (ANAC). These data cover the universe of public procurement contracts exceeding €40,000 awarded between 2012 and 2024. Crucially, each contract record includes identifiers for both the awarding public body and the winning firm, as well as the contract value. This allows us to construct firm-municipality procurement histories and merge them with our matched INVIND-CADS panel.

Municipal income statements Finally, to construct a measure of the municipal public spending shock induced by the reform of interest, we obtain municipal income statements from Aida PA. We extract annual current expenditures and tax revenues for all Italian municipalities from 2009 to 2017, which form the basis for our shock measure.

Summary statistics In our empirical analysis we employ two different samples. To study the effect of government spending on realized outcomes, we construct a large panel by merging CADS and ANAC, covering about 25,000 firms annually. To study forecast errors, we merge our linked CADS-INVIND data with ANAC. This restricts the sample to approximately 800 firms per year but provides the crucial link between expectations, outcomes, and public sales shocks. Detailed summary statistics for both samples are reported in Table A.1 in Appendix A.

2.3 Descriptive statistics

Before turning to our main empirical results, we present descriptive evidence on firms' expectations and on the structure of public procurement.

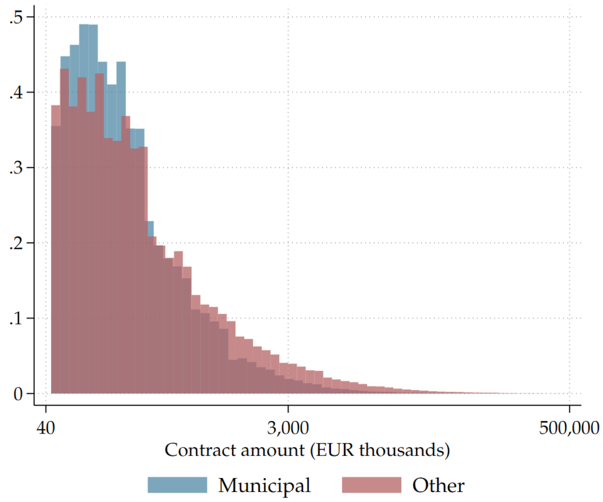
Procurement data We first describe our contract-level procurement data, which form the basis for our measure of firms' exposure to municipal spending. Public procurement constitutes a large share of government activity, representing 36.3% of government consumption and 6.9% of GDP over the period 2012-2022. Of particular relevance for our empirical analysis, the 7,896 Italian municipalities represent an important source of procurement. They account for 21.7% of awarded contracts (by number) and 8.3% (by value), totaling 0.62% of Italy's GDP.

These aggregate patterns are mirrored at the firm level, where public procurement represents a substantial revenue source. Among firms with public contracts, public procurements account for 14.5% of total sales on average. Conditional on contracting with municipalities, municipal procurement alone represents 12.1% of firm sales.[10] These magnitudes underscore why municipal procurement shocks are a meaningful source of demand variation for firms.

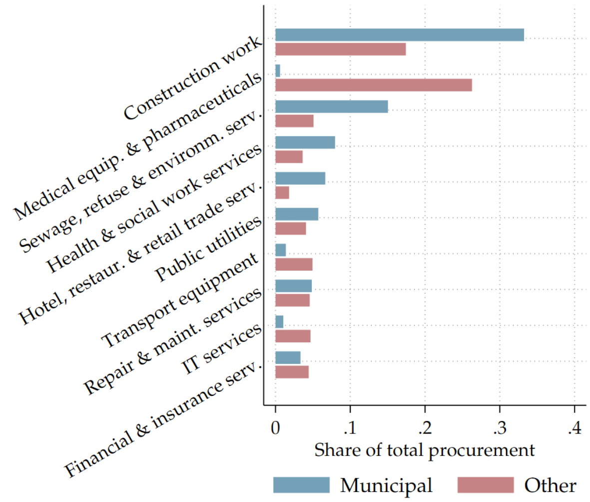
To further illustrate the salience of procurement as a revenue source, Figure 1a shows the distribution of contract values awarded by municipalities and other public bodies. The median contract is sizable—€123, 000 for municipalities and €146, 000 for other public bodies. Average contract values are even higher, driven by the presence of a few "mega-projects". These are more common in non-municipal procurement, where the mean reaches €893, 000 (compared to €396, 000 for municipalities). Such contracts mainly reflect large infrastructure projects and centralized purchases by regional and national authorities, which are largely absent from municipal procurement.

[10]Figure A.1 in Appendix A.1 shows the distribution of the share of total public procurement in firms' revenues, as well as the share specifically attributable to municipal contracts, conditional on receiving at least one procurement contract.

Figure 1: Municipal vs Non-Municipal Procurements



(a) Distribution



(b) Composition

Note:Panel (a) displays the distribution of procurement contracts across ten categories, distinguishing between those issued by municipalities and those issued by other public entities. Panel (b) shows the distribution of contract sizes for municipal and non-municipal contracts. We remove contracts below €40,000, since reporting is non-mandatory below this threshold. For visibility, we only keep the ten most important sectors.

Turning to the sectoral composition of public procurement spending, Figure 1b summarizes the breakdown of procurement contracts in our dataset, distinguishing between those awarded by municipal and non-municipal entities. Compared to other public bodies, municipal procurement is more concentrated in *Construction works*—which includes road paving and building repairs—and *Sewage, refuse, cleaning and environmental services*. These categories account for 33% and 15% of all municipal procurement expenses, respectively, while they only represent 17% and 5% of contracts in the non-municipal sample. On the other hand, municipal procurement is under-represented in *Medical equipment and pharmaceutical products*, which is virtually absent from municipal procurement but constitutes nearly 26% of contracts in the rest of the sample. These patterns reflect the division of government responsibilities: municipalities focus on local infrastructure and sanitation while regional bodies handle healthcare provision. Outside these categories, however, municipal procurement is broadly similar to non-municipal procurement, with roughly half of spending distributed across other sectors such as *Hotel, restaurant and retail trade services*, *Repair & maintenance services*, and *Financial & insurance services*.

Finally, to inform the rest of our empirical analysis, it is useful to discuss the timing of contract execution and payments. Italian accounting standards require firms to record revenues on an accrual basis—that is, when goods or services are delivered rather than when payments are received. Importantly, municipal contracts typically begin shortly after award: the median (mean) lag between award and start dates is 11 (41) days, and between award and completion is 337 (460) days.[11] Payments, however, follow with substantial delay: the median (mean) lag from contract start to first payment is 152 (196) days, and to the last payment 675 (856) days.[12]

Expectations data We construct our main variable of interest, the sales forecast error, using the INVIND survey. Each year, at the beginning of the calendar year, the survey asks managers to predict end-of-year sales. We therefore define the sales forecast error for firm i in year t, denoted $\mathcal{E}_{it}$, as the difference between log realized sales at the end of the year and the log of the sales forecast made at the beginning of the year:

$$\mathcal{E}_{it} = \log\left(Sales_{it}\right) - \log\left(\mathbb{F}_{it-1} Sales_{it}\right), \tag{2}$$

where $\mathbb{F}_{it-1} Sales_{it}$ is the forecast made at the start of the calendar year t for sales over the year t. Under this convention, a positive error ($\mathcal{E}_{it} > 0$) indicates that realized sales exceeded expectations, meaning the firm was overly pessimistic *ex post*.

To provide evidence on the quality of our expectations data, Figure 2a presents a binned scatter plot of realized versus expected sales. Importantly, we residualize both axes by firm fixed effects, thus controlling for persistent differences in firm size. Graphically, the forecast error defined in (2) corresponds to the horizontal distance between each observation and the 45° line. Thus, the tight clustering of observations around the 45° line indicates that managers report meaningful expectations when answering the survey.[13] While this unconditional accuracy lends credibility to the data, it does not imply that expectations are rational. As we show in the next section, expectations can in fact still display systematic biases conditional on realized shocks.

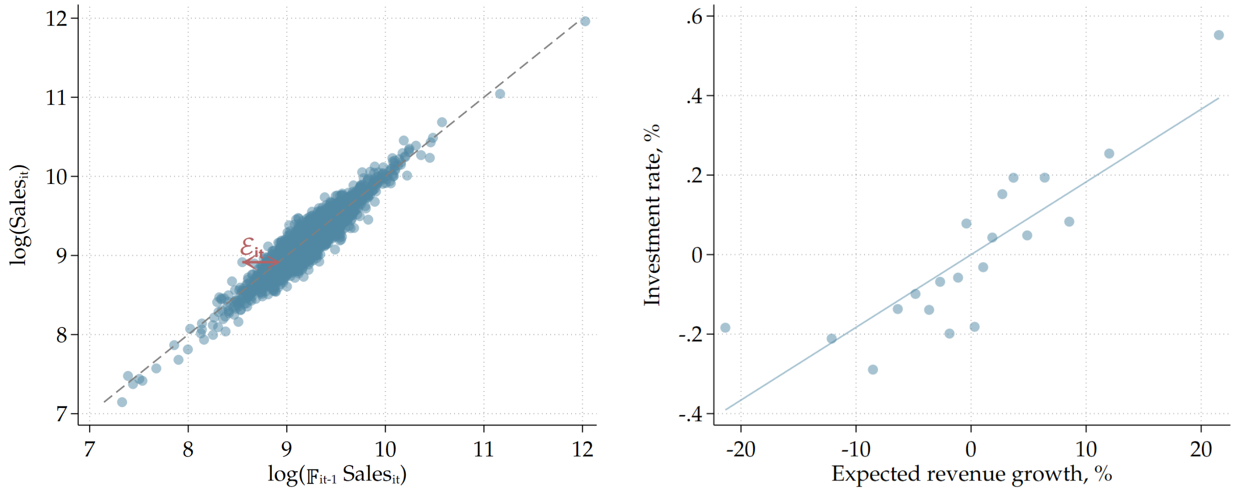
As a further check, Figure 2b examines the relationship between firms' expectations and their actions by plotting the investment rate (investment over capital) against expected future revenue growth. Crucially, we residualize both axes by *realized* future revenue growth and firm fixed effects. Hence, this specification isolates the component of expectations orthogonal to realized outcomes, capturing how optimism or pessimism relative to realizations correlates with actions. The relationship is strongly positive: a 10 percentage-point increase in expected revenue growth is associated with a 0.18

[11] For non-municipal contracts, the corresponding figures are 28 (60) and 265 (358), respectively.

[12] The corresponding figures for non-municipal contracts are 180 (214) and 865 (1015), respectively.

[13] Because the responses are confidential and released only in aggregate form, respondents do not face incentives for strategic misreporting which is typical in other settings, such as public earnings calls.

Figure 2: Forecast Errors, Expectations, and Investment



(a) Forecast error

(b) Investment

Note: Panel (a) plots the binned scatter plot of realized and forecast sales against the 45° *line, residualizing for firm fixed effects. Panel (b) plots a binned scatter plot of the investment rate (defined as investment over capital) against expected revenue growth, defined as* $\log \mathbb{F}_{it-1}\ Sales_{it} - \log\ Sales_{it-1}$, *residualizing for realized future revenue growth and firm fixed effects.*

percentage-point higher investment rate, controlling for firm fixed effects and keeping realized revenue growth fixed. This effect is economically meaningful. Given that the average investment rate in our sample is roughly 6%, a one standard-deviation increase in expected growth corresponds to an investment increase of about 3% relative to the mean. While purely correlational, Figure 2b may be interpreted as providing motivating evidence for a central premise of our analysis: that firms' investment choices are closely linked to their expectations.

Taken together, these patterns establish the economic relevance of our expectations data. Managers' forecasts are accurate on average and correlate with firms' choices. This motivates our next step: studying how these expectations respond to fiscal shocks as a key mechanism through which policy affects real activity.

2.4 Identifying procurement shocks to firms: a shift-share approach

Having assembled our dataset combining firms' forecasts, realizations, and public sales, we now proceed to isolate exogenous variation in government spending. We exploit a 2015 reform to Italian municipal budget laws as a natural experiment. We implement a shift-share design (in which identification comes from the shifts (Borusyak et al., 2022)) that provides plausibly exogenous shocks to firm-level public demand. The institutional context and the construction of the shift-share instrument are described below.

The shifts: 2015 Stability Pact Our source of exogenous variation in public spending comes from a 2015 reform of the Italian *Patto di Stabilità dei Comuni* (henceforth, Stability Pact), the fiscal framework governing municipal budget surpluses.[14]

The Stability Pact was introduced in 1999 as a domestic mechanism to align local finances with EU fiscal rules established in the 1992 Maastricht Treaty and the 1997 Stability and Growth Pact. Starting in 2007, the Stability Pact imposed budget surplus targets on municipalities. In 2013, the government announced the targets for 2014–2017 and set them at 14.07% of average municipal spending in 2009–2011.[15] Because the targets were tied to past spending, surplus goals differed across municipalities.

Our natural experiment comes from a reform announced on October 15, 2014. The government revised the surplus targets for 2015-2018 in two ways: it lowered the required rate to 8.6%, and, importantly, shifted the reference years from 2009-2011 to 2010-2012.[16] The reform was designed to provide additional fiscal space to municipalities. While this aggregate objective was known in advance, the technical details (the change in reference years that forms the basis of our identification strategy) were not publicly debated, rendering them plausibly unanticipated by both municipalities and firms. Together with the mechanical link to past spending, this lack of anticipation makes the reform a source of plausibly exogenous variation in surplus requirements across municipalities.

To capture this variation, we define the shift for municipality m as the change in log municipal expenditures between 2009 and 2012:

$$g_m \equiv \log(Exp_{m2009}) - \log(Exp_{m2012}). \tag{3}$$

Positive values of g_m indicate that municipality m spent more in 2009 than in 2012, and therefore experienced a larger reduction in its surplus target following the reform. Figure A.5 in Appendix A.4 documents the distribution of these shifts across municipalities. The average shift is -2.3%, which shows how municipal spending grew by about half the cumulative inflation rate of 5% over this period. Our identifying variation, however, comes from the cross-sectional dispersion. This is substantial, with a standard deviation of 14.7%.

[14] Coviello et al. (2021) study the 2008 reform to the Stability Pact, which reduced municipal spending. That reform is not suitable for our setting because our procurement data from ANAC begin in 2012.

[15] See art. 31 of *Legge 12 Novembre 2011, n.183* as updated by art .1, comma 532 of *Legge 27 Dicembre 2013, n. 147* in Appendix A.2.

[16] See art. 1, comma 489 of *Legge 23 Dicembre 2014, n.190* in Appendix A.2.

The shift-share instrument Having defined the municipal-level shifts, we translate them into firm-level public demand shocks using a shift-share design. For each firm i, we define its exposure to municipality m as the share of municipal procurement revenues obtained from that municipality during the 2012-2014 pre-reform period:

$$s_{im} = \frac{\sum_{t=2012}^{2014} Sales_{imt}}{\sum_{t=2012}^{2014} \sum_{m'=1}^{M} Sales_{im't}}, \tag{4}$$

where $Sales_{imt}$ denotes firm i's procurement revenues from municipality m in year t. The weights s_{im} capture firms' exposure to the reform through their municipal customer base; by construction, they sum to one for each firm. We then construct the firm-level shift-share instrument as the weighted sum of municipal shifts g_m, using the exposure weights s_{im}:

$$z_i = \sum_{m=1}^{M} s_{im} g_m. \tag{5}$$

Following Borusyak et al. (2022), we assess the effective number of shocks underlying our shift-share instrument by computing the inverse Herfindahl–Hirschman index (HHI) of average firm-level exposures. Formally,

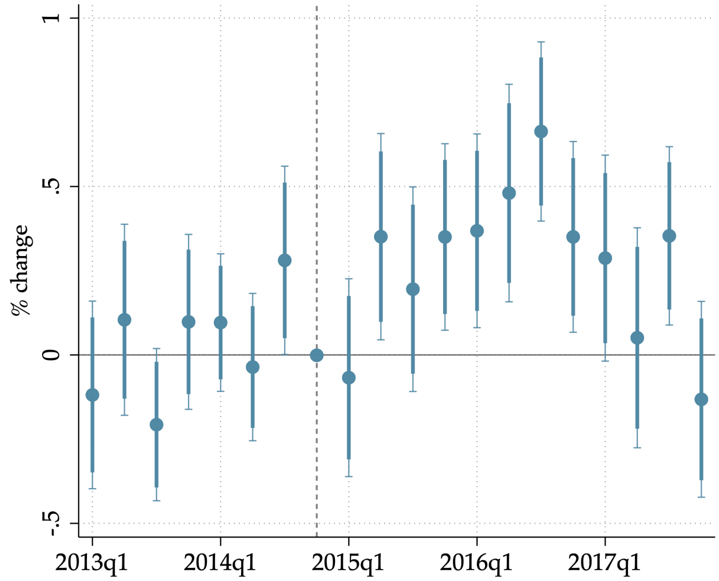
$$\text{inverse HHI} \equiv \frac{1}{\sum_m s_m^2} \quad \text{where} \quad s_m \equiv \frac{1}{N} \sum_{i=1}^{N} s_{im}. \tag{6}$$

Intuitively, the inverse HHI measures how dispersed exposures are across municipalities: larger values indicate identification comes from many small shocks rather than a few dominant ones. In our data, the inverse HHI equals 136.28 in the full CADS sample and 104.01 in the matched INVIND sample, suggesting that our identification rests on a large and reasonably diffuse set of shocks. These magnitudes are well above the heuristic threshold of 20 typically used in the literature.

Figure A.6 and Figure A.7 in Appendix A.4 show the distribution of the average municipal shares s_m and the shift-share instrument z_i across firms, respectively. Firms have an average exposure of 1 to 4% to the few largest Italian municipalities—most notably Milan and Rome—but, as already clear from the large inverse HHI, the average exposure is small, around 0.007%. As for the distribution of z_i, it closely mirrors that of the underlying shifts, with a mean of -2.8% and a standard deviation just above 10%.

Potential threats to identification Before presenting our main empirical results, we discuss some potential threats to identification.

Figure 3: Response of Municipal Procurement Expenses



Note: estimated $\widehat{\beta}_q$ from the event study design in (7). The y-axis denotes the percentage change (relative to the 2014q4 reference quarter) in municipal procurement expenditures. Wide and thin lines represent 90% and 95% confidence bands, respectively. We cluster standard errors at the municipality and year-quarter level. See Table A.3 in Appendix A.6 for more details on the estimates.

A first concern is that the shifts did not affect municipalities only via the reform, as they may capture other persistent determinants of procurement, such as local political cycles or mean-reversion in spending. To address this, we estimate the following event study specification:

$$\log(ProcExp_{mt}) = \alpha_m + \delta_t + \sum_{s=2013q1}^{2017q4} \beta_s \left(\mathbb{1}_{s=t} \times g_m\right) + \log(Pop_{mt}) + \varepsilon_{mt}, \quad (7)$$

where $ProcExp_{mt}$ denotes procurement expenses of municipality m at time t, $\mathbb{1}_{s=t}$ is an indicator for year-quarter s, δ_t is a time fixed effect, α_m a municipality fixed effect, $\log(Pop_{mt})$ denotes the log of municipal population,[17] and g_m is the shift defined in (3).[18] We use the last quarter of 2014 as the base period and cluster standard errors at the municipality and year-quarter level.

Figure 3 plots the estimated coefficients $\widehat{\beta}_q$. The coefficients are flat and insignificant

[17]We control for population to account for heterogeneous demographic trends across municipalities that could affect procurement spending. Figure A.9 in Appendix A.5 shows that our results are robust to excluding this variable.

[18]We winsorize our shift measure at the 0.5% and 99.5% levels.

prior to 2015, consistent with the shifts affecting municipalities only through the reform. Once the reform takes effect, procurement expenses respond strongly: municipalities with larger shifts increase spending as soon as 2015q2. The effect peaks in 2016q3, when a one-percentage-point increase in g_m raises procurement spending by 0.65% relative to 2014q4. The effects persist for over two years, in line with the reform affecting targets for 2015-2018. Figure A.9 in Appendix A.5 shows that our results are robust to the inclusion of region×year, province×year, and population-bin×year fixed effects. These results rule out the possibility that our effects are driven by regional trends or different patterns between urban and rural areas. We also find that our estimates remain unchanged if we remove the population control or use the raw, unwinsorized, shifts.

To further support exogeneity, we show in Figure A.3 that the shifts are uncorrelated with pre-reform municipal characteristics. Figure A.8a further shows that the extent to which the reform relaxed municipal budgets is geographically widespread, with no evidence of spatial clustering.

A related question is whether municipalities responded to the reform on margins other than procurement expenses. To assess this, Figure A.10 in Appendix A.5 plots the behavior of municipal tax revenues around the reform. Tax revenues show no economically or statistically significant differences across municipalities, regardless of the reform's effect on their fiscal space. This suggests that municipalities absorbed the additional fiscal space through higher spending rather than tax cuts, consistent with institutional constraints and their limited autonomy over local tax rates.

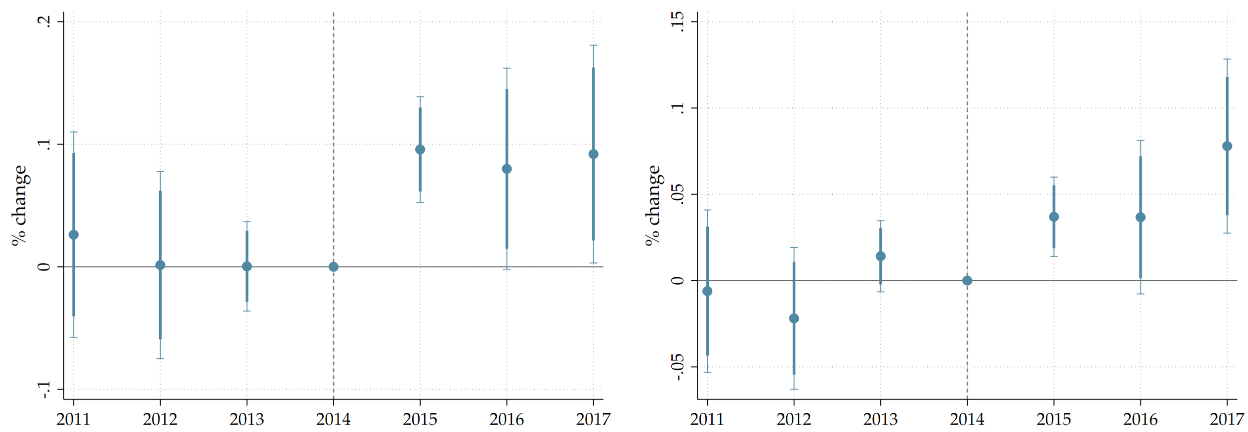
We next examine whether the firm-level shift-share instrument defined in (4) captures meaningful exposure to municipal spending. To do so, we estimate the following event study specification:

$$\log Sales_{it} \;=\; \alpha_i \;+\; \gamma_{s(i)t} \;+\; \sum_{h=2011}^{2017} \beta_h \left(\mathbb{1}_{h=t} \times z_i\right) \;+\; \varepsilon_{it}, \tag{8}$$

where $\log Sales_{it}$ denotes log firm revenues, z_i is the shift-share instrument constructed above, $\gamma_{s(i)t}$ is a 2-digit NACE sector×time fixed effect, and all other variables are defined as in (7). We cluster standard errors at the firm and year level.

Figure 4a plots the estimated coefficients $\widehat{\beta}_h$. The coefficients are flat prior to 2015, consistent with firms with different exposures being on parallel trends. After the reform, revenues rise persistently for firms exposed to municipalities with greater fiscal space. A one-percentage-point increase in z_i raises firm revenues by 0.1% per year throughout 2015-2017. These results can be viewed as a first stage, indicating that exposure to the reform directly translated into higher revenues through procurement spending.

Figure 4: Response of Firms' Revenues and Assets



(a) Revenues

(b) Assets

Note: estimated $\widehat{\beta}_h$ from the event study design in (8). The y-axis denotes the percentage change (relative to the 2014 reference year) in revenues and assets for panels (a) and (b) respectively. Wide and thin lines represent 90% and 95% confidence bands, respectively. We cluster standard errors at the firm and year level. See Table A.4 in Appendix A.6 for more details on the estimates.

Another potential concern is whether firms anticipated the reform. As noted above, the technical details were not publicly debated in advance. In line with this, balance tests in Figure A.4 in Appendix A.3 show that the instrument is uncorrelated with pre-reform firm characteristics, including sales forecast errors, revenues, and profitability. To further rule out anticipation, we re-estimate (8) using firm assets rather than sales as the outcome. Figure 4b plots the estimated $\widehat{\beta}_h$. The coefficients are flat before 2015, indicating that exposed firms did not build capacity in advance. After the reform, investment rises substantially: by 2017, a one-percentage-point increase in the shift-share instrument raises the capital stock by 0.8%.

Is this investment response partly driven by excess optimism triggered by the government spending shock—a sentiment channel of fiscal policy? We turn to this question next, by examining how forecast errors responded to the reform.

2.5 The response of expectations to government spending

We now examine how government spending affects firms' expectations about future demand. To this end, we estimate a specification analogous to (8), using sales forecast errors as the outcome:

$$\mathcal{E}_{it} \;=\; \alpha_i \;+\; \gamma_{s(i)t} \;+\; \sum_{h=2011}^{2017} \beta_h \left(\mathbb{1}_{h=t} \times z_i\right) \;+\; \varepsilon_{it} \tag{9}$$

Where $\mathcal{E}_{it}$ denotes the sales forecast error defined in (2) and all other variables are defined as above.[19] We cluster standard errors at the firm and year level and weight our estimates using survey weights.

Figure 5 presents the results from this regression. Prior to 2015 forecast errors show no differential trends across firms with different exposure. After the reform, forecast errors turn increasingly negative for more exposed firms, reflecting excessive optimism about future sales.[20] The effect is sizable: a one-percentage-point increase in z_i raises expected sales relative to realizations by 0.6 percentage points in both 2016 and 2017. The 2015 estimates, although negative, are not statistically significant. This likely reflects survey timing: because responses were collected in the first half of the year, many exposed firms had not yet experienced higher procurement revenues. By 2016, once these revenues had materialized, the overoptimism is both large and precisely estimated.

Robustness In Figure A.11 in Appendix A.5, we report a series of robustness checks for the results in Figure 5. Most importantly, we show that our estimates are unchanged if we include province×year fixed effects, which suggests that the effects we identify are driven by the direct impact of municipal spending and not by local general equilibrium forces. We also show that our results are not only robust to using untrimmed forecast errors and shift-share measures but also hold without the inclusion of sector×year fixed effects.

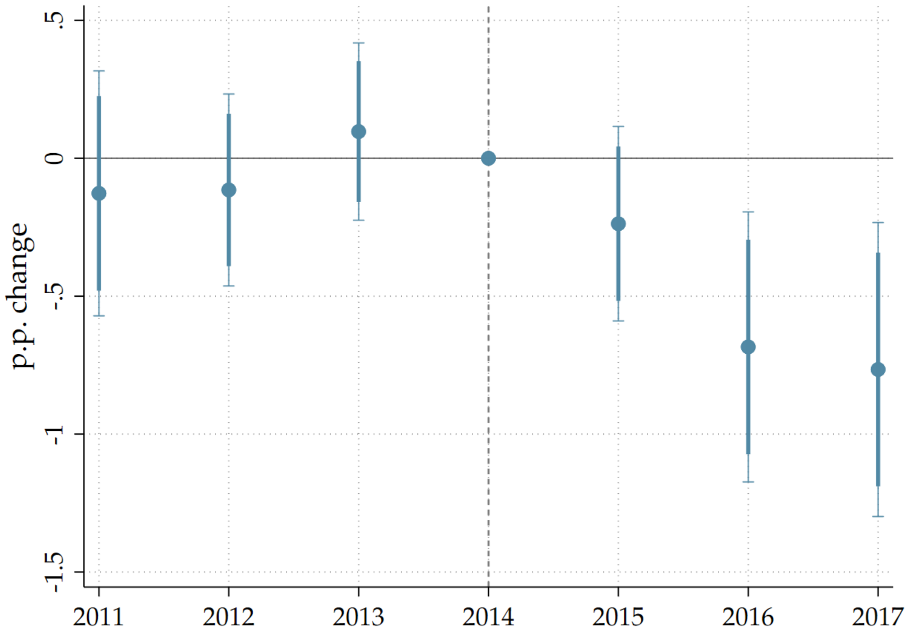
Instrumental variable estimates To quantify the extent of overreaction in economically meaningful terms, we estimate an instrumental variable regression in which past realized sales are instrumented with our shift-share instrument. Specifically, we estimate:

$$\mathcal{E}_{it} = \alpha_i + \delta_t + \beta \, \widehat{\log Sales}_{it-1} + u_{it}, \tag{10}$$

[19] To mitigate the influence of extreme observations, we trim values of the forecast error below the 2.5$^{\text{th}}$ percentile and above the 97.5$^{\text{th}}$ percentile. In Figure A.11 in Appendix A.5 we show results are unchanged when using the raw, untrimmed variable.

[20] Since the average forecast error in our sample is close to zero and slightly negative, the estimates point to excessive optimism rather than a reduction in pessimism.

Figure 5: Response of Forecast Errors



Note: estimated $\widehat{\beta}_h$ from the event study design in (9). The y-axis denotes the change in sales forecast error (relative to the base year 2014) in percentage points. Wide and thin lines represent 90% and 95% confidence bands, respectively. We cluster standard errors at the firm and year level and weight observations using survey weights. See Table A.5 in Appendix A.6 for more details on the estimates.

where $\widehat{\log Sales}_{it-1}$ denotes log sales instrumented with the shift-share instrument interacted with a post-reform dummy, $z_i \times Post_t$.[21] Table 1 shows the results from this regression. The robust first-stage F-statistic exceeds the conventional critical value of 10, consistent with Figure 4a and suggesting no evidence of weak instrument problems. A 1% increase in sales, instrumented by the shift-share instrument, leads to a rise in optimism, making forecast errors more negative by about 0.9 percentage points.[22]

2.6 Cross-domain extrapolation

Our results thus far show that shocks to *public* sales lead firms to overreact in their expectations of *total* sales. Because total sales reflect both public and private revenues, this raises the question of whether the optimism induced by public spending is confined to the di-

[21] We lag realized sales by one survey wave to allow firms time to observe the effect of the reform on sales. We exclude year 2015 from the regression, since past sales were not yet affected by the reform in that year.

[22] Gennaioli et al. (2016) find comparable results for expectations of managers of publicly traded companies. In their sample a one-percentage-point increase of return on assets correlates with a 3-percentage-point reduction in the forecast error.

Table 1: Two-stage least squares regression

	First Stage	Reduced Form	IV
$\text{Post}_t \times z_i$	0.718***	-0.576***	
	(0.203)	(0.169)	
$\log Sales_{it-1}$			-0.863***
			(0.315)
Kleibergen-Paap F-stat.			11.408
Observations	3,580	3,576	3,576
Year FEs	✓	✓	✓
Firm FEs	✓	✓	✓

Note: the first stage reports results from the regression of the endogenous regressor variable $\log(Sales_{it-1})$ *is instrumented with shift-share instrument* z_i *interacted with post-reform dummy* $Post_t$*. The reduced form reports results from the regression of the sales forecast error* $\mathcal{E}_{it}$ *on the shift-share instrument* z_i *interacted with post-reform dummy* $Post_t$*. The IV column reports estimates for the instrumental variable regression in (10). The estimation sample is 2011-2017. Estimates are weighted by survey weights. Robust standard errors in parenthesis.*

rectly affected domain or whether it also distorts expectations of private demand. This question relates to a recent literature documenting cross-domain extrapolation, where salient outcomes in one domain distort expectations in unrelated domains (Binder and Makridis, 2022, Bordalo et al., 2023, 2024, Taubinsky et al., 2024, Cenzon, 2025).

To test this, we use data on export expectations, which provide an ideal testing ground for cross-domain extrapolation since they are orthogonal to domestic (public) procurement.[23] A unique feature of the INVIND survey is that it elicits both expected and realized export revenues, allowing us to construct a measure of export sales forecast error, $\mathcal{E}^X_{it}$, following the same definition as in (2). Because the number of exporting firms is a subset of our main sample, for our baseline specification we rely on a static difference-in-differences design:

$$\mathcal{E}^X_{it} = \alpha_i + \gamma_{s(i)t} + \beta z_i \times Post_t + \varepsilon_{it}, \tag{11}$$

where $\mathcal{E}^X_{it}$ is the export sales forecast error and all other variables are defined as before.[24]

Table 2 reports the estimated $\widehat{\beta}$ from this regression. The first column shows the results for the baseline 2011-2017 time window, while the second column reports results for a

[23] In principle, government spending shocks could reduce exports via price effects if decreasing returns to scale were important. In Figure A.15 in Appendix A.5, however, we show that realized export revenues do not respond to the reform, thus ruling out this channel.

[24] Figure A.14 in Appendix A.5 plots the results for the dynamic event study specification.

Table 2: Response of Export Sales Forecast Errors

	Baseline	Narrow window
$\text{Post}_t \times z_i$	-1.003**	-1.379**
	(0.384)	(0.352)
Observations	493	281
Firm FEs	✓	✓
Sector-Time FEs	✓	✓

Note: estimates from the static difference-in-differences specification (11). The outcome variable is the export sales forecast error $\mathcal{E}_{it}^{X}$. The first column reports results for the baseline 2011-2017 time window, the second column reports results for a narrower time window, defined over 2013-2016. We cluster standard errors at the firm and year level and weight observations using survey weights.

more narrow window around the reform, i.e., 2013-2016. Following the reform, firms exposed to municipalities with more fiscal space become overly optimistic about export revenues. The effect is quantitatively large: a one-percentage-point increase in z_i reduces the export forecast error by about 1 p.p. post-2015.

We interpret this as evidence of cross-domain extrapolation in firms' revenue expectations. Total sales represent a salient variable for managers, so when government spending raises revenues optimism propagates to expectations of unrelated sales streams. Under rational expectations, instead, government spending shocks should have no predictive power for export forecast errors.[25] In the next section, we develop a theory of expectation formation that formalizes this mechanism.

2.7 Taking stock

Taken together, our results point to a sentiment channel of fiscal policy. Government spending raises firm revenues (Figure 4a), which in turn induces managers to become overly optimistic about future demand (Figure 5). The investment behavior in Figure 4b, together with the correlation between expectations and investment in Figure 2b, suggests that this optimism is likely to matter for firms' investment response. Moreover, optimism extends beyond public sales, affecting expectations of unrelated revenue streams (Figure A.14), consistent with cross-domain extrapolation in expectation formation. These findings motivate the model we develop in the next section.

[25] While such results could be consistent with a story of imperfect information, we think that is unlikely since in practice it is easy for managers to observe sales by revenue source.

3 A Theory of Cross-Domain Extrapolation

Our findings in Figures 5 and A.14 call for a framework that can account for two central facts: firms overreact to procurement shocks, raising expectations more than realizations warrant, and they extrapolate these shocks to unrelated domains. Standard models of overreaction, such as diagnostic expectations, can account for the first fact but not for the second. We therefore develop a model of Cross-Domain Extrapolation (XDE), in which recent news about total sales shapes forecasts about the components of sales. We show in B.4 that models of incomplete information can account for some of our findings but they do miss some important moments. Our theory of XDE combines two central ideas from the expectation formation literature: *salience* and *representativeness*.

A variable is *salient* when it stands out in the agent's memory relative to its underlying components, in the spirit of Bordalo et al. (2023). The *representativeness* heuristic instead captures the tendency of agents to overweight the probability of events that have become more likely given recent news (Tversky and Kahneman, 1983).

We present our theory below, with a recursive continuous-time formulation. Appendix B.1 describes the discrete-time case for illustration.

3.1 Continuous-time setup

Time is continuous, $t \geq 0$. The firm observes private demand z_t and public demand g_t, together with total demand, which is defined as the sum of the private and public components, $x_t \equiv z_t + g_t$. We assume z_t and g_t each follow Ornstein-Uhlenbeck processes, the continuous-time analogue of AR(1):

$$dz_t = -\mu_z \, z_t \, dt + dZ_t \tag{12}$$

$$dg_t = -\mu_g \, g_t \, dt + dG_t, \tag{13}$$

where dZ_t and dG_t are independent Brownian motions scaled by the volatility of the shocks σ_z^2 and σ_g^2.

Expectations under cross-domain extrapolation depart from rationality through two biases: representativeness and salience. Representativeness implies that agents overweight the probability of events that become more likely in light of recent news, *relative* to a reference case in which no new information arrives. In this context, salience determines which news will actually trigger this overweighting. In particular, we assume that total demand x_t is salient: although x_t is redundant information for a forecaster observing z_t and g_t, it sways expectations through representativeness.

To formalize this, we need to introduce a notion of background context, i.e., the reference level of x_t used to characterize representativeness. Intuitively, the background is what agents would expect in the absence of innovations (Bordalo et al., 2018). In continuous time, agents are hit with shocks at every instant; we therefore need to define a notion of "recent news" that captures the realization of recent shocks to the salient variable. To do so, we follow Maxted (2023) and introduce a variable $\mathcal{S}_t$ as a measure of recent information, which we define as the exponentially-weighted average of past shocks to the salient variable:

$$\mathcal{S}_t \equiv \int_{-\infty}^{t} e^{-\kappa(t-s)} \left(dZ_s + dG_s\right),$$

where $\kappa > 0$ is a memory parameter governing the rate of decay of past shocks in the agent's memory. We call this variable "sentiment". In this setting, the evolution of sentiment may conveniently be expressed recursively as

$$d\mathcal{S}_t = -\kappa\, \mathcal{S}_t\, dt + dZ_t + dG_t. \tag{14}$$

We then define the background context as the level of x_t in the absence of recent news: $\mathcal{B}_t = x_t - \mathcal{S}_t$. With this structure in place, we can characterize beliefs under XDE. The forecast of future private demand is based on the distorted density:

$$\widetilde{h}_t(z_{t+dt}) \;\propto\; h(z_{t+dt} \mid z_t = \hat{z}_t) \cdot \underbrace{\left(\frac{h(z_{t+dt} \mid x_t = \hat{x}_t)}{h(z_{t+dt} \mid x_t = \mathcal{B}_t)} \right)^{\chi}}_{\text{representativeness ratio}}, \tag{15}$$

where $\widetilde{h}(\cdot|\cdot)$ is the distorted density, $h(\cdot|\cdot)$ represents the correct probability density function, $\hat{z}_t$ and $\hat{x}_t$ represent respectively the current realizations of z_t and x_t, and $\mathcal{B}_t$ is the background context described above.[26] The proportionality constant is chosen to ensure that $\widetilde{h}_t(\cdot)$ integrates to one.

Equation (15) shows that XDE forecasters have in mind a "kernel of truth", that is, the true conditional distribution of future states $h(z_{t+dt} \mid z_t = \hat{z}_t)$, but distort it through the representativeness ratio. After a sales surprise $\hat{x}_t - \mathcal{B}_t \equiv \mathcal{S}_t$, realizations of z_{t+dt} that become more likely are overweighted, while those that become less likely are underweighted. The magnitude of this bias is governed by the behavioral parameter χ. When $\chi = 0$, (15) nests rational expectations. When $\chi > 0$, shocks to public demand g_t spill over into beliefs about private demand—even though g_t is uninformative for z_{t+dt} to a rational

[26] We can equivalently define the distorted beliefs for public demand g_t, given by $\widetilde{h}_t(\hat{g}_{t+dt}) \;\propto\; h(\hat{g}_{t+dt} \mid g_t = \hat{g}_t) \cdot \left(\frac{h(\hat{g}_{t+dt}|x_t=\hat{x}_t)}{h(\hat{g}_{t+dt}|x_t=\mathcal{B}_t)} \right)^{\chi}$

and fully informed agent.

This setup allows us to state how cross-domain extrapolation shapes the perceived evolution of variables.

Proposition 1 (Continuous-time XDE with OU processes)**.** *When the fundamentals follow* (12) *and* (13)*, the perceived laws of motion for z_t and g_t under cross-domain extrapolation are*

$$d\tilde{z}_t = \Big(-\mu_z z_t + \underbrace{\chi_z \mathcal{S}_t}_{\text{XDE bias}} \Big) dt + dZ_t, \quad \chi_z \equiv \chi \, \beta_z \frac{\sigma_z^2}{\sigma_{z|x}^2} \tag{16}$$

$$d\tilde{g}_t = \Big(-\mu_g g_t + \underbrace{\chi_g \mathcal{S}_t}_{\text{XDE bias}} \Big) dt + dG_t, \quad \chi_g \equiv \chi \, \beta_g \frac{\sigma_g^2}{\sigma_{g|x}^2}, \tag{17}$$

where β_z is the coefficient from an OLS regression of z_t on x_t, $\sigma_{z|x}$ is the variance of z_t conditional on x_t, β_g is the coefficient from an OLS regression of g_t on x_t, and $\sigma_{g|x}$ is the variance of g_t conditional on x_t.[27]

Proof. See Appendix B. □

Proposition 1 provides a decomposition of cross-domain beliefs into a rational expectations component and a systematic bias. This bias depends on four components. First, the behavioral parameter χ governs the strength of the representativeness heuristic (i.e., how much the agent lets resemblance override the true conditional distribution). Second, the OLS coefficients, β_z and β_g, capture the co-movement between the forecast variable (z_t or g_t) and the salient outcome (x_t). They summarize the extent to which total sales are representative of public and private sales in the agent's memory. Third, the variance ratio $\sigma_z^2/\sigma_{z|x}^2$ scales the bias by the precision of x as a predictor for z or g. Finally, through its recursive law of motion (14), sentiment then becomes the relevant state variable summarizing agents' optimism or pessimism in expectations. Together, these components map behavioral heuristics into forecast errors.

Finally, cross-domain extrapolation for the two fundamentals generates diagnostic expectations for the salient variable x_t:

$$d\tilde{x}_t = \left(-\mu_z z_t - \mu_g g_t + \chi_x \mathcal{S}_t\right) dt + dZ_t + dG_t, \quad \chi_x \equiv \chi \left(\beta_z \frac{\sigma_z^2}{\sigma_{z|x}} + \beta_g \frac{\sigma_g^2}{\sigma_{g|x}} \right).$$

Taken together, (14), (16) and (17) define the stochastic process governing the perceived evolution of private and public demand under XDE. This process depends on structural

[27] $\beta_z = \frac{\sigma_z^2/\mu_z}{\sigma_z^2/\mu_z + \sigma_g^2/\mu_g}$, $\sigma_{z|x} = \frac{\sigma_z^2/\mu_z}{\sigma_z^2/\mu_z + \sigma_g^2/\mu_g}\left(\sigma_g^2/2\mu_g + \sigma_z^2\right)$, $\beta_g = \frac{\sigma_g^2/\mu_g}{\sigma_z^2/\mu_z + \sigma_g^2/\mu_g}$, $\sigma_{g|x} = \frac{\sigma_g^2/\mu_g}{\sigma_z^2/\mu_z + \sigma_g^2/\mu_g}\left(\sigma_z^2/2\mu_z + \sigma_g^2\right)$.

parameters governing the private and public demand process as well as on the two psychological parameters, κ and χ. In Section 5.2 we leverage our empirical estimates and micro-data to discipline these parameters. The state vector consists of private demand z_t, public demand g_t, and sentiment $\mathcal{S}_t$. To provide some additional intuition for how XDE operates, we illustrate how our theory of expectations works by simulating our model.

3.2 Simulation of cross-domain extrapolation

To illustrate how an XDE agent forms forecasts, the four panels in Figure 6 plot simulated paths for public demand g_t, private demand z_t, total sales x_t and the latent sentiment variable $\mathcal{S}_t$, respectively. To simplify the exposition, we only plot the behavior of the model given a single shock to public sales. In each panel, the solid lines show the realized path of a variable, and the dashed lines show the forecast path of the same variable at different forecasting times.

The first panel depicts a positive shock to public demand raises g_t. Private sales receive no shocks. As total sales are the sum of public and private sales, total sales also jump. Through (14), this shock induce a jump in sentiment $\mathcal{S}_t$ (fourth panel). Elevated sentiment then tilts beliefs upward, implying that the XDE forecast for public demand (solid green) exceeds the rational benchmark (dashed green), which quickly mean-reverts toward the unconditional mean at the correct rate.

The second panel shows how this optimism spills over to beliefs about private demand. Even though z_t experienced no positive shocks, the agent extrapolates the positive sales news across domains, so that the XDE forecast (solid blue line) lies above the rational path (dashed blue).

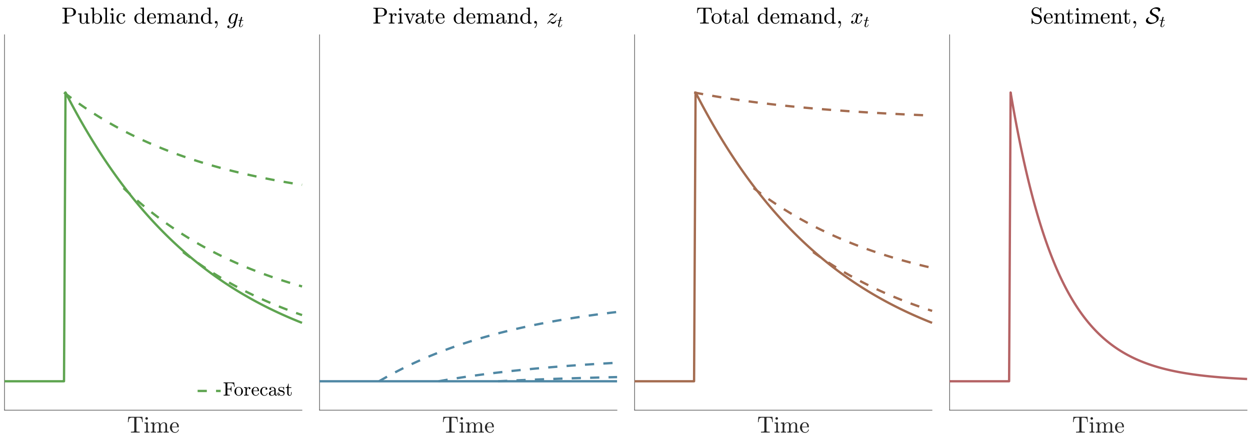
The third panel combines the two components: since expectations for both g_t and z_t are biased upward, the bias in expectations cumulates, so the forecast for total sales (solid red) lies above the rational benchmark by more than either component on its own.

Finally, we show that sentiment gradually wanes as the behavioral memory of the shock fades—a process reflected in the mean reversion of sentiment. Consequently, forecast paths converge to the true realization as sentiment reverts back to zero.

4 A q-theory of Investment with Sentiment

Having developed our model of expectation formation under cross-domain extrapolation, we now examine its implications for firm investment. We embed sentiment into an otherwise standard partial equilibrium q-theory of investment with financial frictions. This

Figure 6: Simulation of cross-domain extrapolation



Note: simulated paths for public, private and total demand, as well as sentiment, given a single shock to public sales. Solid lines denote the realized path of each variable, while dashed lines denote the forecast path of the each variable at different forecasting times.

allows us to characterize analytically not only the impact of sentiment on investment decisions but also its interaction with financial frictions.

4.1 Partial equilibrium model of the firm

Time $t \geq 0$ is continuous and runs forever. There is a continuum of monopolistically competitive firms $i \in [0, 1]$, each producing a differentiated good. Firms face stochastic fluctuations in public and private demand for their good, denoted by g and z, and use capital k as an input.[28] This gives rise to a profit function of the form $\pi(k, g, z)$, which is increasing in g and z and concave and single-peaked in capital.[29]

Firms own their capital stock and accumulate it through investment. To install ι units of capital, a firm must purchase $\iota + \Phi(\iota, k)$ units of the final good, where $\Phi(\iota, k)$ denotes investment adjustment costs. These costs make the investment decision forward-looking. Capital depreciates at rate δ.

Investment is subject to financial frictions in the form of an external-finance premium. We capture this with a cost function $\Gamma(\mathrm{d}) \leq 0$, where $\mathrm{d} \equiv \pi(k, g, z) - \iota - \Phi(\iota, k)$ denotes dividends paid to shareholders. The function $\Gamma(\mathrm{d})$ satisfies $\Gamma(\mathrm{d}) = 0$ when $\mathrm{d} \geq 0$ and $\Gamma(\mathrm{d}) < 0$ when $\mathrm{d} < 0$ (when the firm resorts to external financing). The net dividend paid to shareholders is therefore $\mathrm{d} + \Gamma(\mathrm{d})$.

We depart from the rational expectations benchmark by allowing firms to hold biased

[28] The inclusion of other frictionlessly adjustable inputs does not affect our analysis.
[29] We microfound this profit function in Section 5.1.

expectations about future demand (g,z). To do so, we embed our theory of expectations formation into this environment and let sentiment $\mathcal{S}$ bias firms' expectations, as described in (16) and (17).[30] Throughout this section, we treat sentiment as exogenous, which allows us to cleanly isolate the way in which biased expectations affect firms' investment decisions.

Firms maximize the net present value of dividends and discount the future at the interest rate r.[31] In a stationary economy, they solve the following problem:

$$\begin{aligned} V(k_0,\mathcal{S}_0,g_0,z_0) = \max_{\{\iota_t\}_{t\geq 0}} \quad & \widetilde{\mathbb{E}}_0 \int_0^\infty e^{-rt} \left(\mathrm{d}_t + \Gamma(\mathrm{d}_t)\right) dt \\ s.t. \quad & \dot{k}_t = \iota_t - \delta k_t, \\ & \mathrm{d}_t = \pi(k_t,g_t,z_t) - \iota_t - \Phi\left(\iota_t,k_t\right), \end{aligned} \tag{18}$$

where $\widetilde{\mathbb{E}}_0$ denotes the time-0 expectation over idiosyncratic demand g_t and z_t under firms' subjective beliefs shaped by sentiment $\mathcal{S}$, as described in Proposition 1.

4.2 Sentiment and investment

Armed with our theoretical framework, we now analytically characterize the way in which sentiment affects firms' investment behavior.

Proposition 2 (Euler equation)**.** *Let $\widetilde{q}$ denote the marginal value of capital perceived by the firm, i.e., $\widetilde{q} \equiv V_k$. The evolution of $\widetilde{q}_t$ can be decomposed as*

$$(r+\delta)\widetilde{q}_t = \ \left(\pi_k(k_t,g_t,z_t) - \Phi_k(\iota_t,k_t)\right)\left(1+\Gamma'(\mathrm{d}_t)\right) \ + \ \mathbb{E}_t \ d\widetilde{q}_t/dt \ + \ \left(\widetilde{\mathbb{E}}_t - \mathbb{E}_t\right) \ d\widetilde{q}_t/dt, \tag{19}$$

where $\mathbb{E}_t$ and $\widetilde{\mathbb{E}}_t$ are the rational and biased expectation operators respectively. Under our expectations formation process (14), (16) *and* (17)*, we can rewrite the last term as*

$$\left(\widetilde{\mathbb{E}}_t - \mathbb{E}_t\right) \ d\widetilde{q}_t/dt = \left(\chi_g \frac{\partial \widetilde{q}_t}{\partial g} + \chi_z \frac{\partial \widetilde{q}_t}{\partial z}\right) \mathcal{S}_t.$$

Proof. See Appendix C.2. □

Proposition 2 shows that in the presence of sentiment, the marginal-q condition takes the familiar form of an Euler equation, augmented by a novel expectation term. Absent

[30] Hence, firms are "internally rational" as in Adam and Marcet (2011).

[31] We define this problem in steady-state, generalizing it to the non-stationary case with aggregate shocks in the next section.

financial frictions and under rational expectations, this expression nests the classic Euler equation of neoclassical investment theory (Abel and Eberly, 1994).

The left-hand side represents the required gross return on a marginal unit of capital: the interest rate plus depreciation. The right-hand side is the expected return from that unit, and consists of two terms. The first is the instantaneous cash yield (the marginal increase in profits from an additional unit of capital, net of the adjustment cost and further reduced by the fact that higher investment lowers dividends, increasing the external-finance premium). The second is the expected capital gain (the anticipated appreciation in the value of installed capital, i.e., the change in perceived marginal q).

Under rational expectations, the capital gain component coincides with the true expected path of future returns. The presence of sentiment, however, introduces an additional term: an expectation wedge capturing an "expected forecast error", a predictable bias embedded in firms' subjective beliefs. This wedge is the channel through which sentiment affects investment dynamics. For optimistic firms featuring positive $\mathcal{S}$ the wedge is positive, raising the perceived marginal value of capital and thereby stimulating investment, as the next proposition shows.

Proposition 3 (Sentiment and investment)**.** *The optimal investment decision when firms have sentiment is given by*

$$\Phi_\iota(\iota_t, k_t) = \frac{q_t + \mathscr{F}_t}{1 + \Gamma'(\mathrm{d}_t)} - 1 \tag{20}$$

$$\mathscr{F}_t \equiv \int_0^\infty e^{-(r+\delta)s} \left(\widetilde{\mathbb{E}}_t - \mathbb{E}_t\right) \left(1 + \Gamma'(\mathrm{d}_{t+s})\right) \left(\pi_k(k_{t+s}, g_{t+s}, z_{t+s}) - \Phi_k(\iota_{t+s}, k_{t+s})\right) ds,$$

where $q_t = V_k(k_t, 0, g_t, z_t)$ is the rationally perceived marginal value of capital and $\mathscr{F}_t$ is the forecast error in perceived marginal q and can be expressed in terms of sentiment as

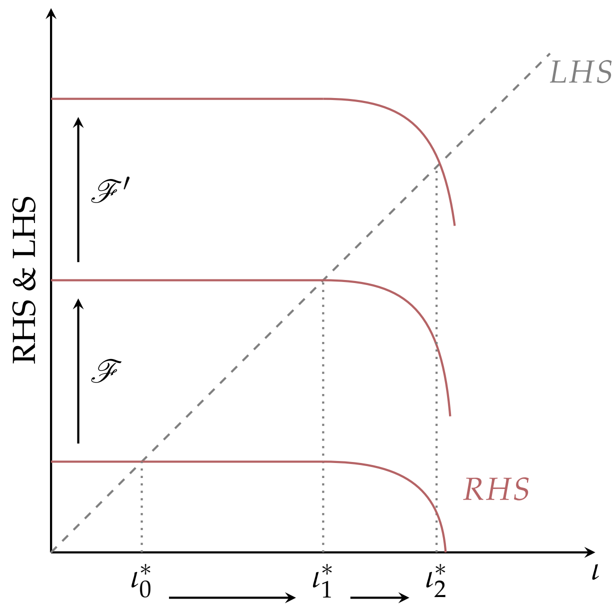
$$\mathscr{F}_t = V_k(k_t, \mathcal{S}_t, g_t, z_t) - V_k(k_t, 0, g_t, z_t). \tag{21}$$

Proof. See Appendix C.2. □

Proposition 3 characterizes how sentiment distorts firms' investment behavior through an additive term $\mathscr{F}_t$, which represents the misperception in the marginal value of capital (the marginal q). As shown in (21), this misperception is increasing in sentiment. Therefore, higher sentiment raises investment, driven by optimistic expectations of future returns.

A key implication of (20) is that financial constraints dampen the relationship between investment and sentiment. When firms are constrained, they must rely on costly external

Figure 7: Effect of sentiment on investment



Note: diagrammatic representation of the effect of an increase in sentiment on investment using Proposition 3. The right hand side (RHS) and left hand side of (20) are depicted in solid red and dashed gray, respectively.

finance to invest more, as reflected in the $\Gamma'(\mathrm{d}_t)$ term in the denominator. This raises the effective marginal cost of investment, thus muting the sensitivity of investment to expectations.

We formalize this intuition by defining the *marginal propensity to invest out of sentiment* (MPIS) as the marginal change in investment induced by an additional unit of sentiment:

$$MPIS(k, \mathcal{S}, g, z) \equiv \frac{\partial \iota}{\partial \mathcal{S}} = \frac{\partial \iota}{\partial \mathscr{F}} \cdot \frac{\partial \mathscr{F}}{\partial \mathcal{S}}. \tag{22}$$

By construction, the MPIS is always positive, since the perceived marginal value of capital is increasing in sentiment, i.e., $V_{k\mathcal{S}} > 0$. Moreover, the MPIS is lower for financially constrained firms, as tighter constraints raise the effective marginal cost of investment through $\Gamma'(\mathrm{d})$.

Figure 7 illustrates the sentiment–investment relationship implied by (20), with investment on the horizontal axis. The grey dashed line plots the marginal cost of investment (left-hand side of (20)), which is increasing in investment. The red solid lines plot the perceived marginal benefit of capital (right-hand side of (20)) at different sentiment levels. The marginal benefit is flat in investment when firms are unconstrained, but declines with investment once external finance is needed, as lower dividends raise funding costs. The intersection of these two curves determines the optimal investment level.

To see how financial frictions affect the MPIS, start from some baseline level of investment ι_0^* and consider an increase in sentiment which shifts up the marginal benefit schedule (red line). When the firm is unconstrained, this leads to a large increase in investment to ι_1^*. When investment exceeds internal cash flows, the firm must turn to outside funding. From that point, the marginal benefit schedule slopes downward, so that further increases in sentiment yield smaller increases in investment, to ι_2^*. The MPIS is therefore lower for constrained firms, whose limited scope for intertemporal decisions makes beliefs about the future less relevant for current behavior. The interaction between financial frictions and the MPIS will play a central role in Section 5, where we study how the sentiment channel shapes the aggregate transmission of fiscal policy.

Testing the theory in the data A central prediction of Proposition 3 is that financially constrained firms exhibit a flatter investment-sentiment relationship. Under a standard power form for the investment function, $\Phi(\iota, k) = \frac{\phi}{a}\left(\frac{\iota}{k} - \delta\right)^a k$, (20) maps directly into a regression specification of the log investment-capital ratio on marginal q_t and $\mathscr{F}_t$. Since $\mathscr{F}_t$ is the present value of forecast errors about future profitability, our survey data allow us to construct an empirical proxy for it and test this prediction.

Since most firms in our sample are privately owned, market-based measures of q are not readily available. Following Asker et al. (2014), we therefore proxy q_t with sales growth, for which both realizations and expectations are available in our data. Because $\mathscr{F}_t$ captures misperceptions in q_t, we construct its empirical counterpart as the forecast error in sales growth between t and $t+1$, which we denote by $\widehat{\mathscr{F}_t}$.

To measure financial frictions, we use the CADS credit score, which is constructed based on Altman's Z-score. Within each sector-year, we split firms into two groups: those in the bottom half of the credit score distribution (corresponding to high default risk and strong frictions), and those in the top half (corresponding to low default risk and weak frictions).[32] We then define an indicator variable D_{it}^{score} equal to 1 if firm i in year t is in the high-frictions bin.

Having constructed empirical counterparts to the objects defined in Proposition 3, we estimate the following regression:

$$\log \frac{Inv_{it}}{K_{it}} = \gamma \Delta \log Sales_{it+1} + \delta \widehat{\mathscr{F}}_{it+1} + \beta \widehat{\mathscr{F}}_{it+1} \times D_{it}^{\text{score}} + \psi D_{it}^{\text{score}} + \Xi \boldsymbol{X}_{it}' + u_{it}, \quad (23)$$

where $\frac{Inv_{it}}{K_{it}}$ is total investment over total material assets in period t, $\Delta \log Sales_{it+1}$ denotes

[32] We construct groups within 2-digit NACE sectors and year to prevent industry composition or aggregate fluctuations driving our results.

Table 3: Financial frictions and sentiment in the data

	Dependent variable: log $\frac{Inv_{it}}{K_{it}}$				
	(1)	(2)	(3)	(4)	(5)
$\Delta \log Sales_{it+1}$	1.207***	1.415***	0.690***	1.099***	1.189***
	(0.339)	(0.171)	(0.115)	(0.136)	(0.152)
$\widehat{\mathscr{F}}_{it+1}$	-1.119***	-1.130***	-0.712***	-0.847***	-0.850***
	(0.168)	(0.136)	(0.103)	(0.115)	(0.148)
Low score$_{it}$	-0.151***	-0.163***	-0.074***	-0.194***	-0.162***
	(0.052)	(0.049)	(0.024)	(0.044)	(0.045)
$\widehat{\mathscr{F}}_{it+1} \times$ Low score$_{it}$	0.386***	0.379***	0.413***	0.283**	0.199*
	(0.135)	(0.131)	(0.104)	(0.123)	(0.106)
$\log \frac{Inv_{it-1}}{K_{it-1}}$			0.690***		
			(0.017)		
Observations	45,590	45,590	43,952	45,401	42,635
R-squared	0.01	0.04	0.49	0.20	0.21
Within R-squared	0.01	0.02	0.48	0.01	0.01
Year FEs		✓	✓		
Sector $\times$ Year FEs				✓	✓
Year of birth FEs					✓

Note: estimates from (23). $\widehat{\mathscr{F}}_t$ denotes the forecast error in sales growth between t and $t+1$. $\Delta \log Sales_{it+1}$ is realized sales growth between t and $t+1$. Low score is a dummy for firms in the bottom 50% of the credit score distribution within a 2-digit NACE sector and year. Standard errors are clustered at the firm and year levels. All estimates are weighted using survey weights.

the growth rate in sales of firm i between years t and $t+1$, $\widehat{\mathscr{F}}_{it+1}$ and D^{score}_{it} were defined above, and $\boldsymbol{X}_{it}$ denotes a vector of controls, specified below.

Table 3 reports the results from this regression. The different columns report specifications with different sets of controls in $\boldsymbol{X}_{it}$: no controls, year fixed effects, lagged log investment, sector-year fixed effects, and firm-cohort fixed effects. We find that a one-percentage-point increase in realized sales growth—our proxy for marginal q—is associated with an increase in the investment rate between 0.7 and 1.4%. Similarly, firms that turn out to be ex-post overoptimistic about sales growth also invest more, with a 1 percentage point optimistic forecast error associated with a 0.7 to 1.1% increase in the investment rate. The estimated coefficients on realized growth and forecast errors are remarkably similar in magnitude, consistent with the prediction of (20) that q_t and $\mathscr{F}_t$ enter the investment equation symmetrically.

Our main coefficient of interest, $\widehat{\beta}$, captures the differential sensitivity of investment to sentiment for financially constrained firms relative to unconstrained ones. Across all specifications, we find consistently positive and statistically significant estimates. A one-percentage-point optimistic forecast error is associated with a 0.2 to .38 % smaller investment rate for firms in the bottom half of the credit score distribution, relative to a baseline of 0.7 to 1.1% for firms in the top 50% of the distribution. These findings confirm our theoretical prediction that financial constraints dampen the transmission of sentiment to investment.

4.3 Taking stock

In this section we have analytically characterized how sentiment shapes investment by embedding our model of expectation formation in a q-theory of investment with financial frictions. After showing that investment is increasing in sentiment, and that the sensitivity of investment to sentiment is muted for firms that face tighter financial constraints, we confirmed this prediction using our firm-level survey data. We now proceed to embed this model of investment with sentiment in a full general equilibrium environment, which will enable us to study the role of the sentiment channel for the aggregate transmission of fiscal policy.

5 The Sentiment Channel in General Equilibrium

Having characterized the effect of sentiment on investment, we now turn to the quantification of our framework. To do so, we first embed both the firm investment problem of Section 4 and the XDE model of Section 3 within a general equilibrium heterogeneous-firm New Keynesian model. We calibrate the model to our data, quantify the sentiment channel and run policy experiments.

5.1 General equilibrium model

We now proceed to embed the problem of the firm (detailed in Section 4.1) in a general equilibrium model and describe the different blocks of the model.

Public and private demand There are two final goods: a private good (used for household consumption and investment), and public goods (consumed by the government). Let Y_t^z and G_t denote, respectively, the aggregators for these two goods. Both are CES composites of the differentiated intermediate varieties y_{it}:

$$Y_t^z = \left(\int_0^1 e^{\frac{z_{it}}{\varepsilon}} y_{it}^{z\,\frac{\varepsilon-1}{\varepsilon}}\, di\right)^{\frac{\varepsilon}{\varepsilon-1}} \quad \text{and} \quad G_t = \left(\int_0^1 e^{\frac{g_{it}}{\varepsilon}} y_{it}^{g\,\frac{\varepsilon-1}{\varepsilon}}\, di\right)^{\frac{\varepsilon}{\varepsilon-1}}, \tag{24}$$

where z_{it} and g_{it} denote, respectively, idiosyncratic taste shifters of private and public demand for variety i. These aggregators deliver standard downward-sloping firm-level demands:

$$y_{it}^z = e^{z_{it}} \left(\frac{p_{it}}{P_t^z}\right)^{-\varepsilon} Y_t^z \quad \text{and} \quad y_{it}^g = e^{g_{it}} \left(\frac{p_{it}}{P_t^g}\right)^{-\varepsilon} G_t, \tag{25}$$

with corresponding price indices

$$P_t^z = \left(\int_0^1 e^{z_{it}} p_{it}^{1-\varepsilon} di\right)^{\frac{1}{1-\varepsilon}}, \quad P_t^g = \left(\int_0^1 e^{g_{it}} p_{it}^{1-\varepsilon} di\right)^{\frac{1}{1-\varepsilon}}.$$

Total demand for firm i is therefore $y_{it} = y_{it}^z + y_{it}^g$. To map this structure to our theory of expectation formation, we let the taste shifters z_{it} and g_{it} evolve stochastically according to the OU processes described in (12) and (13).

Production Each firm is a monopolistically competitive producer of a differentiated variety i and uses a constant-returns-to-scale technology that combines k_{it} units of capital and ℓ_{it} units of labor:

$$y_{it} = k_{it}^{\alpha} \ell_{it}^{1-\alpha},$$

where $\alpha \in (0,1)$. Labor is hired on a competitive market at the real wage w_t and can be adjusted frictionlessly within a period. Capital is owned by the firm and evolves according to the law of motion described in (18). Normalizing the price index of the private good to one, the firm's static optimization over labor and prices yields a profit function of the form

$$\pi_t(g,z,k) = \Omega_t(k) \cdot \left(e^z + \alpha_{gt} e^g\right)^{\frac{1}{1-\alpha+\varepsilon\alpha}}, \tag{26}$$

where $\Omega_t(k)$ is a function of capital, aggregate variables (captured by the time index) and structural parameters, and $\alpha_{gt} \equiv G_t (P_t^g)^{\varepsilon} / Y_t^z$.[33] This expression incorporates the firm's optimal static labor and pricing decisions.

[33] See Appendix C.1 for derivations.

The problem of the firm and expectations As the time-dependence of the profit function (26) makes clear, solving the dynamic capital accumulation problem requires firms to form expectations over sequences of aggregate variables $\{\Upsilon_t\}_{t\geq 0} = \{w_t, r_t, Y_t^z, G_t, P_t^g\}_{t\geq 0}$. Regarding these aggregate variables, we maintain the assumption of rational expectations.[34]

We introduce departures from rational expectations in the way firms forecast their own idiosyncratic private and public demand shifters z_{it} and g_{it}. We model expectations over these variables using the recursive formulation of cross-domain extrapolation defined in (14), (16) and (17).[35] This formulation introduces sentiment $\mathcal{S}$ as an additional state, which means that the firm's idiosyncratic state vector becomes $\boldsymbol{s} = (k, \mathcal{S}, g, z)$.

A key challenge when departing from rational expectations is that the *perceived* and *true* law of motion of the states no longer coincide. Our recursive formulation with sentiment provides a tractable way to capture this discrepancy in the Hamilton-Jacobi-Bellman (HJB) equation through an additive term, which we label the rationality wedge:

$$
\begin{aligned}
r_t V_t(\boldsymbol{s}) - \partial_t V_t(\boldsymbol{s}) \;=\; & \max_{\iota} \; \underbrace{\pi_t(k,g,z) - \iota - \Phi(\iota,k) + \Gamma(\mathrm{d}_t)}_{\text{flow to households}} \\
& + \underbrace{(\iota - \delta k)\, \partial_k V_t(\boldsymbol{s}) - \mu_z \partial_z V_t(\boldsymbol{s}) + \frac{\sigma_z^2}{2}\partial_{zz}^2 V_t(\boldsymbol{s}) - \mu_g \partial_g V_t(\boldsymbol{s}) + \frac{\sigma_g^2}{2}\partial_{gg}^2 V_t(\boldsymbol{s})}_{\text{continuation}} \\
& + \underbrace{\chi_z \mathcal{S}\, \partial_z V_t(\boldsymbol{s}) \;+\; \chi_g \mathcal{S}\, \partial_g V_t(\boldsymbol{s})}_{\text{rationality wedge}}, \qquad (27)
\end{aligned}
$$

where $\mathrm{d}_t = \pi_t(k,g,z) - \iota - \Phi(\iota,k)$ denotes dividends. The rationality wedge in (27) reflects the behavioral distortions introduced by sentiment, leading firms to misperceive the future evolution of z and g. Note that the law of motion for $\mathcal{S}$ itself is absent from the HJB, since we assume that firms are naïve: they do not internalize their sentiment bias, and therefore do not track its dynamics.

The solution to the HJB (27) yields a sequence of policy functions for firm investment, labor demand and dividends: $\{\iota_t(\boldsymbol{s}), \ell_t(\boldsymbol{s}), \mathrm{d}_t(\boldsymbol{s})\}_{t\geq 0}$. Together with the sequence of aggregate variables $\{\Upsilon\}_{t\geq 0}$, these fully characterize the evolution of the distribution of firms over the state space, $\mu_t(\boldsymbol{s})$, as described by the Kolmogorov Forward equation:

[34] In what follows we focus on a so-called MIT shock, which implies that after date 0 firms have perfect foresight with respect to sequences of aggregate variables.

[35] To cast the firm's problem in the log-additive setting of Section 3, we take a first-order approximation of the profit function (26) around steady-state, which shows that the salient variable shifting total profits is $z + \alpha_g g$.

$$\partial_t \mu_t(\boldsymbol{s}) = \underbrace{-\partial_k[(\iota_t(\boldsymbol{s}) - \delta k)\mu_t(\boldsymbol{s})] - \mu_z \partial_z[z\mu_t(\boldsymbol{s})] - \mu_g \partial_g[g\mu_t(\boldsymbol{s})] - \kappa \partial_{\mathcal{S}}[\mathcal{S}\mu_t(\boldsymbol{s})]}_{\text{true drifts}} \tag{28}$$
$$\underbrace{+\frac{1}{2}\sigma_z^2 \partial_{zz}^2\, \mu_t(\boldsymbol{s}) + \frac{1}{2}\sigma_g^2 \partial_{gg}^2\, \mu_t(\boldsymbol{s}) + \frac{\sigma_z^2 + \sigma_g^2}{2}\partial_{\mathcal{S}\mathcal{S}}^2 \mu_t(\boldsymbol{s}) + \sigma_z^2 \partial_{z\mathcal{S}}^2\, \mu_t(\boldsymbol{s}) + \sigma_g^2 \partial_{g\mathcal{S}}^2\, \mu_t(\boldsymbol{s})}_{\text{true diffusion}}$$

Because the Kolmogorov Forward equation describes the *true* evolution of the distribution, it omits the rationality wedge present in the HJB, while instead featuring the law of motion for sentiment. Misperceptions of z and g enter only indirectly through the biased policy functions $\iota_t(\boldsymbol{s})$, while the processes for z, g and $\mathcal{S}$ remain themselves correct.

This concludes the description of the firms' problem. We now turn to the remaining blocks of the model to characterize general equilibrium.

Household There is a representative Ricardian household that discounts the future at rate ρ and has preferences over hours worked and consumption. The household owns the firms and can save in a riskless real bond issued by the government A_t. The household solves the following problem:

$$\begin{aligned} \max_{\{C_t, \dot{A}_t\}_{t\geq 0}} \quad & \mathbb{E}_0 \int_0^\infty e^{-\rho t}\ (u(C_t) - v(L_t))\ dt \\ s.t. \quad & \dot{A}_t = w_t L_t + r_t A_t + D_t - T_t - C_t, \end{aligned} \tag{29}$$

where C_t is consumption, L_t is hours worked, D_t are aggregate dividends rebated by firms, and T_t are government lump-sum taxes. Labor supply is determined by unions, which we describe next.

Unions We introduce nominal rigidities in the form of sticky wages. As is standard in the New Keynesian sticky-wage literature, we assume that households' hours worked L_t are determined by labor unions (Erceg et al., 2000, Schmitt-Grohé and Uribe, 2005, Auclert et al., 2024). Specifically, the household supplies a measure of labor ℓ_{kt} to a continuum of monopolistically competitive unions indexed by $k \in [0, 1]$, with $\int_0^1 \ell_{kt} dk = L_t$. Each union provides a differentiated labor service, which is combined into aggregate labor by a competitive labor packer using a CES aggregator with elasticity of substitution ϵ. Unions set wages for their variety to maximize lifetime utility of the household, subject to Rotemberg adjustment costs:

$$\max_{\{\pi^w_{kt}\}} \mathbb{E}_0 \int_0^\infty e^{-\rho t}\Big(u(C_t) - v(\ell_{kt}) - \frac{\psi}{2}\,(\pi^w_{kt})^2 L_t\Big) dt,$$

where π^w_{kt} denotes nominal wage inflation. This problem gives rise to the following non-linear wage New Keynesian Phillips Curve:[36]

$$\epsilon\, v'(L_t)L_t - (\epsilon-1)u'(C_t)\, w_t\, L_t + \psi\big(\dot{\pi}^w_t - \rho\, \pi^w_t\big) = 0.$$

Government The government finances G_t via lump-sum taxes on households T_t and debt issuance. The budget constraint of the government is

$$\dot{B}_t = r_t B_t + G_t - T_t \tag{30}$$

We assume the government runs a balanced budget in every period.[37]

Government spending shocks A convenient feature of our heterogeneous-firm framework is that we can model government spending shocks not as a homogeneous disturbance to aggregate demand G_t, but as a collection of correlated shocks to individual firms' procurement. Specifically, following a procurement shock, firm i's public demand shifter becomes $g_{it} + \widehat{g}_i$, reverting thereafter at the exponential rate μ_g. Importantly, these shocks operate only through firm-level demand functions (25) and do not directly shift the aggregator governing government preferences (24). In the special case of a homogeneous procurement shock, aggregate government consumption jumps on impact to $G_t = e^{\widehat{g}}G$, where G denotes steady-state government spending, and then mean reverts at rate μ_g.

According to our equation (14), procurement shocks affect sentiment by shifting the demand of individual firms. A shock $\widehat{g}_i$ raises sentiment of firm i by $\alpha_g \widehat{g}_i$, thereby making the firm optimistic about its future private and public demand z and g. We denote by $\mu_0(\boldsymbol{s})$ the post-shift distribution of firms.

Monetary authority Monetary policy follows a standard Taylor rule:

$$i_t = r + \phi^\pi \pi_t,$$

[36] See Appendix D.1 for details on the derivation.

[37] Because our model features a Ricardian household, the financing mix between taxes and debt is irrelevant for equilibrium outcomes.

where r is the steady-state interest rate and we assume $\phi^{\pi} > 1$ to ensure determinacy. Given inflation and the nominal interest rate, the real interest rate is then given by the continuous-time Fisher equation $r_t = i_t - \pi_t$.

Equilibrium We are now ready to define the equilibrium of our model.

Definition 1 (Equilibrium)**.** *Given a government spending shock $\{\widehat{g}_i\}_i$, an equilibrium is a sequence of firm policy functions $\{\iota_t(\boldsymbol{s}), \ell_t(\boldsymbol{s}), \mathrm{d}_t(\boldsymbol{s})\}_{t\geq 0}$, firm distributions $\{\mu_t(\boldsymbol{s})\}_{t\geq 0}$, prices $\{w_t, r_t, P_t^g\}_{t\geq 0}$, and aggregate quantities $\{G_t, C_t, L_t, A_t, I_t, D_t, T_t, B_t, Y_t\}_{t\geq 0}$ such that the post-shock distribution of firms $\mu_0(\boldsymbol{s})$ is consistent with the endogenous response of sentiment to the shocks $\{\widehat{g}_i\}_i$, the path of Government expenditures G_t is consistent with the initial government demand shock, firms optimize given prices and sentiment, the Fisher and Taylor equations hold, the government runs a balanced budget, the firm distribution evolves according to the Kolmogorov forward equation with initial condition $\mu_0(\boldsymbol{s})$, and the goods, labor, and asset markets clear:*

$$Y_t = C_t + I_t + P_t^G G_t, \tag{31}$$

$$L_t = \int \ell_t(\boldsymbol{s})\, d\mu_t(\boldsymbol{s}), \tag{32}$$

$$A_t = B_t. \tag{33}$$

Aggregate investment is defined as $I_t \equiv \int \left[\iota_t(\boldsymbol{s}) + \Phi\big(\iota_t(\boldsymbol{s}), k(\boldsymbol{s})\big) + \Gamma\big(\mathrm{d}_t(\boldsymbol{s})\big)\right] d\mu_t(\boldsymbol{s})$, where $k(\boldsymbol{s})$ is the capital stock component of the firm state, and nominal output is $Y_t \equiv \int p_t(\boldsymbol{s})\, y_t(\boldsymbol{s})\, d\mu_t(\boldsymbol{s})$.

5.2 Calibration

We now turn to the calibration of the model. Parameters are set at an annual frequency, as reported in Table 4, and we describe our calibration strategy below.

Household preferences We externally calibrate a number of parameters governing households' preferences. Following standard values in the literature, we set the household discount rate $\rho = 0.02$ to target a 2% yearly real interest rate. We specify a standard separable CRRA functional form for households' utility $u(C_t, L_t) = \frac{C_t^{1-\gamma}-1}{1-\gamma} - \vartheta \frac{L_t^{1+\varphi}}{1+\varphi}$, setting the Frisch elasticity of labor supply to 1 in line with the estimates of Chetty et al. (2011), and the elasticity of intertemporal substitution γ to 1. We internally calibrate the ϑ parameter to hit a steady-state number of hours worked equal to one third. Finally, we set the demand elasticity ε to 10, a standard value in the literature. We set government spending to 20% of GDP.

Table 4: Model calibration

	Description	Value	Comment
		Externally Calibrated	
ρ	Household discount rate	0.02	Target 2% steady-state real rate
γ	EIS	1	Standard
φ	Frisch elasticity	1	Chetty et al. (2011)
ϑ	Household labor disutility	1	Standard
α	Capital share	0.33	Standard
ε	Demand elasticity	10	Standard
ϵ	Union market power	21	Schmitt-Grohé and Uribe (2005)
ϕ^{π}	Taylor coefficient	1.5	Standard
		Internally Calibrated	
δ	Depreciation rate	0.07	Target aggregate investment rate
δ^{exit}	Firm exit rate	0.065	Target exit rate
μ_z	Id. private demand persistence	0.654	Match estimated revenue process
σ_z	Id. private demand volatility	0.318	Match estimated revenue process
μ_g	Id. public demand persistence	0.788	Match estimated procurement process
σ_g	Id. public demand volatility	1.315	Match estimated procurement process
ϕ	Investment adjustment costs	10	Target investment-q relation in Peters and Taylor (2017)
ψ	Phillips curve parameter	35	Target slope of Phillips Curve 0.18 in Beraja et al. (2019)
ζ	External finance cost	.81	Target 3.16% excess bond premium in Gilchrist and Zakrajšek (2012)
χ	Cross-domain extrapolation	0.94	Target IV results in Table 1
κ	Sentiment memory	1.72	Target estimated autocorrelation of forecast error

Note: calibration parameters are expressed at an annual frequency.

Technology parameters We calibrate the capital share in production α to 0.33, as standard. We posit a quadratic function for investment adjustment costs, as is standard in the literature, $\Phi(\iota, k) = \frac{\phi}{2}\left(\frac{\iota}{k} - \delta\right)^2 k$. We also assume a quadratic cost of external finance, i.e., $\Gamma(\mathrm{d}) = -\frac{\zeta}{2}\mathrm{d}^2 \cdot \mathbb{1}(\mathrm{d} < 0)$. We calibrate ϕ and ζ jointly to match an external finance premium of 3.16% (Gilchrist and Zakrajšek, 2012) and a regression coefficient between the investment rate and marginal q of 0.08 as estimated in (Peters and Taylor, 2017).[38] We set the depreciation rate δ to 0.07 to match a ratio of aggregate investment to GDP of 22%. Finally, firms exit the economy at a Poisson rate $\delta^{\text{exit}} = 0.065$, calibrated to match the empirical exit rate. To preserve the total mass of firms, each exit is offset by the entry of a new firm, which is initialized at the lowest capital level, with midpoint demand and no sentiment.

[38]The literature on adjustment costs reports a wide range of estimates (Chodorow-Reich, 2025), with the value in Peters and Taylor (2017) lying near the midpoint of this range.

Demand process We internally calibrate the idiosyncratic demand processes for private and public demand, z_{it} and g_{it} in (12) and (13) using our income statement and procurement data. From the contract-level procurement data we construct an annual measure of firm-level public sales, $Sales^g_{it}$, and define private sales as the difference between total sales, as reported in the income statement, and public sales:

$$Sales^z_{it} = Sales_{it} - Sales^g_{it}.$$

Using these two measures, we then estimate an AR(1) process for both public and private sales:

$$\log Sales^z_{it} = \alpha^z_i + \rho^z \log Sales^z_{it-1} + u^z_{it} \tag{34}$$

$$\log Sales^g_{it} = \alpha^g_i + \rho^g \log Sales^g_{it-1} + u^g_{it}, \tag{35}$$

where α^z_i, α^g_i are firm fixed effects. We then set our model parameters μ_z, μ_g and σ_z, σ_g so that the simulated revenue dynamics match our estimates $\hat{\rho}^z, \hat{\rho}^g$ and $\hat{\sigma}^z_u, \hat{\sigma}^g_u$. See Appendix D.2 for more details.

Cross-domain extrapolation We calibrate the two parameters χ and κ governing expectation formation using our survey data and our reduced-form estimates.

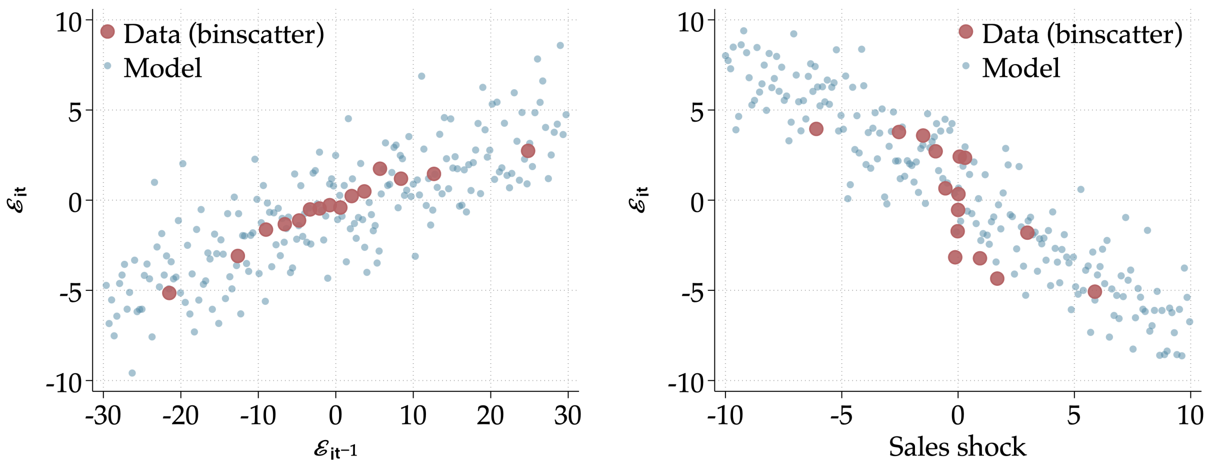
Since κ governs the persistence of sentiment, we discipline it using the autocorrelation of forecast errors in the data. After residualizing forecast errors from firm fixed effects, we estimate an autocorrelation of 0.17.[39] Figure 8a plots the empirical autocorrelation as a binned scatter, alongside the simulated counterpart from the model. Matching this empirical target yields a value of $\kappa = 1.72$, implying short-lived dynamics for sentiment, with a half-life of about two quarters. Maxted (2023) estimates a comparable parameter in the context of forecasts of capital returns. In that setting, sentiment is a lot more persistent, with a half-life of about six years.

With κ pinned down, we calibrate χ to match the IV estimate of the semi-elasticity of forecast errors with respect to sales of -0.86, as reported in Table 1. Figure 8b illustrates this moment by plotting the partial regression underlying our IV estimate—the residualized forecast error against instrumented sales—together with the corresponding simulated relationship in the calibrated model. Matching this slope yields a value of $\chi = 0.94$.

Given this calibration, our estimates of extrapolative behavior are generally conserva-

[39]We residualize for firm fixed effects to avoid persistent optimistic and pessimistic types driving the result. To mitigate Nickell bias, we implement a two-step GMM estimator as in Arellano and Bover (1995) and restrict the sample to firms with at least eight observations.

Figure 8: Simulation of forecast errors



(a) Autocorrelation in forecast error

(b) Forecast error vs sales shocks

Note: the large red dots are binned scatters from our data, while the small blue dots are scatters from model simulation. Panel (a) plots the autocorrelation of the sales forecast error $\mathcal{E}_{it}$, using the full sample from 1995 (42,713 observations) and restricting the sample to firms that appear in at least eight survey waves. The x-axis denotes the sales forecast error in $t-1$ and the y-axis the sales forecast error in t. Panel (b) visualizes the results from the IV specification (10), based on 3,576 observations. The x-axis denotes the instrumented sales shock $\widehat{\log Sales}_{it-1}$ and the y-axis the sales forecast error $\mathcal{E}_{it}$.

tive compared to the literature.

Figure D.2 in Appendix D.3 shows that our two model parameters are well identified by the chosen empirical targets. Further details on the calibration strategy are provided in Appendix D.3.

Stationary equilibrium Appendix D.6 presents stationary graphs of the economy's steady-state. It displays the distributions of capital, private demand, public demand, and sentiment, as well as the policy function with respect to capital. It also shows the dynamics of firm growth in our model (not targeted in our calibration), which is in line with the data. The policy functions illustrate how sentiment distorts investment decisions: positive sentiment stimulates additional investment, while negative sentiment dampens it.

5.3 The sentiment channel and the government spending multiplier

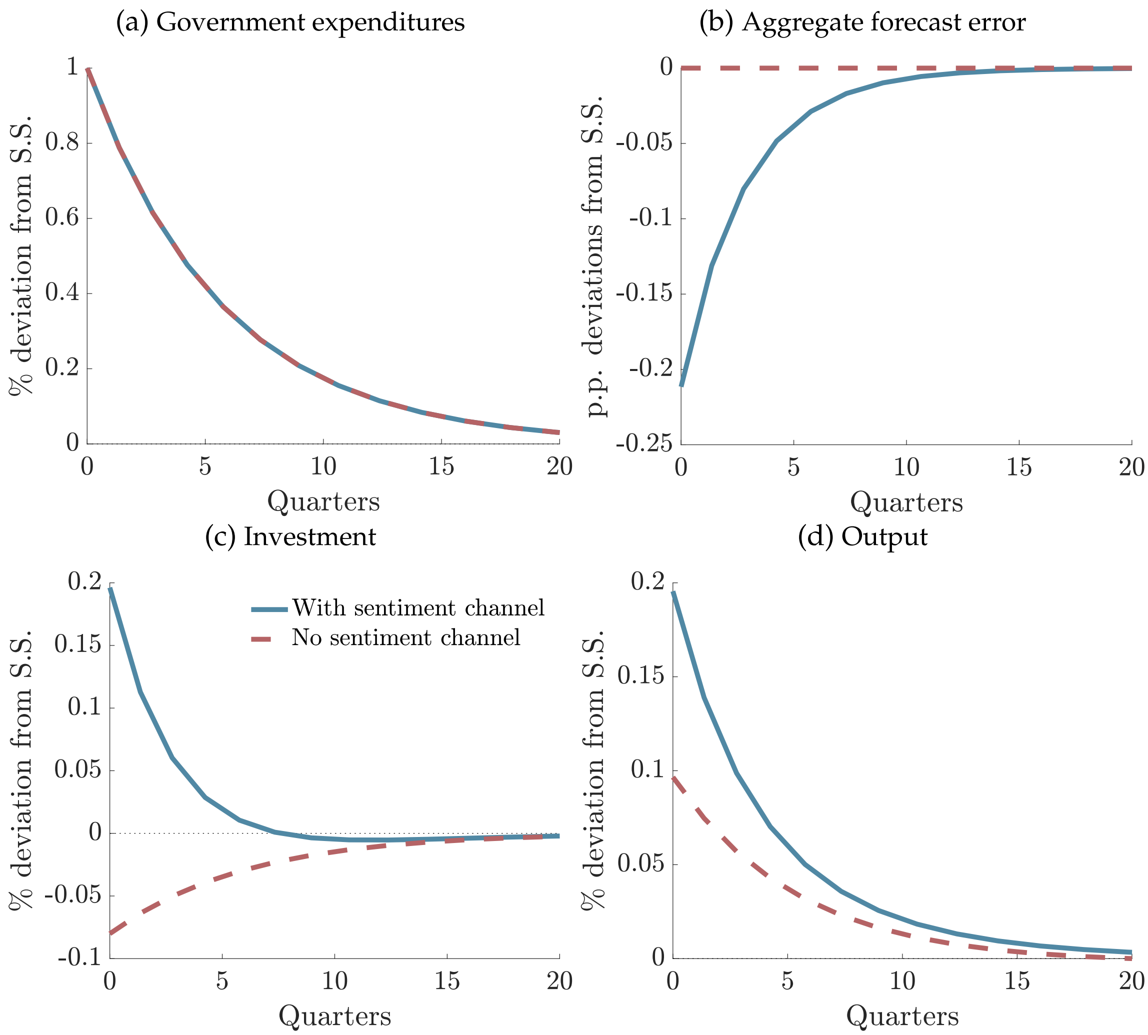
We now study the role of the sentiment channel in the transmission of fiscal policy. To do so, we consider a uniform procurement shock, as described in Section 5.1, which raises aggregate government consumption G_t by 1% on impact and with a half-life of one year, as depicted in Figure 9a.[40]

The transmission of fiscal policy with sentiment Figure 9 plots the impulse responses to this shock. The red dotted line shows the benchmark case where sentiment does not respond to government spending. Because of this, the average forecast error that firms make in forecasting their own future sales does not respond to the government spending shock—as is clear from Figure 9b. Hence, the usual transmission mechanism is at play: a rise in Government expenditures induces an income effect on households' labor supply, thus increasing output. At the same time, as households try to smooth their consumption path, the interest rate rises, leading to a fall in investment. Overall, this crowding-out of investment makes the fiscal multiplier small: around 0.5 on impact. This is the standard value of fiscal multiplier in a New Keynesian model with capital and a Taylor rule; see for example Christiano et al. (2011).

The blue solid line depicts the responses in our baseline model, where sentiment endogenously responds to government spending. Government procurement induces firms to become overoptimistic about future private and public idiosyncratic demand, as depicted in Figure 9b. This boost in optimism is sufficiently strong to overturn the crowding-out effect of government spending on investment, yielding a positive investment response in general equilibrium: Sentiment has a "*crowding-in*" effect on investment, as is clear from Figure 9c. In turn, the positive investment response boosts aggregate demand and increases the fiscal multiplier substantially, nearly doubling it, as depicted in Figure 9d. Overall, the sentiment channel substantially amplifies the government spending multiplier predicted by the model, from 0.49 in the baseline without sentiment to 1 once we introduce the sentiment channel. Figure 10 shows how our channel brings the model's predictions closely in line with the empirical consensus on multipliers, even though we do not target this moment in our calibration. The figure reports the four-year cumulative multipliers obtained from three leading methodologies summarized in Ramey (2016), together with the model's counterparts with and without sentiment. Empirical estimates cluster around a value of one—-precisely the magnitude generated by the model with sentiment, but well above that implied by the baseline specification without sentiment. Therefore, our sentiment channel offers an alternative to Heterogeneous Agents New Keynesian (HANK) models—which rely on incomplete markets and household borrowing constraints—to generate large multipliers (Auclert et al., 2024, Hagedorn et al., 2019). Our

[40] A half-life of one year is the persistence of the process for public expenditure we estimate in our microdata.

Figure 9: The sentiment channel of fiscal policy

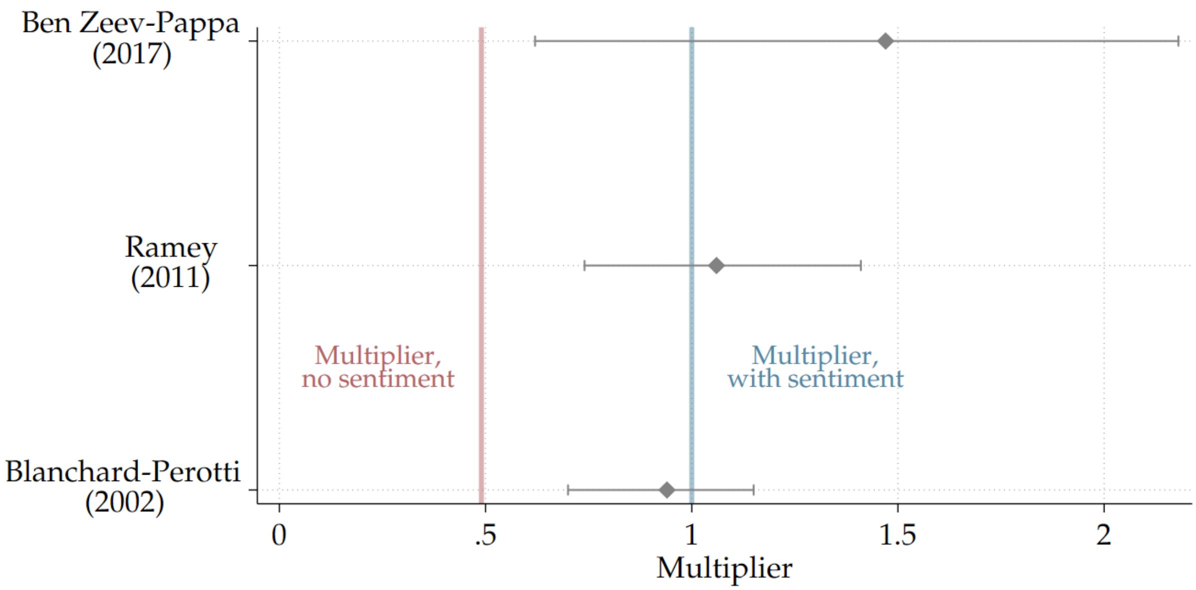


Note: IRFs to a 1% aggregate government spending shock distributed homogeneously across all firms, with quarterly mean reversion of 0.82. The dashed lines plot the response in the case where sentiment does not respond to the government spending shock. The x-axis denotes quarters elapsed since the shock, while the y-axis denotes percent deviation from steady state for panels (a), (c), and (d) and percentage point deviation from steady state for panel (b).

mechanism is instead rooted in firms' belief formation, and has different implications for the behavior of investment in response to government spending shocks.

Figure D.6 in Appendix D.6 shows additional impulse responses for the other variables in the model. A noteworthy implication of the sentiment channel concerns consumption. In the benchmark New Keynesian economy, a rise in government spending raises the natural real interest rate; under a Taylor rule, policy tightens, and private consumption is crowded out. With sentiment, firms' optimism raises investment; in the presence of nominal rigidities this extra spending feeds back into income and demand for all goods, implying that the crowding-out of consumption is attenuated relative to the rational benchmark.

Figure 10: Estimates of the government spending multiplier in the empirical literature



Note: estimates of 4-year cumulative government spending multipliers in the empirical literature and predictions from our model with and without the sentiment channel. The gray diamonds denote the point estimates for the 4-year cumulative multiplier from the three identification strategies replicated in Ramey (2016). Bands denote 68% bootstrapped confidence intervals. The blue and red vertical lines denote the cumulative multiplier in the version of our model with and without sentiment, respectively. See Appendix D.7 for more details.

To isolate the role of nominal frictions, we solve the model with flexible wages and plot the corresponding impulse responses in Figure D.10, with and without the sentiment channel. When wages are flexible, higher investment induced by optimism mainly raises the real interest rate without boosting demand, so consumption is crowded out more strongly. In other words, when nominal frictions are absent, the sentiment channel has no multiplier effects on demand; it therefore reduces, rather than mitigates, the crowding-out of consumption.

Decomposing the investment response To inspect the sentiment channel further, we leverage the investment policy function $\iota(k, \mathcal{S}, g, z; \{G_s, \Upsilon_s\}_{s\geq t})$ derived from the firm's problem in (27). Combined with the evolution of the firm distribution in (28), we can express aggregate investment as a function of aggregate sequences only (Kaplan et al., 2018, Auclert et al., 2021):

$$I_0 = \int \iota_0(k, \mathcal{S}, g, z; \{G_s, \Upsilon_s\}_{s\geq 0}) d\mu_0(k, \mathcal{S}, g, z) = \mathcal{I}\left(\{G_s, \Upsilon_s\}_{s\geq 0}\right). \tag{36}$$

We then differentiate (36) to characterize the aggregate investment response to a government spending shock:

$$dI_0 = \underbrace{\int_0^\infty \frac{\partial I_0}{\partial G_s} dG_s ds}_{\text{Direct effect}} + \underbrace{\int_0^\infty \frac{\partial I_0}{\partial \Upsilon_s} d\Upsilon_s ds}_{\text{Indirect effects}} + \underbrace{\int \frac{\partial \iota}{\partial \mathcal{S}} \frac{\partial \mathcal{S}}{\partial G_0} dG_0 d\mu_0}_{\text{Sentiment channel}} . \tag{37}$$

Equation (37) decomposes the response of aggregate investment into three terms.[41] First, the direct effect reflects the impact of increase public demand on firms' investment. Second, the indirect effects operating through general equilibrium forces, most notably the real interest rate. Finally, the sentiment channel, the central object of our analysis, captures how fiscal policy transmits to firm investment through its effect on expectations.

It is also clear in (37) that the strength of the sentiment channel depends critically on the MPIS introduced in Section 4.1. When the sensitivity of investment to expectations is low, this channel is muted. As emphasized in Section 4.1, financial frictions are a key determinant of the MPIS. This observation motivates the next section, where we study how variations in the economy-wide average MPIS shape the magnitude of the sentiment channel and, in turn, the transmission of government spending.

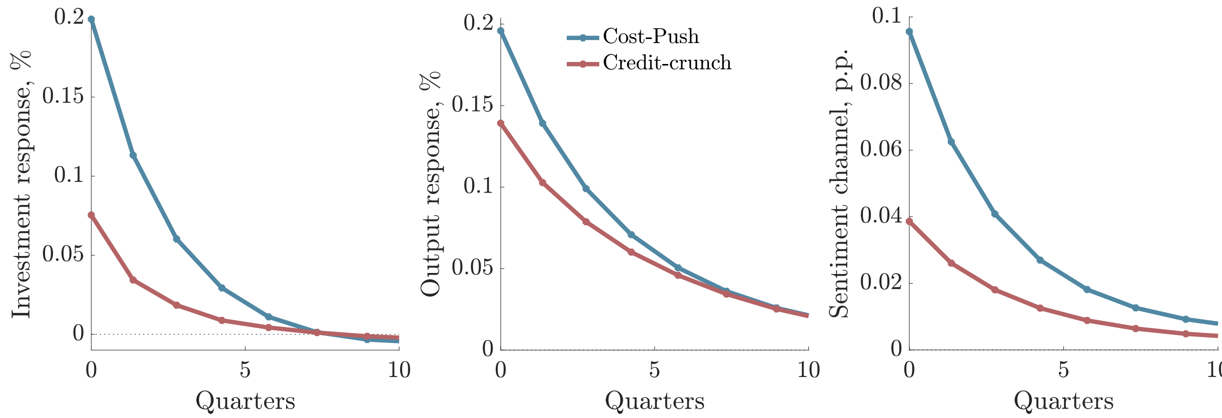
5.4 State dependency

We now study how the effectiveness of government spending depends on the state of the economy, and in particular the type of recession the economy is in. We focus on two shocks that have figured prominently in recent macroeconomic events. The first is a credit-crunch shock, capturing episodes in which firms face a sudden tightening of external finance, as in the 2008–09 Global Financial Crisis. We model this as an exogenous increase in the external-finance-premium parameter ζ. The second is a cost-push shock, interpreted as an adverse movement in real marginal costs that appears as an additive wedge in the wage Phillips curve.

Starting from steady-state, we hit the economy with each shock separately, normalizing their size so that, absent fiscal intervention, both generate the same net-present-value contraction in output. We then introduce a government spending shock of 1% of steady-state government consumption and compute Generalized Impulse Response Functions (GIRFs). For each shock, these GIRFs are computed as the difference between the economy's response with and without fiscal intervention, and hence measure the marginal effect of government spending on the propagation of each shock.

[41] Figure D.7 in Appendix D.6 plots this decomposition for the response in Figure 9.

Figure 11: State dependency of the sentiment channel



(a) Generalized IRFs, investment (b) Generalized IRFs, output (c) Generalized IRFs, sentiment channel

Note: generalized impulse responses to a 1% government spending shock. Each line shows the difference between the economy's response to a given shock with and without fiscal intervention. The blue line shows the case of a cost-push shock, while the red line that of a credit crunch, which we model as an exogenous increase in the external finance premium parameter ζ. The size of the shocks is normalized so that they induce the same net-present-value contraction in output absent government intervention.

Figure 11a shows the GIRFs for investment. The blue line corresponds to the cost-push shock, while the red line represents the credit-crunch case. The investment response to government spending is almost three times stronger in a cost-push recession than in a credit-crunch recession. This is consistent with our MPIS mechanism: tighter financial conditions lower the MPIS and mute the sentiment-induced boost to investment. Figure 11b displays the corresponding GIRFs for output, showing that government spending is up to 34% more effective during a cost-push recession than during a financial crisis. Finally, Figure 11c isolates the role of the sentiment channel by comparing GIRFs for output with and without endogenous sentiment. It reveals that the entire gap in the government spending multiplier across the two shocks is accounted for by sentiment.

The underlying mechanism follows from Proposition 3. When credit constraints are tight, the optimism generated by fiscal expansions has little effect on investment, since the high external-finance premium restricts the ability of firms to make intertemporal decisions via borrowing. Investment is instead tied to internal cash-flow, weakening the role of forward-looking expectations. Figure D.8 in Appendix D.6 confirms this by showing that, once sentiment is neutralized, the marginal effect of government spending is equalized across the two recessions.

Taken together, these results indicate that government spending is least potent in periods of financial crisis, precisely those times when policymakers may want to rely on it the most. A model abstracting from the sentiment channel, even if disciplined to match the multipliers in Figure 10, would miss this mechanism and the resulting state dependence of fiscal policy.

6 Conclusion

This paper identifies and formalizes a sentiment channel of fiscal policy, in which government spending boosts business optimism, which leads firms to invest more and thereby raises aggregate demand and output.

Empirically, we assemble a novel dataset linking Italian firms' sales forecasts, realizations, and balance sheets with the universe of procurement contracts. We show that government spending shocks make firms systematically overoptimistic about their future sales. We develop a heterogeneous-firm New Keynesian model in which firms' beliefs about future demand overreact to government spending shocks, and use this framework to quantify the aggregate importance of the sentiment channel of fiscal policy. After calibrating the expectation formation process to our micro estimates, the model implies that government spending raises optimism by enough to crowd-in private investment, amplifying the fiscal multiplier by twofold. This amplification is state-dependent: during financial crises, when constraints bind more tightly, the sentiment channel weakens and government spending is as much as 34% less effective at stabilizing output. Our results speak to longstanding debates about the role of "sentiment" in fiscal policy. They emphasize the importance of incorporating empirically-disciplined behavior of firms' beliefs into standard macroeconomic models.

Several avenues for future research remain open. First, while we focus on public demand shocks, other fiscal instruments such as taxation or credit subsidies may shape expectations in systematically different ways. Understanding whether tax cuts generate optimism about private demand, or instead induce caution by raising concerns about fiscal sustainability, would shed light on the broader belief-based transmission of fiscal policy. Second, our results raise normative questions. When expectations deviate from rationality, should policy aim to harness optimism as a stabilizing force, or should it counteract distorted beliefs to prevent inefficient investment booms and busts? More generally, how can we use the empirical responses to policy shocks to inform the design of optimal policy, in an environment where agents may overreact to unexpected disturbances but respond more weakly to systematic policy interventions?

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The Sentiment Channel of Fiscal Policy

Online Appendix

Marco Bellifemine **Adrien Couturier** **Jacopo Tozzo**

Table of Contents

A Appendix to Section 2

In this Section we present additional figures and tables, as well as robustness exercises, related to our empirical exercises. We also report the details of the reform to the Italian internal stability pact that we use as our natural experiment.

A.1 Summary statistics

Table A.1 presents summary statistics for the two merged datasets used in our analysis: the CADS-ANAC dataset (Panel A) and the INVIND-CADS-ANAC dataset (Panel B).

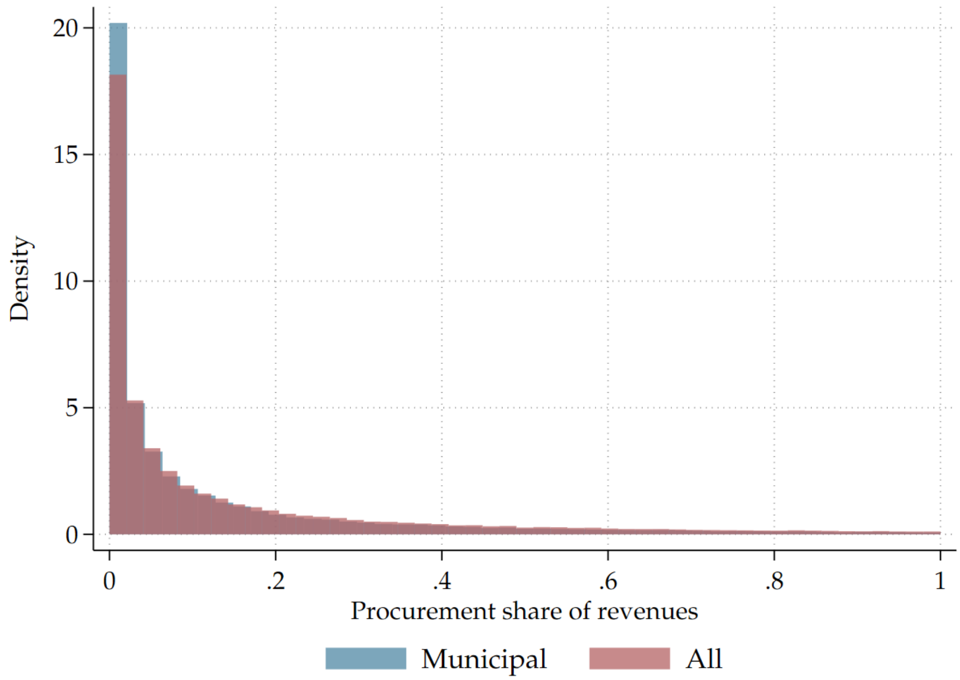
Figure A.1 plots the histogram of the share of procurement revenues over total revenues for all firms in our sample, respectively for procurements awarded by municipalities versus other public bodies. Figure A.2 plots the distribution (interquartile range and mean) of firms engaging in municipal and non-municipal procurement, versus that of firms that do not obtain procurement contracts from the public sector.

Table A.1: Summary statistics

	Obs.	Mean	p25	p50	p75
	Panel A: ANAC-CERVED sample				
Employment	173,261	71	4	10	24
Revenues	175,541	17231	434	1,150	3202
Assets	175,541	26,415	519	1,352	3846
Leverage	175,149	12.57	2.57	5.08	11.33
Age	175,541	19	9	16	26
	Panel B: ANAC-CERVED-INVIND sample				
Employment	5,344	134	16	27	49
Revenues	5,345	33,046	2,386	4,637	10,390
Assets	5,345	51,300	3,282	7,372	16,478
Leverage	5,345	10	3	5	8
Age	5,345	31	16	29	37
Sales forecast error	3,761	-2.47	-19.68	-1.60	15.58

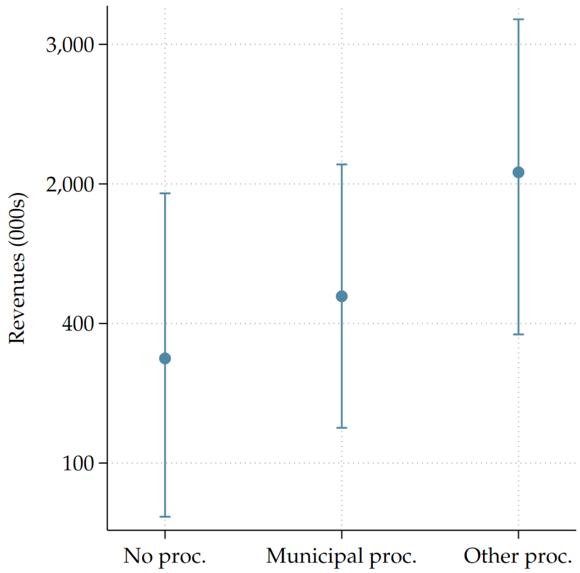
Note: Panel A reports summary statistics of the merged CADS and ANAC datasets, while panel B for the merged CADS, INVIND and ANAC datasets. Revenues and Assets are expressed in thousands of euros, employment in units,

Figure A.1: Share of Procurement in Firms' Revenues

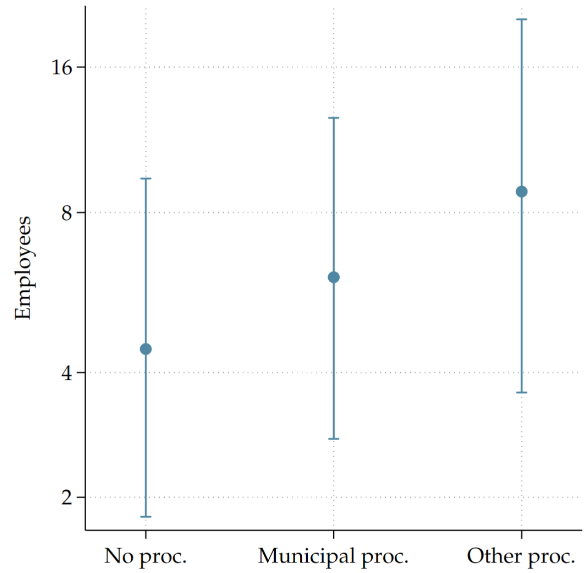


Note: Share of procurement contracts in firms' revenues for municipal contracts and all contracts. The sample of firms is conditional on having a public procurement contract for the red bars and conditional on having a municipal public procurement contract for the blue bars.

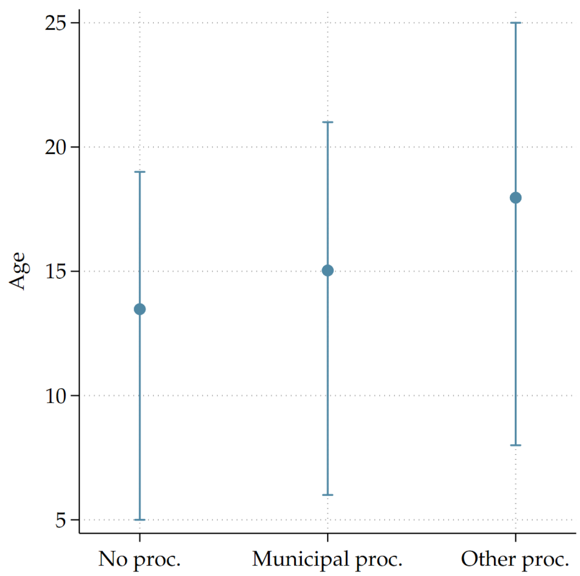
Figure A.2: Characteristics of Municipal Procurement, Non-Procurement and Non-Municipal procurement Firms



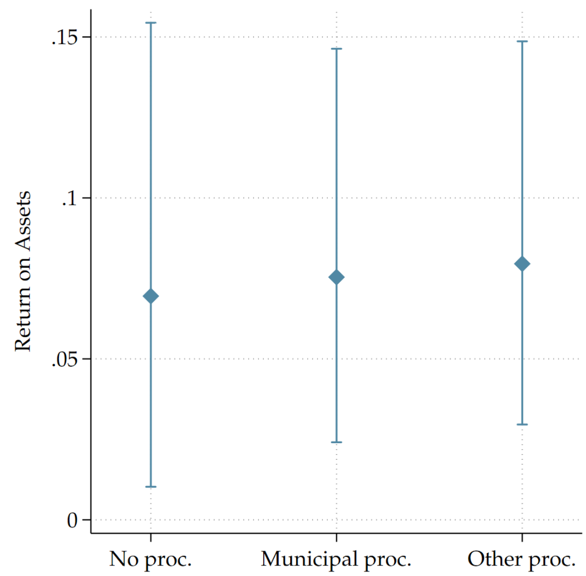
(a) Revenues



(b) Employment



(c) Age



(d) Leverage

Note: characteristics of firms without procurement contracts, with municipal procurement contracts and with procurement contracts with non municipal bodies. The bands show the interquartile range, while squares show the mean and diamonds the median of the distribution. For readability, we plot the distribution of revenues and employment in logarithms. We define return on assets as Earnings Before Interest, Taxation, Depreciation and Amortization over total assets.

A.2 Stability Pact

Art. 31 of *Legge 12 Novembre 2011, n.183*

> *For the purposes of determining the specific financial balance objective, provinces and municipalities with a population greater than 1,000 inhabitants shall apply, to the average current expenditure recorded in the years 2006–2008 (for the year 2012), in the years 2007–2009 (for the year 2013), in the years 2009–2011 (for the year 2014), and in the years 2010–2012 (for the years 2015 to 2018), as derived from the certified financial statements, the following percentages:*
> *a) for provinces, the percentages are 16.5 percent for the year 2012, 18.8 percent for the year 2013, 19.25 percent for the year 2014, 17.20 percent for the year 2015, and 18.03 percent for the years 2016, 2017, and 2018;*
> *b) for municipalities with a population greater than 5,000 inhabitants, the percentages are 15.6 percent for the year 2012, 14.8 percent for the year 2013, 14.07 percent for the year 2014, 8.60 percent for the year 2015, and 9.15 percent for the years 2016, 2017, and 2018;*
> *c) for municipalities with a population between 1,001 and 5,000 inhabitants, the percentages are 12.0 percent for the year 2013, 14.07 percent for the year 2014, 8.60 percent for the year 2015, and 9.15 percent for the years 2016, 2017, and 2018.*

Art .1, comma 532 of *Legge 27 Dicembre 2013, n. 147*

> *In paragraph 2 of Article 31 of Law No. 183 of 12 November 2011, the following amendments are made:*
> *a) the words: "and recorded in the years 2007–2009, for the years from 2013 to 2016," are replaced by the following: ", recorded in the years 2007–2009, for the year 2013, and recorded in the years 2009–2011 for the years from 2014 to 2017,";*
> *b) the words: "and 18.8 percent for the year 2013 and thereafter" are replaced by the following: ", 18.8 percent for the year 2013, 19.25 percent for the years 2014 and 2015, and 20.05 percent for the years 2016 and 2017";*
> *c) the words: "and 14.8 percent for the year 2013 and thereafter" are replaced by the following: ", 14.8 percent for the year 2013, 14.07 percent for the years 2014 and 2015, and 14.62 percent for the years 2016 and 2017";*
> *d) the words: "and 14.8 percent for the years from 2014 to 2016" are replaced by the following: ", 14.07 percent for the years 2014 and 2015, and 14.62 percent for the years 2016 and 2017."*

Art. 1, comma 489 of *Legge 23 Dicembre 2014, n.190*

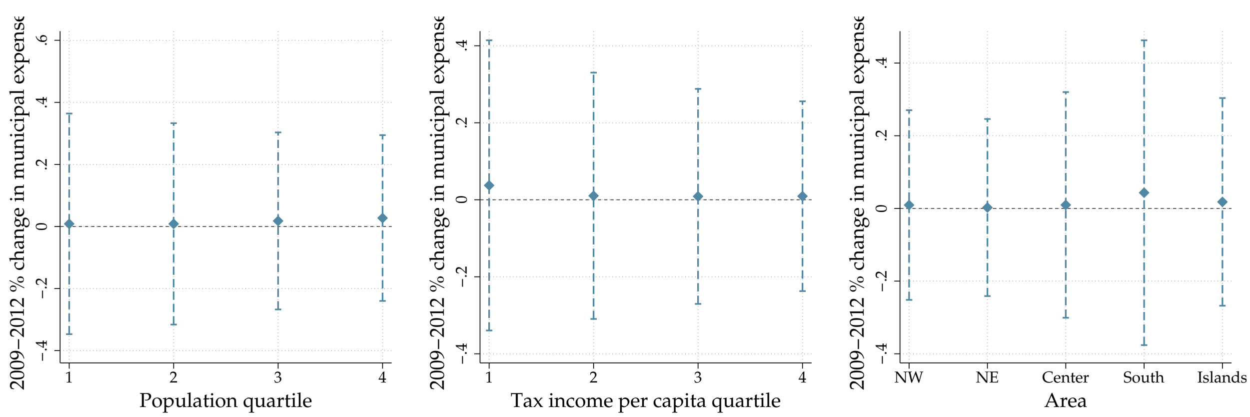
In paragraph 2 of Article 31 of Law No. 183 of 12 November 2011, as subsequently amended, the following modifications are made:
a) the words: "and recorded in the years 2009–2011 for the years from 2014 to 2017" are replaced by the following: "recorded in the years 2009–2011, for the year 2014, and recorded in the years 2010–2012, for the years from 2015 to 2018";
b) in letter a), the words: ", 19.25 percent for the years 2014 and 2015, and 20.05 percent for the years 2016 and 2017" are replaced by the following: ", 19.25 percent for the year 2014, 17.20 percent for the year 2015, and 18.03 percent for the years 2016, 2017, and 2018";
c) in letter b), the words: ", 14.07 percent for the years 2014 and 2015, and 14.62 percent for the years 2016 and 2017" are replaced by the following: ", 14.07 percent for the year 2014, 8.60 percent for the year 2015, and 9.15 percent for the years 2016, 2017, and 2018";
d) in letter c), the words: ", 14.07 percent for the years 2014 and 2015, and 14.62 percent for the years 2016 and 2017" are replaced by the following: ", 14.07 percent for the year 2014, 8.60 percent for the year 2015, and 9.15 percent for the years 2016, 2017, and 2018";
e) the following sentences are added at the end: "By decree of the Minister of Economy and Finance, subject to agreement within the State-City and Local Autonomies Conference, the objectives of each entity referred to in this paragraph may be redefined—on the proposal of ANCI and UPI—by 31 January 2015, without prejudice to the overall objective for the sector, also taking into account the additional functions assigned to metropolitan cities and the increased costs related to natural disasters, safety interventions for school buildings and the territory, the exercise of the role of lead entity, as well as the costs arising from final judgments resulting from expropriation procedures or disputes related to structural failures. After this deadline, the objectives of each entity shall be those determined by applying the percentages referred to in letters a), b), and c) of this paragraph."

A.3 Balance tests

We run balance tests for our shift-share instrument. We check whether the municipal-level shift that we construct in Section 2 shows any systematic difference across observable characteristics of municipalities. To do so, we split municipalities into groups based on three observables: population, taxable income per capita, and geographical area. We then compute the average shift for municipalities in each group, as well as 95% confidence intervals. The results are reported in Figure A.3. There is no evidence of any systematic difference in the distribution of the shift along municipal observables—both in terms of averages and confidence bands.

Figure A.4 reports results for balance tests at the firm level. We split firms into quartiles based on their pre-2015 forecast error, revenues and return on assets. For each group we then compute the average shift-share z_i, as well as the 95% confidence bands. Overall, there is no evidence of any systematic difference in the distribution of our shift-share measure across firms' observables.

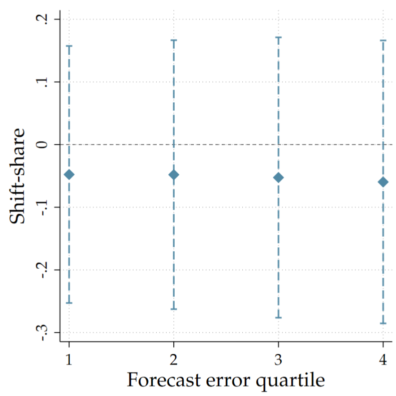
Figure A.3: Balance test—municipal level



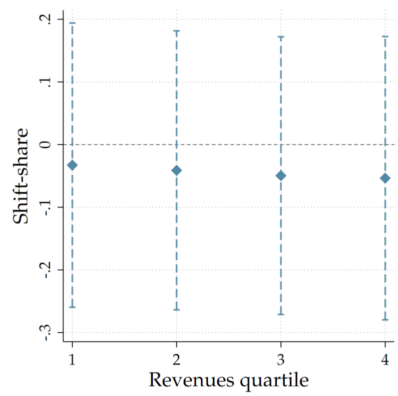
(a) Population (b) Taxable income per capita (c) Geographical area

Note: balance test for the shift g_m defined in (3). We split municipalities in quartiles based on population, taxable income per capita and geographical area. For each quartile we plot the average shift, together with 95% confidence bands.

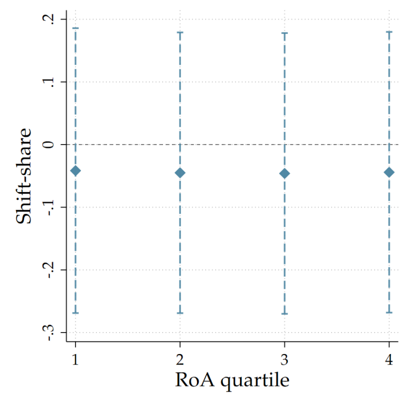
Figure A.4: Balance test—firm level



(a) Forecast Error



(b) Revenues



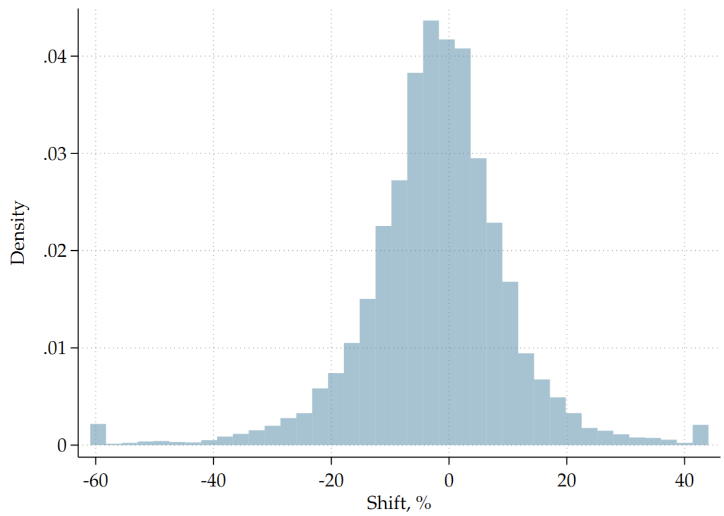
(c) Return on Assets

Note: balance test for the shift-share z_i defined in Section 2.4. We split firms in quartiles based on pre-2015 forecast error, revenues and return on assets. For each quartile we plot the average shift-share, together with 95% confidence bands.

A.4 Descriptive statistics for the shift-share

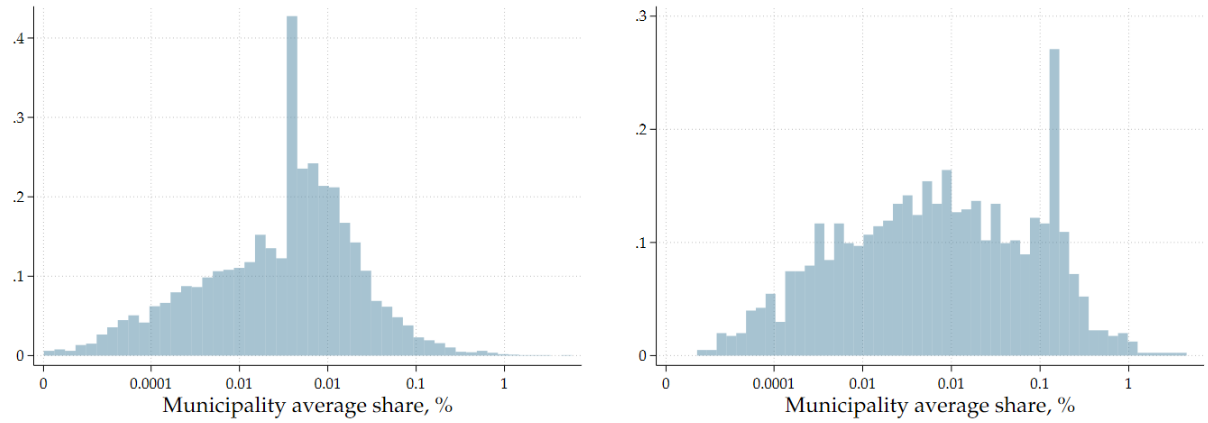
In this Section we report some descriptive statistics for the shifts discussed in Section 2, as well as the associated weights. Figure A.5 plots the distribution of municipal shifts g_m as defined in (3). Figure A.6 plots the distribution of average municipal shares s_m defined in (6) for both our CADS and CADS-INVIND merged datasets. Figure A.7 does the same for the distribution of the shift-share instrument z_i, as defined in (5). Finally, Figure A.8 plots the geographical distribution of our shifts, as well as the average exposure shares, over space.

Figure A.5: Distribution of Shifts



Note: distribution of shifts g_m defined in Section 2.4 across municipalities. Shifts are winsorized at the 1% level.

Figure A.6: Distribution of Average Municipal Shares

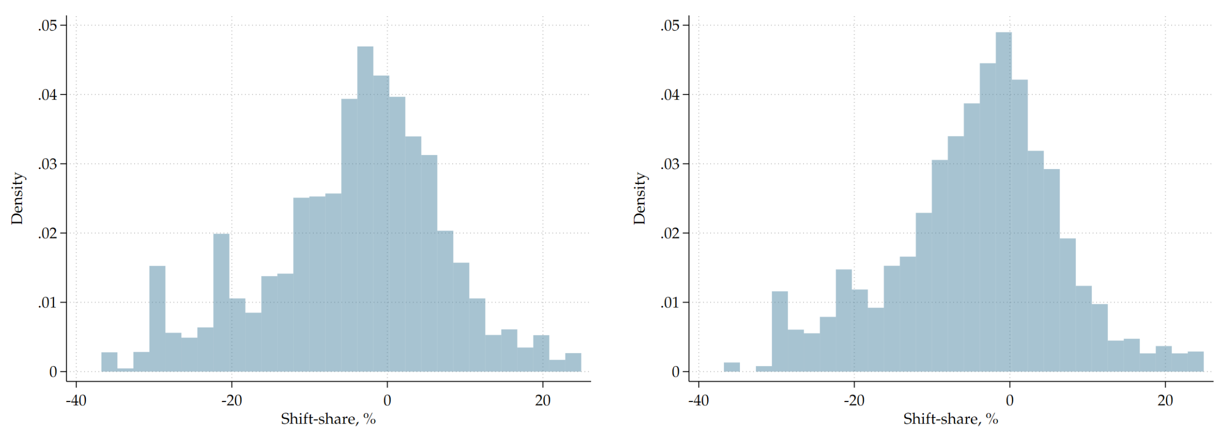


(a) CADS dataset

(b) CADS-INVIND dataset

Note: distribution of average municipal shares s_m defined in (6) for the merged CADS-ANAC sample and the merged CADS-INVIND-ANAC sample.

Figure A.7: Distribution of Shift-Share

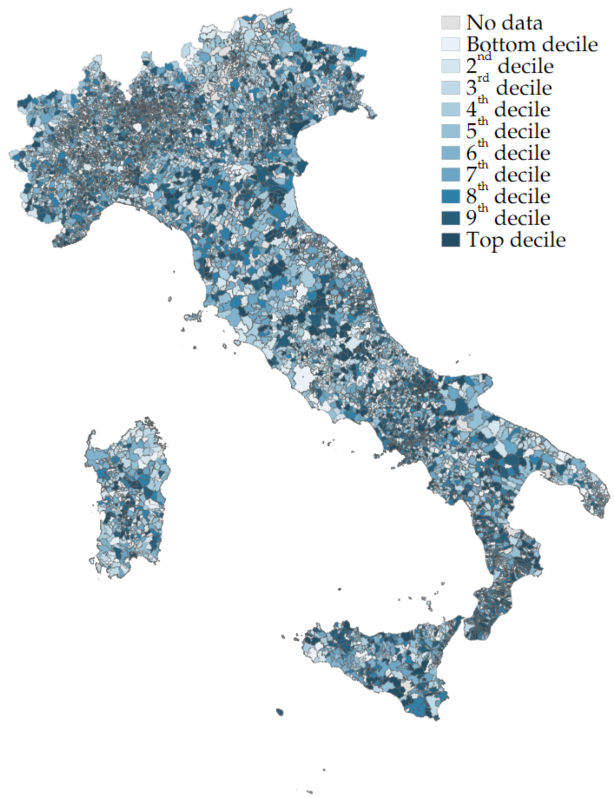


(a) CADS dataset

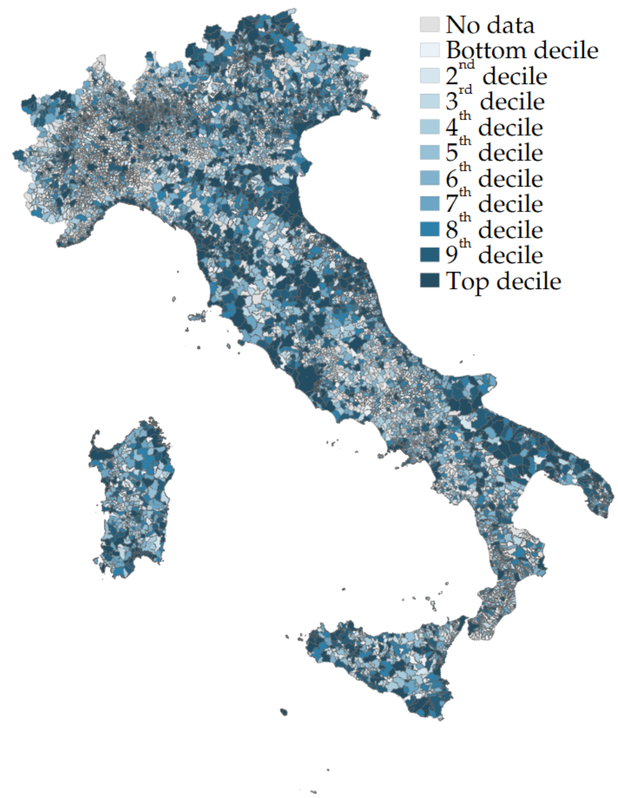
(b) CADS-INVIND dataset

Note: distribution of shift-share instrument z_i defined in (5) across firms for the merged CADS-ANAC sample and the merged CADS-INVIND-ANAC sample.

Figure A.8: Geographical Distribution of our Shift-Share Instrument



(a) Shifts distribution



(b) Shares distribution

Note: panel (a) plots the geographical distribution of the shifts g_m across municipalities. Panel (b) plots the geographical distribution of the share s_{im} across municipalities.

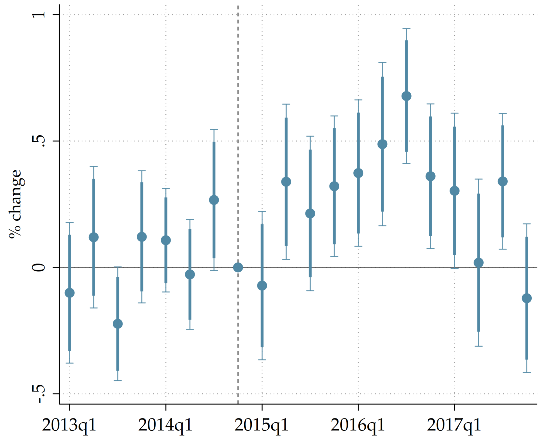
A.5 Robustness

Municipal response We now perform a battery of robustness tests for the response of municipal procurement expenses presented in Figure 3. In Figure A.9a, we run the same event study as in (7), but remove all controls except municipality and time fixed effects. Figure A.9b and Figure A.9c introduce progressively more stringent fixed effects by having region$\times$time and province$\times$time interactions, respectively. Figure A.9d accounts for differential trends by municipality size, controlling for population bin$\times$time fixed effects. In Figure A.9e, we remove winsorization of the shift variable to test the sensitivity of our results to outliers. Across all specifications, the core pattern we identify in the main text remains robust.

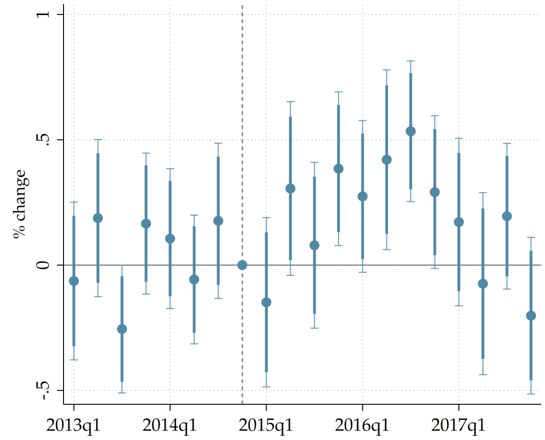
In Figure A.10 we run the same specification as in (7), but use log municipal tax revenues as the dependent variable.[42] The behavior of tax revenues is both economically and statistically insignificant both before and after the reform. First, this lends further support to the parallel trend identifying assumption. Second, it also confirms that the main way via which municipalities responded to less stringent surplus targets is via increased expenses, as opposed to reduced taxes.

[42]Because municipal revenues are only available at yearly frequency, we run the regression at annual level.

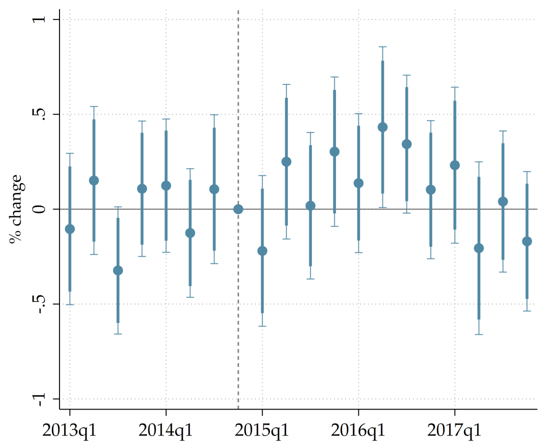
Figure A.9: Response of Municipal Expenses—Robustness



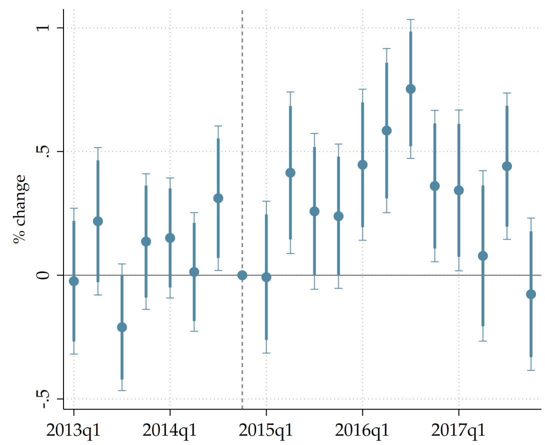
(a) No Controls



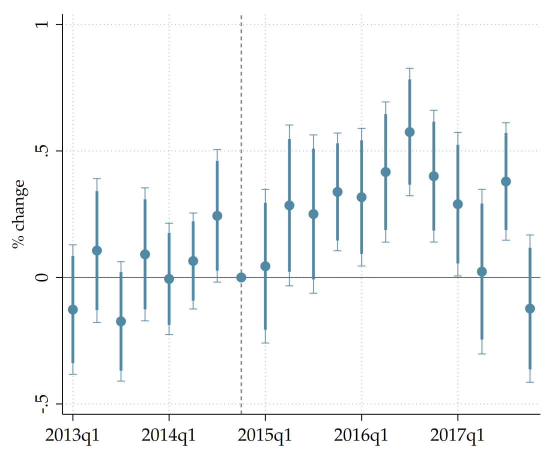
(b) Region×Time Fixed Effects



(c) Province×Time Fixed Effects



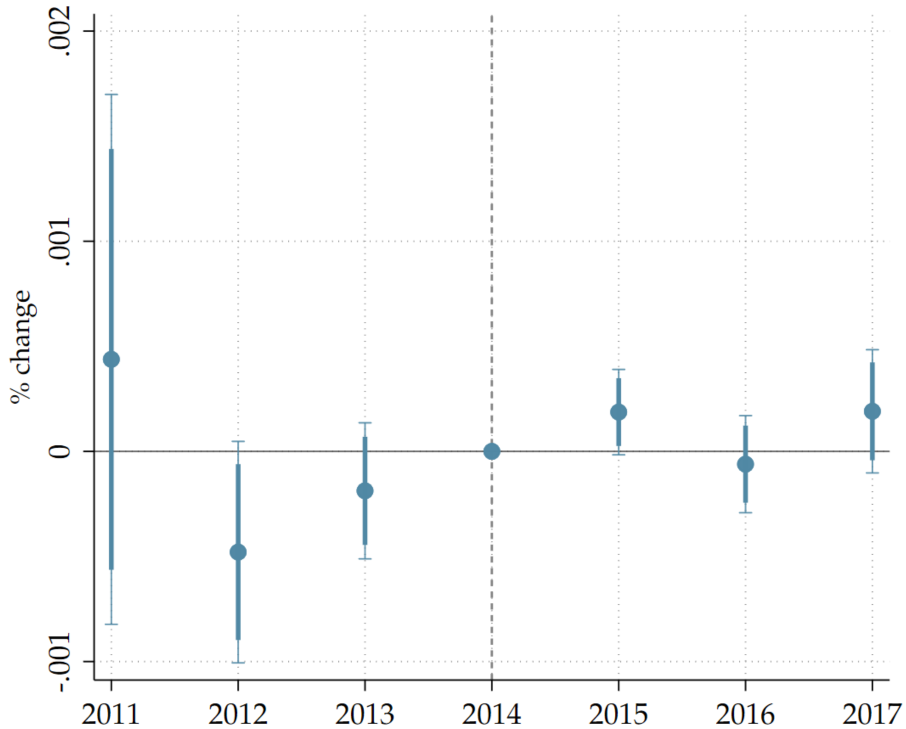
(d) Pop. Bin×Time Fixed Effects



(e) Unwinsorized Shifts

Note: robustness for the estimated $\hat{\beta}_q$ from the event study design in (7). The y-axis denotes the percentage change (relative to the 2014q4 reference quarter) in municipal procurement expenditures. Wide and thin lines represent 90% and 95% confidence bands, respectively. We cluster standard errors at the municipality and year-quarter level. See Table A.3 in Appendix A.6 for more details on the estimates.

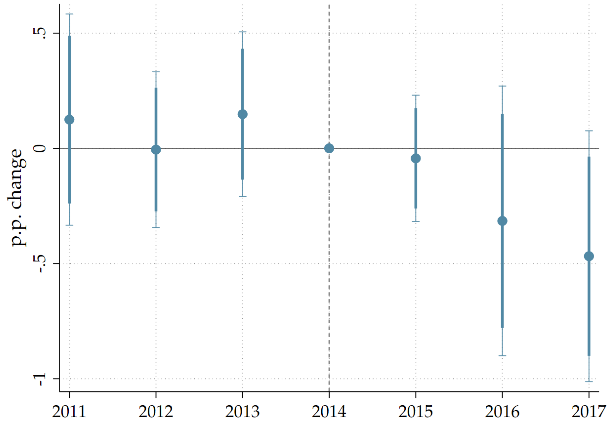
Figure A.10: Response of Municipal Tax Revenues



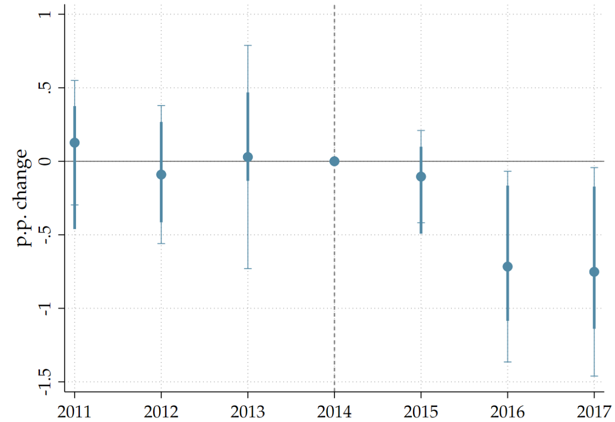
Note: response of tax revenues from a yearly specification of the event study design in (7). The y-axis denotes the percentage change (relative to the 2014 reference year) in municipal tax revenues. Wide and thin lines represent 90% and 95% confidence bands, respectively. We cluster standard errors at the municipality and year level.

Forecast error response We now perform a battery of robustness tests for the response of municipal procurement expenses presented in Figure 5. In Figure A.11a, we run the same event study as in (9), but include province×year fixed effects, to control for potential local general equilibrium spillovers. Figure A.11b shows results for the case in which we do not trim our dependent variable (the forecast error). Similarly, in Figure A.11c we plot our estimates when we do not winsorize the shift-share variable. Figure A.11d reports the results of (9) when we remove the sector×year fixed effects from the controls. Finally, Figures A.11e and A.11f show results for the case in which we define our sectortime fixed effects at the 2 and 4 digits level, respectively. Across all specifications, the core pattern we identify in the main text remains robust.

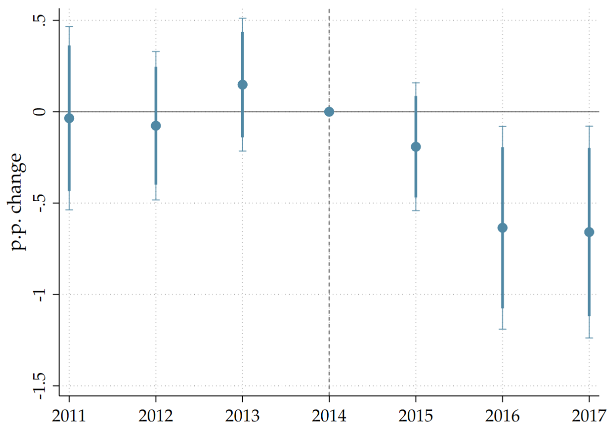
Figure A.11: Response of Forecast Error—Robustness



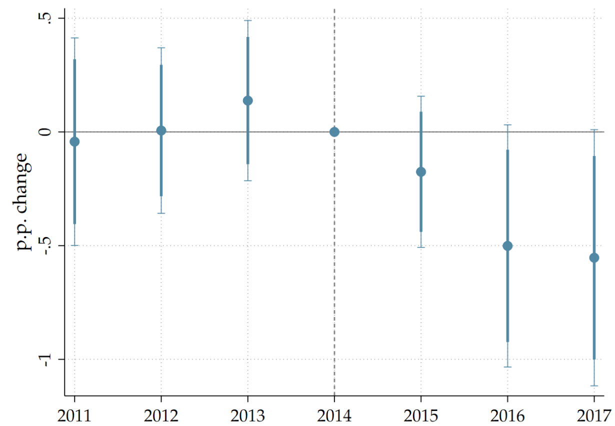
(a) Province×Time Fixed Effects



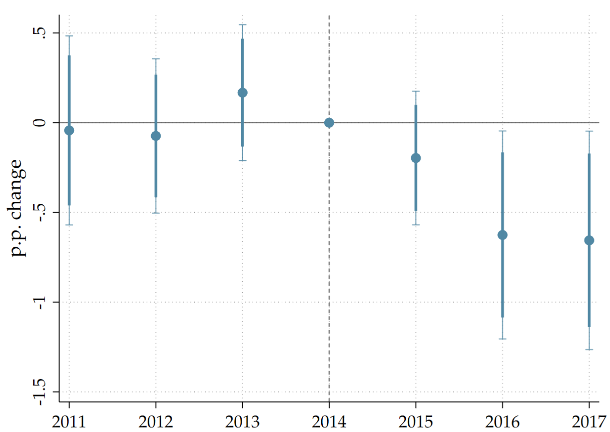
(b) No Trimming of Forecast Errors



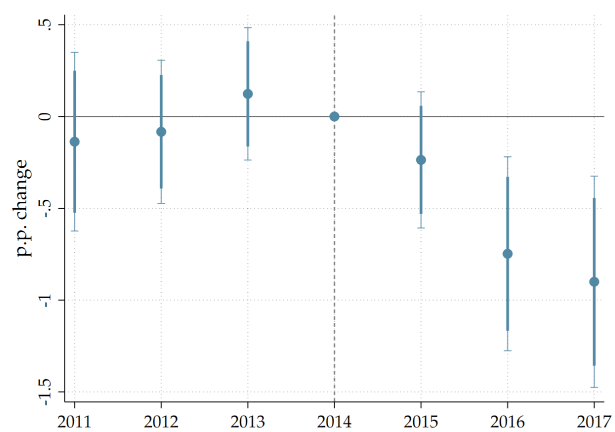
(c) No Trimming of Shift-Share



(d) No Sector×Time Fixed Effects



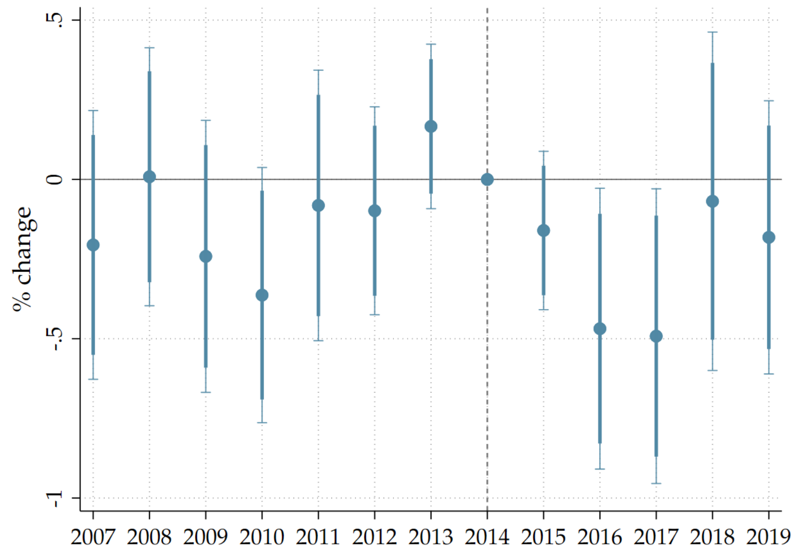
(e) 2-digit Sector×Time Fixed Effects



(f) 4-digit Sector×Time Fixed Effects

Note: robustness for the estimated $\widehat{\beta}_h$ from the event study design in (9). The y-axis denotes the change in forecast error (relative to the base year 2014) in percentage points. Wide and thin lines represent 90% and 95% confidence bands, respectively. We cluster standard errors at the firm and year level and weight observations using survey weights. See Table A.5 in Appendix A.6 for more details on the estimates.

Figure A.12: Different Time Window for Estimation



Note: estimated $\widehat{\beta}_h$ from the event study design in (9) with a wider time-window. The y-axis denotes the change in forecast error (relative to the base year 2014) in percentage points. Wide and thin lines represent 90% and 95% confidence bands, respectively. We cluster standard errors at the firm and year level and weight observations using survey weights.

Decomposition of price and quantity forecast error response Table A.2 reports the estimates of the price and quantity forecast error response to our government spending shock. It does so by reporting the $\widehat{\beta}$ from the following regression:

$$\mathcal{E}^{y}_{it} = \alpha_i + \gamma_{s(i)t} + \beta z_i \times Post_t + \varepsilon_{it}, \ \ y \in \{P, Q\} \tag{A.1}$$

where $\mathcal{E}^{P}_{it}$ are the forecast errors on the price and quantity changes, respectively, defined along the same lines of (2). To construct the forecast error on the price change we leverage a feature of our survey, that directly asks respondents to report their expectation of the change in the average price over the next 12 months as well as the realized average change in prices over the past 12 months. To construct a measure of forecast error on the quantity change we do a log-linearization of firms realized and expected sales and extract realized and expected quantity changes as a residual:

$$\widehat{Q}_t = \widehat{Y}_t - \widehat{P}_t \tag{A.2}$$

Where $\widehat{X}_t$ denotes the log change in variable X and Y_t denotes revenues.

Figures A.13a and A.13b do the same in a dynamic event study way, along the lines of

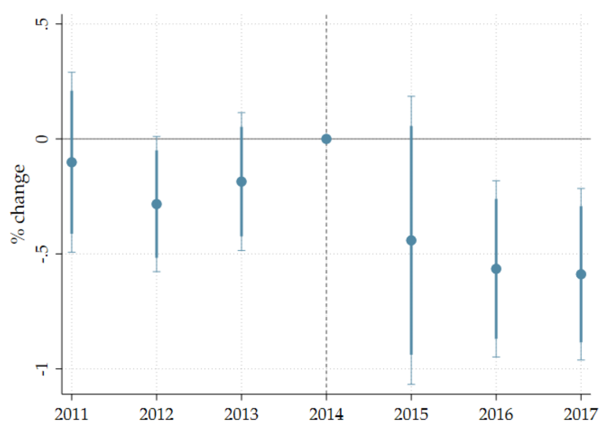
(9) in the main text.

Table A.2: Response of Quantity and Price Forecast Errors

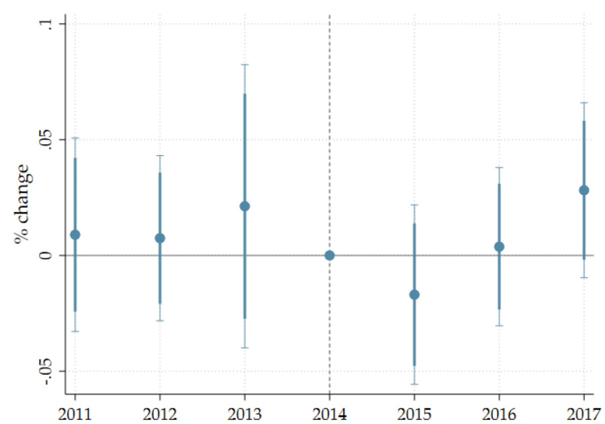
	Quantities	Prices
$\text{Post}_t \times z_i$	-0.407**	-0.005
	(0.121)	(0.012)
Observations	869	872
Firm FEs	✓	✓
Sector-Time FEs	✓	✓

Note: estimates from (A.1). Standard errors are clustered at the year and firm level and we weight our estimates using survey weights. See the discussion of (A.2) for a description of how we construct the price and quantity forecast error.

Figure A.13: Response of Quantity and Price Forecast Errors



(a) Quantity forecast error



(b) Price forecast error

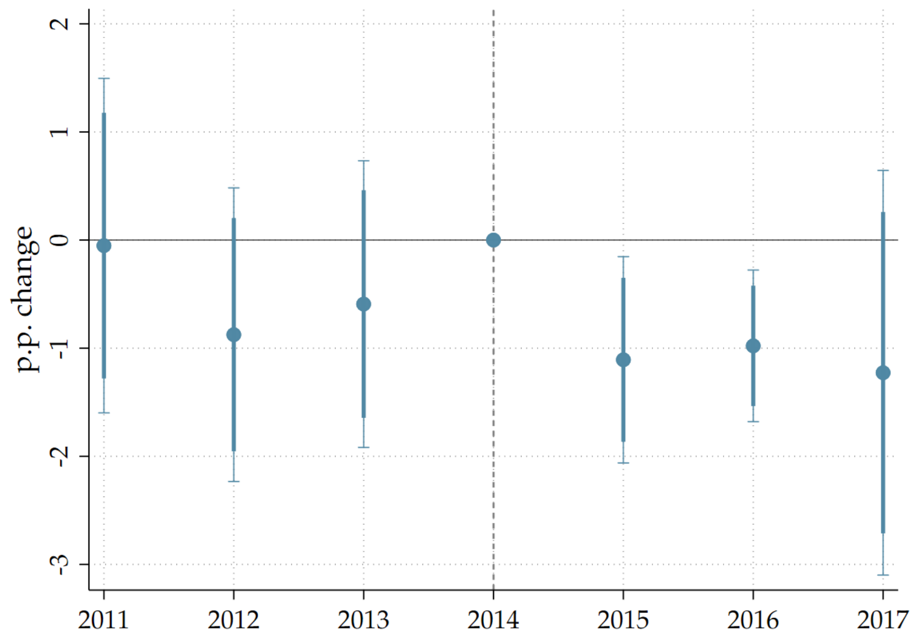
Note: estimated $\widehat{\beta}_h$ from the event study design in (9), where we use the quantity (panel a) and price (panel b) forecast errors as outcome variables. The y-axis denotes the change in forecast error (relative to the base year 2014) in percentage points. Wide and thin lines represent 90% and 95% confidence bands, respectively. We cluster standard errors at the firm and year level and weight observations using survey weights. See the discussion of (A.2) for a description of how we construct the price and quantity forecast error.

Export forecast error event study Figure A.14 estimates the dynamic response of the export forecast error by plotting the estimated $\widehat{\beta}_h$ coefficients from the following regression:

$$\mathcal{E}_{it}^{X} = \alpha_i + \gamma_{s(i)t} + \sum_{h=2011}^{2017} \beta_h \left(\mathbb{1}_{h=t} \times z_i\right) + \varepsilon_{it}, \tag{A.3}$$

where $\mathcal{E}_{it}^{X}$ is the export sales forecast error and all other variables are defined as in Section 2 in the main text.

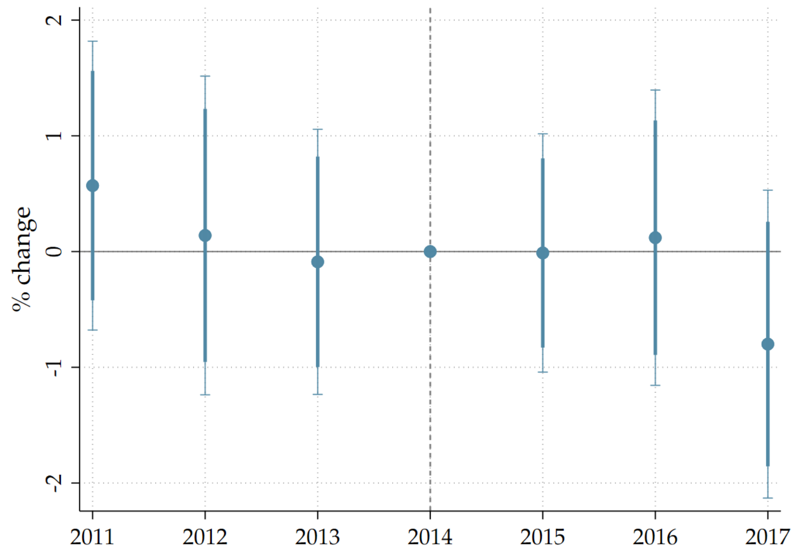
Figure A.14: Response of Forecast Error on Exports



Note: estimated $\widehat{\beta}_h$ from the event study design in (A.3). The y-axis denotes the change in export sales forecast error (relative to the base year 2014) in percentage points. Wide and thin lines represent 90% and 95% confidence bands, respectively. We cluster standard errors at the firm and year level and weight observations using survey weights. See Table A.6 in Appendix A.6 for more details on the estimates.

Export revenues response Figure A.15 plots the estimates from (A.3) using actual export sales as outcome variable. It shows that export revenues are not crowded-out following a government spending shock.

Figure A.15: Response of Export Revenues



Note: estimated $\widehat{\beta}_h$ from the event study design in (A.3), with realized export revenues as the dependent variable. The y-axis denotes the change in export sales (relative to the base year 2014) in percent. Wide and thin lines represent 90% and 95% confidence bands, respectively. We cluster standard errors at the firm and year level and weight observations using survey weights.

A.6 Estimation tables

Table A.3: Response of Municipal Procurement Expenses

	Dependent variable: $\log(ProcExp_{mt})$					
	(1)	(2)	(3)	(4)	(5)	(6)
$D_{2013q1t} \times g_m$	-0.107	-0.063	-0.104	-0.024	-0.101	-0.127
	(0.133)	(0.150)	(0.191)	(0.141)	(0.133)	(0.122)
$D_{2013q2t} \times g_m$	0.095	0.188	0.151	0.218	0.119	0.106
	(0.135)	(0.150)	(0.186)	(0.142)	(0.134)	(0.136)
$D_{2013q3t} \times g_m$	-0.226*	-0.255**	-0.323*	-0.210	-0.223*	-0.174
	(0.108)	(0.122)	(0.160)	(0.122)	(0.108)	(0.113)
$D_{2013q4t} \times g_m$	0.114	0.166	0.108	0.136	0.121	0.091
	(0.124)	(0.134)	(0.170)	(0.131)	(0.125)	(0.126)
$D_{2014q1t} \times g_m$	0.077	0.106	0.124	0.151	0.108	-0.006
	(0.098)	(0.133)	(0.168)	(0.116)	(0.098)	(0.105)
$D_{2014q2t} \times g_m$	-0.019	-0.057	-0.125	0.013	-0.028	0.065
	(0.104)	(0.123)	(0.162)	(0.115)	(0.104)	(0.091)
$D_{2014q3t} \times g_m$	0.261*	0.177	0.106	0.311**	0.267*	0.244*
	(0.134)	(0.148)	(0.187)	(0.140)	(0.133)	(0.125)
$D_{2015q1t} \times g_m$	-0.076	-0.148	-0.220	-0.008	-0.072	0.044
	(0.140)	(0.161)	(0.190)	(0.147)	(0.140)	(0.145)
$D_{2015q2t} \times g_m$	0.335**	0.306*	0.250	0.415**	0.339**	0.285*
	(0.146)	(0.166)	(0.195)	(0.156)	(0.147)	(0.152)
$D_{2015q3t} \times g_m$	0.210	0.079	0.018	0.258	0.213	0.251
	(0.145)	(0.158)	(0.184)	(0.150)	(0.146)	(0.150)
$D_{2015q4t} \times g_m$	0.359**	0.385**	0.303	0.239	0.321**	0.338***
	(0.132)	(0.147)	(0.188)	(0.139)	(0.133)	(0.111)
$D_{2016q1t} \times g_m$	0.371**	0.274*	0.137	0.447***	0.373**	0.317**
	(0.137)	(0.145)	(0.175)	(0.146)	(0.138)	(0.130)
$D_{2016q2t} \times g_m$	0.482***	0.421**	0.433**	0.585***	0.488***	0.417***
	(0.154)	(0.171)	(0.202)	(0.159)	(0.154)	(0.132)
$D_{2016q3t} \times g_m$	0.674***	0.534***	0.343*	0.753***	0.678***	0.575***
	(0.127)	(0.134)	(0.174)	(0.134)	(0.127)	(0.120)
$D_{2016q4t} \times g_m$	0.362**	0.291*	0.103	0.361**	0.361**	0.400***
	(0.135)	(0.146)	(0.174)	(0.146)	(0.137)	(0.124)
$D_{2017q1t} \times g_m$	0.304*	0.172	0.232	0.343**	0.303*	0.290**
	(0.146)	(0.160)	(0.196)	(0.155)	(0.147)	(0.136)
$D_{2017q2t} \times g_m$	0.037	-0.074	-0.205	0.078	0.019	0.023
	(0.156)	(0.173)	(0.217)	(0.165)	(0.158)	(0.155)
$D_{2017q3t} \times g_m$	0.346**	0.195	0.040	0.441***	0.340**	0.379***
	(0.127)	(0.139)	(0.178)	(0.141)	(0.128)	(0.111)
$D_{2017q4t} \times g_m$	-0.124	-0.202	-0.169	-0.077	-0.122	-0.123
	(0.139)	(0.149)	(0.176)	(0.147)	(0.141)	(0.139)
Observations	41,672	41,672	41,672	41,672	41,690	41,672
Year FEs	✓					✓
Region × Year FEs		✓				
Province × Year FEs			✓			
Population Decile × Year FEs				✓		

Note: estimates from (7). (1) is our baseline specification. (2), (3), and (4) introduce region×year, province×year and population decile×year fixed effects respectively. (5) is the specification without population controls, (6) with unwinsorized shifts. Errors clustered at the municipality and quarter level. * denotes significance at the 10% level, ** at the 5% and *** at the 1%.

Table A.4: Response of Firm Revenues and Assets

Dependent var.:	Revenues	Assets
	(1)	(2)
$D_{2011t} \times z_i$	0.026	-0.006
	(0.034)	(0.019)
$D_{2012t} \times z_i$	0.001	-0.022
	(0.031)	(0.017)
$D_{2013t} \times z_i$	0.000	0.014
	(0.015)	(0.008)
$D_{2015t} \times z_i$	0.096***	0.037***
	(0.018)	(0.009)
$D_{2016t} \times z_i$	0.080*	0.037*
	(0.034)	(0.018)
$D_{2017t} \times z_i$	0.092**	0.078***
	(0.036)	(0.021)
Observations	165568	165917
Firm FEs	✓	✓
Sector × Year FEs	✓	✓

Note: details on the estimates from (8). Errors are clustered at the firm and quarter level. * denotes significance at the 10% level, ** at the 5% level and *** at the 1% level.

Table A.5: Response of Sales Forecast Errors

	Dependent variable: sales forecast error, $\mathcal{E}_{it}$						
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
$D_{2011t} \times z_i$	-0.127	0.125	0.127	-0.035	-0.043	-0.052	-0.137
	(0.227)	(0.187)	(0.173)	(0.205)	(0.187)	(0.213)	(0.199)
$D_{2012t} \times z_i$	-0.114	-0.006	-0.090	-0.076	0.006	-0.082	-0.083
	(0.178)	(0.138)	(0.192)	(0.166)	(0.149)	(0.173)	(0.159)
$D_{2013t} \times z_i$	0.097	0.148	0.029	0.148	0.138	0.159	0.123
	(0.164)	(0.146)	(0.310)	(0.148)	(0.144)	(0.150)	(0.147)
$D_{2015t} \times z_i$	-0.237	-0.043	-0.104	-0.191	-0.175	-0.206	-0.236
	(0.180)	(0.112)	(0.128)	(0.143)	(0.136)	(0.149)	(0.152)
$D_{2016t} \times z_i$	-0.683***	-0.315	-0.716**	-0.635**	-0.501*	-0.630**	-0.748**
	(0.249)	(0.239)	(0.265)	(0.227)	(0.218)	(0.233)	(0.216)
$D_{2017t} \times z_i$	-0.766***	-0.468*	-0.752**	-0.658**	-0.553*	-0.661**	-0.900***
	(0.272)	(0.222)	(0.290)	(0.237)	(0.230)	(0.245)	(0.235)
Observations	4048	4028	4127	4048	4048	3898	3623
R-squared	0.50	0.66	0.46	0.47	0.50	0.53	0.54
Within R-squared	0.01	0.01	0.01	0.01	0.01	0.01	0.02
Firm FEs	✓	✓	✓	✓	✓	✓	✓
2D Sector × Year FEs						✓	
3D Sector × Year FEs	✓			✓	✓		
4D Sector × Year FEs							✓
Province × Year FEs		✓					

Note: details on the estimates from (9). Column 1 is our baseline specification. Columns 2 reports results with province×year fixed effects. Columns 3 and 4 respectively report results with untrimmed dependent variable and unwinsorized shift-share instrument. Column 5 reports results without sector×year fixed effects. Columns 6 and 7 report results for the case in which we define the sector×year fixed effect at the 2 digits and 4 digits level, respectively. We weight estimates using survey weights and cluster standard errors at the firm and year level. * denotes significance at the 10% level, ** at the 5% level and *** at the 1% level.

Table A.6: Response of Export Forecast Errors

	Dependent var.: Export forecast error, $\mathcal{E}^z_{it}$
	(1)
$D_{2011t} \times z_i$	-0.051
	(0.632)
$D_{2012t} \times z_i$	-0.875
	(0.555)
$D_{2013t} \times z_i$	-0.592
	(0.542)
$D_{2015t} \times z_i$	-1.108**
	(0.390)
$D_{2016t} \times z_i$	-0.979**
	(0.287)
$D_{2017t} \times z_i$	-1.227
	(0.765)
Observations	635
R-squared	0.67
Within R-squared	0.03
Firm FEs	✓
Sector × Year FEs	✓

Note: details on the estimates from (A.3). We weight estimates using survey weights and cluster standard errors at the firm and year level. * denotes significance at the 10% level, ** at the 5% level and *** at the 1% level.

B Appendix to Section 3

B.1 A simple AR(1) example

To build intuition, we consider a simple case in discrete time where both fundamentals follow AR(1) processes:

$$z_{t+1} = \rho_z z_t + \varepsilon^z_{t+1}, \quad \varepsilon^z_{t+1} \overset{iid}{\sim} \mathcal{N}\left(0, \sigma^2_z\right) \tag{B.1}$$

$$g_{t+1} = \rho_g g_t + \varepsilon^g_{t+1}, \quad \varepsilon^g_{t+1} \overset{iid}{\sim} \mathcal{N}\left(0, \sigma^2_g\right) \tag{B.2}$$

Where $x_t = \alpha_z z_t + \alpha_g g_t$ like in the main text. In a similar way to Bordalo et al. (2018), we define in the rest of our analysis the background context as the value x_t would take absent current shocks, so that $\hat{x} - \mathcal{B} = \alpha_z \varepsilon^z_t + \alpha_g \varepsilon^g_t$, and $\hat{x}$ is the value of x in period t. We define the distortion in a similar fashion to the continuous time version

$$\widetilde{h}_t(y_{t+1}) \;=\; h(y_{t+1} \mid y_t = \hat{y}_t) \cdot \underbrace{\left(\frac{h(y_{t+1} \mid x_t = \hat{x}_t)}{h(y_{t+1} \mid x_t = \mathcal{B}_t)}\right)^{\chi}}_{\text{representativeness ratio}} \frac{1}{Z_x(\chi)} \tag{B.3}$$

Given this Gaussian setting, we can now derive the perceived distributions under XDE.

Proposition 4 (Cross-Domain Extrapolation with AR(1) Fundamentals)**.** *When the fundamentals z_t and g_t follow the process in (B.1) and (B.2), the perceived distribution of z_{t+1} and g_{t+1} under cross-domain extrapolation are:*

$$z_{t+1}|z_t, g_t \overset{\chi}{\sim} \mathcal{N}\left(\rho_z z_t + \frac{\sigma^2_z}{\sigma^2_{z|x}} \chi \beta_z (\alpha_z \varepsilon^z_t + \alpha_g \varepsilon^g_t), \sigma^2_z\right) \tag{B.4}$$

$$g_{t+1}|z_t, g_t \overset{\chi}{\sim} \mathcal{N}\left(\rho_g g_t + \frac{\sigma^2_g}{\sigma^2_{g|x}} \chi \beta_g (\alpha_z \varepsilon^z_t + \alpha_g \varepsilon^g_t), \sigma^2_g\right) \tag{B.5}$$

where $\beta_z = \frac{\mathbb{C}ov(z_{t+1}, x_t)}{\mathbb{V}ar(x_t)}$ is the OLS coefficient of a regression of z_{t+1} on x_t, $\sigma^2_{z|x} = \frac{\sigma^2_z}{1-\rho^2_z}\left(1 - \frac{\rho^2_z \alpha^2_z \sigma^2_z/1-\rho^2_z}{\alpha^2_z \sigma^2_z/1-\rho^2_z + \alpha^2_g \sigma^2_g/1-\rho^2_g}\right)$ is the variance of z_{t+1} conditional on x_t, $\beta_g = \frac{\mathbb{C}ov(g_{t+1}, x_t)}{\mathbb{V}ar(x_t)}$, and $\sigma^2_{g|x} = \frac{\sigma^2_g}{1-\rho^2_g}\left(1 - \frac{\alpha^2_g \rho^2_g \sigma^2_g/1-\rho^2_g}{\alpha^2_z \sigma^2_z/1-\rho^2_z + \alpha^2_g \sigma^2_g/1-\rho^2_g}\right)$ is the variance of g_{t+1} conditional on y_t.

Proof. See Appendix B. □

Proposition 4 delivers three key results. First, cross-domain extrapolation preserves normality of perceived distributions. Second, it does not distort perceived variances.

Third, the only bias introduced by XDE is in the perception of expected values. The following corollary shows that this first-moment distortion admits a useful decomposition.

Corollary 1 (Perceived First-Moment under XDE with AR(1) Fundamentals)**.** *When the fundamentals z_t and g_t follow the process in* (B.1) *and* (B.2)*, the perceived expected value of z_{t+1} and g_{t+1} under cross-domain extrapolation can be decomposed as:*

$$\mathbb{E}_t^{\chi}(z_{t+1}) = \mathbb{E}_t(z_{t+1}) + \chi \beta_z \frac{\sigma_z^2}{\sigma_{z|x}^2} \left(\alpha_z \varepsilon_t^z + \alpha_g \varepsilon_t^g\right) \tag{B.6}$$

$$\mathbb{E}_t^{\chi}(g_{t+1}) = \mathbb{E}_t(g_{t+1}) + \chi \beta_g \frac{\sigma_g^2}{\sigma_{g|x}^2} \left(\alpha_z \varepsilon_t^z + \alpha_g \varepsilon_t^g\right) \tag{B.7}$$

Where $\mathbb{E}_t(z_{t+1}) \equiv \rho_z z_t$ and $\mathbb{E}_t(g_{t+1}) \equiv \rho_g g_t$ denote expectations under rational expectations.

Corollary 1 provides a decomposition of cross-domain beliefs into the rational expectations benchmark plus an additive bias. This bias is the product of four components: (i) the behavioral parameter χ, governing the degree of cross-domain extrapolation, (ii) the OLS coefficient β capturing the strength of the co-movement between the target variable and the salient outcome, (iii) a variance ratio adjusting for relative precisions, and (iv) the total surprise in the salient variable relative to the background, $\alpha_z \varepsilon_t^z + \alpha_g \varepsilon_t^g$. Each component has a distinct psychological or statistical meaning. χ is the behavioural parameter: it measures the strength of the representativeness heuristic, i.e. how much the agent lets resemblance override the objective conditional distribution. β is purely mnemonic: it is the slope the agent has learned from past co-movements between x and the target variable z; it captures how diagnostic the agent's memory of x is for z. Finally, the variance ratio $\frac{\sigma_z^2}{\sigma_{z|x}^2}$ is a purely statistical scaling that tells the agent how precisely x pins down z: the more precise the link, the larger the leverage of any surprise in x.

Finally, note that cross-domain extrapolation for the fundamentals z_t, g_t generates standard diagnostic expectations for the salient outcome x_t. (B.4) and (B.5) together imply that the perceived expected value of x_{t+1} can be expressed as:

$$\mathbb{E}_t^{\chi}(x_{t+1}) = \mathbb{E}_t(x_{t+1}) + \tilde{\chi}(\alpha_z \varepsilon_t^z + \alpha_g \varepsilon_t^g)$$

Where $\tilde{\chi} \equiv \chi \left[\alpha_z \frac{\sigma_z^2}{\sigma_{z|x}^2} \beta_z + \alpha_g \frac{\sigma_g^2}{\sigma_{g|x}^2} \beta_g\right]$.

B.2 Proof of Proposition 4

In equation (B.3) we define Z as

$$Z_x(\chi) \equiv \int h_x(s|x) \left(\frac{h_x(s|y=\hat{y})}{h_x(s|y=y_0)} \right)^{\chi} ds$$

Is a normalizing constant that guarantees the density integrates to 1. For convenience we can rewrite the expression above as

$$h_x^{\chi}(x'|\mathcal{I}) = \frac{1}{Z_x(\chi)} h_x(x'|x) e^{\chi T_x(x')}$$

Where

$$\begin{aligned} T_x(x') &\equiv \log \left(\frac{h_x(x'|y=\hat{y})}{h_x(x'|y=y_0)} \right) \\ &= \log \left(h_x(x'|y=\hat{y}) \right) - \log \left(h_x(x'|y=y_0) \right) \end{aligned}$$

Now consider the discrete-time AR(1) gaussian setting in the main text:

$$\begin{aligned} z_{t+1} &= \rho_z z_t + \varepsilon_{t+1}^z \\ g_{t+1} &= \rho_g g_t + \varepsilon_{t+1}^g \\ x_t &= \alpha_z z_t + \alpha_g g_t \end{aligned}$$

Where $\varepsilon_{t+1}^z \sim \mathcal{N}\left(0, \sigma_z^2\right)$ iid and $\varepsilon_{t+1}^g \sim \mathcal{N}\left(0, \sigma_g^2\right)$. From this it follows that $\mathbb{V}_z = \frac{\sigma_z^2}{1-\rho_z^2}$ and $\mathbb{V}_g = \frac{\sigma_g^2}{1-\rho_g^2}$. Moreover $\mathbb{C}\mathrm{ov}(z_t, x_t) = \alpha_z \mathbb{V}_z$. Then it follows that

$$\begin{bmatrix} z_t \\ x_t \end{bmatrix} \sim \mathcal{N}\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \mathbb{V}_z & \alpha_z \mathbb{V}_z \\ \alpha_z \mathbb{V}_z & \alpha_z^2 \mathbb{V}_z + \alpha_g^2 \mathbb{V}_g \end{bmatrix} \right)$$

Remember that if $\begin{bmatrix} X_1 \\ X_2 \end{bmatrix} \sim \mathcal{N}\left(\begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix}, \begin{bmatrix} \Sigma_{11} & \Sigma_{12} \\ \Sigma_{21} & \Sigma_{22} \end{bmatrix} \right)$. Then

$$X_1 \mid X_2 = a \ \sim \ \mathcal{N}\big(\mu_1 + \Sigma_{12}\Sigma_{22}^{-1}(a - \mu_2),\ \Sigma_{11} - \Sigma_{12}\Sigma_{22}^{-1}\Sigma_{21}\big).$$

Thus, it follows that

$$z_t \mid x_t \ \sim \ \mathcal{N}\left(\frac{\alpha_z \mathbb{V}_z}{\alpha_z^2 \mathbb{V}_z + \alpha_g^2 \mathbb{V}_g} x_t,\ \mathbb{V}_z - \frac{\alpha_z^2 \mathbb{V}_z^2}{\alpha_z^2 \mathbb{V}_z + \alpha_g^2 \mathbb{V}_g}\right)$$

Now let $\beta = \frac{\alpha_z \mathbb{V}_z}{\alpha_z^2 \mathbb{V}_z + \alpha_g^2 \mathbb{V}_g}$. Then

$$z_t \,\big|\, x_t \;\sim\; \mathcal{N}\big(\beta x_t,\; \mathbb{V}_z(1-\alpha_z\beta)\big)$$

Moreover

$$z_{t+1} \,\big|\, x_t \;\sim\; \mathcal{N}\big(\rho_z \beta x_t,\; \mathbb{V}_z(1-\rho_z^2\alpha_z\beta)\big)$$

So we can see that the variance does not depend on y_t. Let $\mathcal{F}_t \equiv \sigma\{y_s : s \leq t\}$. The smallest σ-field in which the realized y_t is measurable is $\mathcal{F}_t$, while the smallest σ-field in which the no-news counterfactual y_t^0 is measurable is $\mathcal{F}_{t-1} \subset \mathcal{F}_t$. Crucially, both y_t and y_t^0 become deterministic elements of $\mathcal{F}_t$ once t is reached so they can be plugged into any $\mathcal{F}_t$-measurable function without ambiguity. The likelihood tilt is computed within a single filtration, $\mathcal{F}_t$, the numerator and denominator densities condition on the same σ-field'at the moment they are compared, even though the benchmark argument was chosen using the smaller σ-field $\mathcal{F}_{t-1}$. See footnote 8 in the original Journal of Finance diagnostic expectation paper to see that this is indeed what Bordalo et al. assume. So that means that our numerator and denominator share equal variances by design.

Given this we can define

$$\begin{aligned} T_y(y') &\equiv \log\left(\frac{h_y(y'|x=x_t)}{h_y(y'|x=\mathcal{B})}\right) \\ &= \log\big(h_y(y'|x=x_t)\big) - \log\big(h_y(y'|x=\mathcal{B})\big) \end{aligned}$$

In our specific AR(1) Gaussian case. The tilting is:

$$\begin{aligned} T_y(y') &= -\frac{(y'-\rho_z\beta x_t)^2}{2\mathbb{V}_z(1-\rho_z^2\alpha_z\beta)} + \frac{(y'-\rho_z\beta\mathcal{B})^2}{2\mathbb{V}_z(1-\rho_z^2\alpha_z\beta)} \\ &= \frac{1}{2\mathbb{V}_z(1-\rho_z^2\alpha_z\beta)}\left[-y'^2 + 2y'\rho_z\beta x_t - \rho_z^2\beta^2 x_t^2 + y'^2 - 2y'\rho_z\beta\mathcal{B} + \rho_z^2\beta^2\mathcal{B}^2\right] \\ &= \frac{1}{2\mathbb{V}_z(1-\rho_z^2\alpha_z\beta)}\left[2y'(\rho_z\beta x_t - \rho_z\beta\mathcal{B}) + (\rho_z^2\beta^2\mathcal{B}_0^2 - \rho_z^2\beta^2 x_t^2)\right] \\ &= \frac{\rho_z\beta(x_t-\mathcal{B})}{\mathbb{V}_z(1-\rho_z^2\alpha_z\beta)}y' + \frac{\rho_z^2\beta^2}{2\mathbb{V}_z(1-\rho_z^2\alpha_z\beta)}(\mathcal{B}^2 - x_t^2) \end{aligned}$$

And the rational density is

$$h_y(y'|z_t) = \frac{1}{\sqrt{2\pi\sigma_z^2}} e^{-\frac{(y'-\rho_z z_t)^2}{2\sigma_z^2}} \tag{B.8}$$

From which it follows that $Z_y(\chi)$ is

$$\begin{aligned}
Z_y(\chi) &\equiv \int h_y(s|z_t) \left(\frac{h_y(s|x=x_t)}{h_y(s|x=\mathcal{B})}\right)^{\chi} ds \\
&= e^{\chi \frac{\rho_z^2\beta^2}{2\mathbb{V}_z(1-\rho_z^2\alpha_z\beta)}(\mathcal{B}^2-x_t^2)} \int \frac{1}{\sqrt{2\pi\sigma_z^2}} e^{-\frac{(s-\rho_z z_t)^2}{2\sigma_z^2}} e^{\chi \frac{\rho_z\beta(x_t-\mathcal{B})}{\mathbb{V}_z(1-\rho_z^2\alpha_z\beta)} s} ds
\end{aligned}$$

Note that

$$\frac{1}{\sqrt{2\pi\sigma_z^2}} e^{-\frac{(s-\rho_z z_t)^2}{2\sigma_z^2}} e^{\chi \frac{\rho_z\beta(x_t-\mathcal{B})}{\mathbb{V}_z(1-\rho_z^2\alpha_z\beta)} s}$$

Is a moment generating function, and it's integral is equal to

$$\int \frac{1}{\sqrt{2\pi\sigma_z^2}} e^{-\frac{(s-\rho_z z_t)^2}{2\sigma_z^2}} e^{\chi \frac{\rho_z\beta(x_t-\mathcal{B})}{\mathbb{V}_z(1-\rho_z^2\alpha_z\beta)} s} ds = e^{\chi \frac{\rho_z\beta(x_t-\mathcal{B})\rho_z z_t}{\mathbb{V}_z(1-\rho_z^2\alpha_z\beta)} + \frac{1}{2}\left(\chi \frac{\rho_z\beta(x_t-\mathcal{B})}{\mathbb{V}_z(1-\rho_z^2\alpha_z\beta)}\right)^2 \sigma_z^2}$$

From which it follows that the normalizing term is

$$Z_y(\chi) = e^{\chi \frac{\rho_z^2\beta^2}{2\mathbb{V}_z(1-\rho_z^2\alpha_z\beta)}(\mathcal{B}^2-x_t^2)} e^{\chi \frac{\rho_z\beta(x_t-\mathcal{B})\rho_z z_t}{\mathbb{V}_z(1-\rho_z^2\alpha_z\beta)} + \frac{1}{2}\left(\chi \frac{\rho_z\beta(x_t-\mathcal{B})}{\mathbb{V}_z(1-\rho_z^2\alpha_z\beta)}\right)^2 \sigma_z^2}$$

Then

$$\begin{aligned}
h_y^{\chi}(y'|\mathcal{I}) =& \frac{1}{\sqrt{2\pi\sigma_z^2}} e^{-\frac{(y'-\rho_z z_t)^2}{2\sigma_z^2}} e^{\chi \frac{\rho_z\beta(x_t-\mathcal{B})}{\mathbb{V}_z(1-\rho_z^2\alpha_z\beta)} y' + \chi \frac{\rho_z^2\beta^2}{2\mathbb{V}_z(1-\rho_z^2\alpha_z\beta)}(\mathcal{B}^2-x_t^2)} e^{-\chi \frac{\rho_z^2\beta^2}{2\mathbb{V}_z(1-\rho_z^2\alpha_z\beta)}(\mathcal{B}^2-x_t^2)} \\
& e^{-\chi \frac{\rho_z\beta(x_t-\mathcal{B})\rho_z z_t}{\mathbb{V}_z(1-\rho_z^2\alpha_z\beta)} - \frac{1}{2}\left(\chi \frac{\rho_z\beta(x_t-\mathcal{B})}{\mathbb{V}_z(1-\rho_z^2\alpha_z\beta)}\right)^2 \sigma_z^2}
\end{aligned}$$

After simplifying:

$$h_y^{\chi}(y'|\mathcal{I}) = \frac{1}{\sqrt{2\pi\sigma_z^2}} e^{-\frac{(y'-\rho_z z_t)^2}{2\sigma_z^2}} e^{\chi \frac{\rho_z\beta(x_t-\mathcal{B})}{\mathbb{V}_z(1-\rho_z^2\alpha_z\beta)} y'} e^{-\chi \frac{\rho_z\beta(x_t-\mathcal{B})\rho_z z_t}{\mathbb{V}_z(1-\rho_z^2\alpha_z\beta)} - \frac{1}{2}\left(\chi \frac{\rho_z\beta(x_t-\mathcal{B})}{\mathbb{V}_z(1-\rho_z^2\alpha_z\beta)}\right)^2 \sigma_z^2}$$

Let $A = \chi \frac{\rho_z \beta(x_t - \mathcal{B})}{\mathbb{V}_z(1-\rho_z^2 \alpha_z \beta)}$. Rewriting we get:

$$h_y^{\chi}(y'|\mathcal{I}) = \frac{1}{\sqrt{2\pi\sigma_z^2}} e^{-\frac{1}{2\sigma_z^2}\left[(y'-\rho_z z_t)^2 - 2A\sigma_z^2(y'-\rho_z z_t)\right] - \frac{1}{2}A^2\sigma_z^2}$$

Add and subtract $A^2\sigma_z^4$ inside the square brackets

$$\begin{aligned} h_y^{\chi}(y'|\mathcal{I}) &= \frac{1}{\sqrt{2\pi\sigma_z^2}} e^{-\frac{1}{2\sigma_z^2}\left[y'-\rho_z z_t - A\sigma_z^2\right]^2 + \frac{1}{2}A^2\sigma_z^2 - \frac{1}{2}A^2\sigma_z^2} \\ &= \frac{1}{\sqrt{2\pi\sigma_z^2}} e^{-\frac{1}{2\sigma_z^2}\left[y'-\rho_z z_t - A\sigma_z^2\right]^2} \end{aligned}$$

This is just the pdf of a normal distribution with mean $\rho_z z_t + A\sigma_z^2 = \rho_z \left(z_t + \frac{\sigma_z^2}{\mathbb{V}_z(1-\rho_z^2\alpha_z\beta)} \chi\beta(x_t - \mathcal{B}) \right)$ and variance σ_z^2. We can rewrite the mean as $\rho_z \left(z_t + \frac{(1-\rho_z^2)}{(1-\rho_z^2\alpha_z\beta)} \chi\beta(x_t - \mathcal{B}) \right)$. Remember that the reason why we have some objects premultiplying $\beta\chi$ is that the variance of the tilted likelihood is larger than the variance of the rational markovian forecast, so you take this into account when updating your mean.

B.3 Proof of Proposition 1

Consider the following true environment:

$$dz_t = -\mu_z\, z_t\, dt + \sigma_z\, dW_t^z, \qquad dg_t = -\mu_g\, g_t\, dt + \sigma_g\, dW_t^g, \qquad W^z \perp W^g, \qquad \mu_z, \mu_g > 0.$$

We'll prove the result under the more general setup where $x = \alpha_g g + \alpha_z z$, this will be convenient later on in the paper. Note that this implies that the long-run variances are $\mathrm{Var}(z_t) = \sigma_z^2/(2\mu_z)$ and $\mathrm{Var}(g_t) = \sigma_g^2/(2\mu_g)$. Hence

$$\beta_z \equiv \frac{\mathrm{Cov}(z_t, x_t)}{\mathrm{Var}(x_t)} = \frac{\alpha_z\sigma_z^2/\mu_z}{\alpha_z^2\sigma_z^2/\mu_z + \alpha_g^2\sigma_g^2/\mu_g}, \qquad 0 < \beta_z < 1,$$

with companion weight β_g for g.

Define the background context as

$$\mathcal{B}_t \equiv x_t - \mathcal{S}_t$$

And

$$\mathcal{S}_t \equiv \int_0^t e^{-\kappa(t-s)}\, d\Delta_s, \qquad \kappa \geq 0.$$

Where

$$d\Delta_t = \alpha_z \sigma_z \, dW_t^z + \alpha_g \sigma_g \, dW_t^g.$$

Note that the limit $\kappa \to 0$ we get

$$\mathcal{S}_t = d\Delta_t$$

which brings us back to our discrete-time benchmark where there is no memory. In fact, in this limit case the background just boils down to the rationally predicted x_t, i.e., x_t in the absence of shocks. It's just the realization minus the shock which by definition is the rationally predicted FIRE realization of x_t.

Because (z_t, g_t) is Gaussian, the joint process for (z_t, y_t) is also Gaussian with

$$\begin{bmatrix} z_t \\ x_t \end{bmatrix} \sim \mathcal{N}\Big(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \sigma_z^2/(2\mu_z) & \alpha_z \sigma_z^2/(2\mu_z) \\ \alpha_z \sigma_z^2/(2\mu_z) & \alpha_z^2 \sigma_z^2/(2\mu_z) + \alpha_g^2 \sigma_g^2/(2\mu_g) \end{bmatrix} \Big)$$

Remember that if $\begin{bmatrix} X_1 \\ X_2 \end{bmatrix} \sim \mathcal{N}\Big(\begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix}, \begin{bmatrix} \Sigma_{11} & \Sigma_{12} \\ \Sigma_{21} & \Sigma_{22} \end{bmatrix} \Big)$. Then

$$X_1 \,\big|\, X_2 = a \;\sim\; \mathcal{N}\big(\mu_1 + \Sigma_{12}\Sigma_{22}^{-1}(a - \mu_2),\; \Sigma_{11} - \Sigma_{12}\Sigma_{22}^{-1}\Sigma_{21}\big).$$

Thus, it follows that

$$z_t \,\big|\, x_t \;\sim\; \mathcal{N}\big(\beta_z x_t,\; \sigma_z^2/(2\mu_z) - \frac{\big(\alpha_z \sigma_z^2/(2\mu_z)\big)^2}{\alpha_z^2 \sigma_z^2/(2\mu_z) + \alpha_g^2 \sigma_g^2/(2\mu_g)}\big)$$

That is

$$z_t \,\big|\, x_t \;\sim\; \mathcal{N}\big(\beta y_t,\; \frac{\sigma_z^2}{2\mu_z}(1 - \alpha_z \beta)\big)$$

For simplicity, call

$$\Sigma_{z,0} \equiv \frac{\sigma_z^2}{2\mu_z}(1 - \alpha_z \beta).$$

Propagating one step of length dt forward, we can write

$$z_{t+dt} = z_t + dz_t = (1 - \mu_z dt) z_t + \sigma_z (W_{t+dt}^z - W_t^z)$$

Notice that $W^z_{t+dt} - W^z_t \sim \mathcal{N}(0, dt)$ and is independent of $\mathcal{F}_t$. Thus it follows that

$$z_{t+dt} \mid z_t \sim \mathcal{N}\big(m^z_t,\ \mathrm{Var}\big(z_{t+dt} \mid z_t\big)\big), \quad m^z_t \equiv (1-\mu_z dt) z_t, \quad \mathrm{Var}\big(z_{t+dt} \mid z_t\big) = \frac{\sigma_z^2}{2\mu_z}\Big(1 - e^{-2\mu_z\,dt}\Big).$$

Which for small dt boils down to

$$z_{t+dt} \mid z_t \sim \mathcal{N}\big(m^z_t,\ \mathrm{Var}\big(z_{t+dt} \mid z_t\big)\big), \quad m^z_t \equiv (1-\mu_z dt) z_t, \quad \mathrm{Var}\big(z_{t+dt} \mid z_t\big) = \sigma_z^2 dt.$$

Now however we want to change the conditioning set from z_t to either x_t or $\mathcal{B}_t$. To do so, we'll use the law of total expectations and the law of total variance. By the law of total expectations we have that

$$\begin{aligned}
m^x_t &= E(z_{t+dt}|x_t) \\
&= E(E(z_{t+dt}|z_t, x_t)|x_t) \\
&= E((1-\mu_z dt)z_t|x_t) \\
&= (1-\mu_z dt)E(z_t|x_t) \\
&= (1-\mu_z dt)\beta_z x_t
\end{aligned}$$

Now law of total variance

$$\begin{aligned}
Var(z_{t+dt}|x_t) &= E(Var(z_{t+dt}|z_t, x_t)|x_t) + Var(E(z_{t+dt}|z_t, x_t)|x_t) \\
&= E(\frac{\sigma_z^2}{2\mu_z}\Big(1 - e^{-2\mu_z\,dt}\Big)|x_t) + Var(e^{-\mu_z dt} z_t|x_t) \\
&= \frac{\sigma_z^2}{2\mu_z}\Big(1 - e^{-2\mu_z\,dt}\Big) + e^{-2\mu_z dt} Var(z_t|x_t) \\
&= \frac{\sigma_z^2}{2\mu_z}\Big(1 - e^{-2\mu_z\,dt}\Big) + e^{-2\mu_z dt}\frac{\sigma_z^2}{2\mu_z}(1-\alpha_z\beta_z)
\end{aligned}$$

Which for small dt becomes

$$\begin{aligned}
Var(z_{t+dt}|x_t) &= \frac{\sigma_z^2}{2\mu_z}\Big(2\mu_z dt\Big) + (1-2\mu_z dt)\frac{\sigma_z^2}{2\mu_z}(1-\alpha_z\beta_z) \\
&= \sigma_z^2 dt - \sigma_z^2(1-\alpha_z\beta)dt + \underbrace{\frac{\sigma_z^2}{2\mu_z}(1-\alpha_z\beta_z)}_{=\Sigma_{z,0}} \\
&= \Sigma_{z,0} + \beta_z\alpha_z\sigma_z^2 dt
\end{aligned}$$

So now it immediately follows that

$$z_{t+dt} \mid x_t \sim \mathcal{N}\big(m_t^x,\, \Sigma_z(dt)\big), \quad m_t^x \equiv (1-\mu_z dt)\beta_z x_t, \quad \Sigma_z(dt) = \Sigma_{z,0} + \beta_z \alpha_z \sigma_z^2 dt + o(dt).$$

Similarly,

$$z_{t+dt} \mid \mathcal{B}_t \sim \mathcal{N}\big(m_t^{\mathcal{B}},\, \Sigma_z(dt)\big), \quad m_t^{\mathcal{B}} \equiv (1-\mu_z dt)\beta_z \mathcal{B}_t, \quad \Sigma_z(dt) = \Sigma_{z,0} + \beta \alpha_z \sigma_z^2 dt + o(dt).$$

Now we have all the results needed to proceed with our calculation of the XDE density. First, note that

$$h_z\big(z_{t+dt} \mid z_t\big) = \frac{1}{\sqrt{2\pi\,\sigma_z^2\,dt}} \exp\Big(-\frac{\big(z_{t+dt} - (1-\mu_z\,dt)\,z_t\big)^2}{2\,\sigma_z^2\,dt}\Big).$$

Moreover

$$h_z\big(z_{t+dt} \mid x_t\big) = \frac{1}{\sqrt{2\pi\,\Sigma_z(dt)}} \exp\Big(-\frac{\big(z_{t+dt} - (1-\mu_z dt)\beta_z x_t,\big)^2}{2\,\Sigma_z(dt)}\Big).$$

And similarly

$$h_z\big(z_{t+dt} \mid \mathcal{B}_t\big) = \frac{1}{\sqrt{2\pi\,\Sigma_z(dt)}} \exp\Big(-\frac{\big(z_{t+dt} - (1-\mu_z dt)\beta_z \mathcal{B}_t,\big)^2}{2\,\Sigma_z(dt)}\Big).$$

Now let's compute the tilting

$$\begin{aligned}
T_{z,t}(z') &\equiv \log \frac{h_z(z' \mid x_t)}{h_z(z' \mid \mathcal{B}_t)} \\
&= -\log(\sqrt{2\pi\,\Sigma_z(dt)}) - \frac{\big(z_{t+dt} - (1-\mu_z dt)\beta_z x_t,\big)^2}{2\,\Sigma_z(dt)} + \log(\sqrt{2\pi\,\Sigma_z(dt)}) \\
&+ \frac{\big(z_{t+dt} - (1-\mu_z dt)\beta_z \mathcal{B}_t,\big)^2}{2\,\Sigma_z(dt)} \\
&= \frac{\big(z_{t+dt} - (1-\mu_z dt)\beta_z \mathcal{B}_t,\big)^2 - \big(z_{t+dt} - (1-\mu_z dt)\beta_z x_t,\big)^2}{2\,\Sigma_z(dt)} \\
&= \frac{(1-\mu_z dt)^2 \beta_z^2 \mathcal{B}_t^2 - (1-\mu_z dt)^2 \beta_z^2 x_t^2 + 2 z_{t+dt}(1-\mu_z dt)\beta_z (x_t - \mathcal{B}_t)}{2\,\Sigma_z(dt)} \\
&= \frac{z_{t+dt}(1-\mu_z dt)\beta_z (x_t - \mathcal{B}_t)}{\Sigma_z(dt)} + \frac{(1-\mu_z dt)^2 \beta_z^2 (\mathcal{B}_t^2 - x_t^2)}{2\,\Sigma_z(dt)}
\end{aligned}$$

Then, the normalizing constant is

$$Z_z(\chi) = \int \frac{1}{\sqrt{2\pi\,\sigma_z^2\,dt}} \exp\Big(-\frac{\big(s-(1-\mu_z\,dt)\,z_t\big)^2}{2\,\sigma_z^2\,dt} + \frac{\chi s(1-\mu_z dt)\beta_z(x_t-\mathcal{B}_t)}{\Sigma_z(dt)}$$
$$+ \frac{\chi(1-\mu_z dt)^2\beta_z^2(\mathcal{B}_t^2-x_t^2)}{2\,\Sigma_z(dt)}\Big)ds$$
$$= \exp\Big(\frac{\chi(1-\mu_z dt)^2\beta_z^2(\mathcal{B}_t^2-x_t^2)}{2\,\Sigma_z(dt)}\Big) \int \frac{1}{\sqrt{2\pi\,\sigma_z^2\,dt}} \exp\Big(-\frac{\big(s-(1-\mu_z\,dt)\,z_t\big)^2}{2\,\sigma_z^2\,dt}$$
$$+ \frac{\chi s(1-\mu_z dt)\beta_z(x_t-\mathcal{B}_t)}{\Sigma_z(dt)}\Big)ds$$

Notice that

$$\frac{1}{\sqrt{2\pi\,\sigma_z^2\,dt}} \exp\Big(-\tfrac{(s-(1-\mu_z\,dt)\,z_t)^2}{2\,\sigma_z^2\,dt}\Big) \exp\Big(\chi\,\frac{(1-\mu_z\,dt)\,\beta_z\,(x_t-\mathcal{B}_t)}{\Sigma_z(dt)}\,s\Big)$$

Is a moment generating function for the moment $w = \chi\,\frac{(1-\mu_z\,dt)\,\beta_z\,(x_t-\mathcal{B}_t)}{\Sigma_z(dt)}$. Remember that if $x \sim \mathcal{N}(m,\sigma^2)$ then

$$E(e^{ax}) = \exp(am + \frac{1}{2}\sigma^2 a^2)$$

Then it follows that

$$\int \frac{1}{\sqrt{2\pi\,\sigma_z^2\,dt}} \exp\Big(-\tfrac{(s-(1-\mu_z\,dt)\,z_t)^2}{2\,\sigma_z^2\,dt}\Big) \exp\Big(\chi\,\frac{(1-\mu_z\,dt)\,\beta_z\,(x_t-\mathcal{B}_t)}{\Sigma_z(dt)}\,s\Big)ds =$$
$$\exp\Big(\chi\,\frac{(1-\mu_z\,dt)\,\beta_z\,(x_t-\mathcal{B}_t)\,(1-\mu_z\,dt)\,z_t}{\Sigma_z(dt)} \;+\; \tfrac{1}{2}\Big(\chi\,\frac{(1-\mu_z\,dt)\,\beta_z\,(x_t-\mathcal{B}_t)}{\Sigma_z(dt)}\Big)^2\sigma_z^2\,dt\Big)$$

Plugging all of this in we have that

$$Z_z(\chi) = \exp\Big[\chi\,\frac{(1-\mu_z\,dt)^2\,\beta_z^2}{2\,\Sigma_z(dt)}\,(\mathcal{B}_t^2-x_t^2) + \chi\,\frac{(1-\mu_z\,dt)^2\,\beta_z\,(x_t-\mathcal{B}_t)\,z_t}{\Sigma_z(dt)}$$
$$+ \tfrac{1}{2}\,\chi^2\frac{(1-\mu_z\,dt)^2\,\beta_z^2\,(x_t-\mathcal{B}_t)^2}{\Sigma_z(dt)^2}\,\sigma_z^2\,dt\Big].$$

So now we can bring everything together

$$h_z^\chi\big(z_{t+dt}\mid\mathcal{F}_t\big) = \frac{1}{Z_z(\chi)}\,h_z\big(z_{t+dt}\mid z_t\big)\,\exp\big(\chi\,T_{z,t}(z_{t+dt})\big)$$

$$
\begin{aligned}
&= \frac{1}{\sqrt{2\pi\,\sigma_z^2\,dt}}\exp\Big(-\frac{\big(z_{t+dt}-(1-\mu_z\,dt)\,z_t\big)^2}{2\,\sigma_z^2\,dt}\\
&+\frac{\chi z_{t+dt}(1-\mu_z dt)\beta_z(x_t-\mathcal{B}_t)}{\Sigma_z(dt)}+\frac{\chi(1-\mu_z dt)^2\beta_z^2(\mathcal{B}_t^2-x_t^2)}{2\,\Sigma_z(dt)}\Big)\times\\
&\times\exp\Big[-\chi\,\frac{(1-\mu_z\,dt)^2\,\beta_z^2}{2\,\Sigma_z(dt)}\,(\mathcal{B}_t^2-x_t^2)-\chi\,\frac{(1-\mu_z\,dt)^2\,\beta_z\,(x_t-\mathcal{B}_t)\,z_t}{\Sigma_z(dt)}\\
&-\tfrac{1}{2}\,\chi^2\frac{(1-\mu_z\,dt)^2\,\beta_z^2\,(x_t-\mathcal{B}_t)^2}{\Sigma_z(dt)^2}\,\sigma_z^2\,dt\Big].
\end{aligned}
$$

Now let's simplify terms to get the kernel of the normal distribution. First, define

$$
m \equiv (1-\mu_z\,dt)\,z_t, \qquad \sigma^2 \equiv \sigma_z^2\,dt, \qquad a \equiv \chi\,\frac{(1-\mu_z\,dt)\,\beta_z\,(x_t-\mathcal{B}_t)}{\Sigma_z(dt)}.
$$

With these abbreviations the tilted density reads

$$
h_z^\chi(z_{t+dt}\mid\mathcal{F}_t) \;=\; \frac{1}{\sqrt{2\pi\sigma^2}}\;\exp\Big(-\tfrac{(z_{t+dt}-m)^2}{2\sigma^2}+a\,z_{t+dt}-\big[m\,a+\tfrac{1}{2}a^2\sigma^2\big]\Big).
$$

Now complete the squares

$$
\begin{aligned}
-\frac{(z_{t+dt}-m)^2}{2\sigma^2}\;+\;a\,(z_{t+dt}-m)\;-\;\frac{1}{2}a^2\sigma^2 &= -\frac{1}{2\sigma^2}\Big[(z_{t+dt}-m)^2-2a\sigma^2(z_{t+dt}-m)+a^2\sigma^4\Big]\\
&= -\frac{1}{2\sigma^2}\big(z_{t+dt}-m-a\sigma^2\big)^2.
\end{aligned}
$$

Insert the right-hand side above into the exponential:

$$
h_z^\chi(z_{t+dt}\mid\mathcal{F}_t)=\frac{1}{\sqrt{2\pi\sigma^2}}\;\exp\Big(-\tfrac{1}{2\sigma^2}\big(z_{t+dt}-m-a\sigma^2\big)^2\Big).
$$

Write

$$
\mu_{t+dt}^{(\chi)}\equiv m+a\sigma^2=(1-\mu_z\,dt)\,z_t+\chi\,(1-\mu_z\,dt)\,\beta_z\,(x_t-\mathcal{B}_t)\,\frac{\sigma_z^2 dt}{\Sigma_z(dt)}=(1-\mu_z\,dt)\Big(z_t+\frac{\sigma_z^2 dt}{\Sigma_z(dt)}\chi\beta_z\mathcal{S}_t\Big)
$$

Then

$$
h_z^\chi(z_{t+dt}\mid\mathcal{F}_t)\;=\;\frac{1}{\sqrt{2\pi\sigma^2}}\;\exp\Big(-\frac{\big(z_{t+dt}-\mu_{t+dt}^{(\chi)}\big)^2}{2\sigma^2}\Big).
$$

Because the right-hand side is the canonical form of a normal pdf with variance σ^2, we have

$$
z_{t+dt}\,\big|\,\mathcal{F}_t,\chi\;\sim\;\mathcal{N}(\mu_{t+dt}^{(\chi)},\,\sigma^2),\qquad \sigma^2=\sigma_z^2\,dt.
$$

Finally, it follows that

$$d\widetilde{z}_t = \left[-\mu_z\, z_t + \chi\, \beta_z\, \mathcal{S}_t \frac{\sigma_z^2}{\Sigma_z(dt)}\right] dt + \sigma_z\, dW_t^z$$

Where, using the definition $\mathcal{S}_t = \int_0^t e^{-\kappa(t-s)}\, d\Delta_s$ and Ito's rule for deterministic kernels one obtains

$$d\mathcal{S}_t = -\kappa\, \mathcal{S}_t\, dt + d\Delta_t, \qquad d\Delta_t = \alpha_z \sigma_z\, dW_t^z + \alpha_g \sigma_g\, dW_t^g.$$

B.4 Models of imperfect information

In this section, we present models of overreaction based on limited information, rather than on deviations from rational expectations.

In this section, we present models of overreaction based on limited information, rather than on deviations from rational expectations. Overreaction arises as a byproduct of a signal extraction problem rather than behavioral bias. We explain why, in the main analysis of this paper, we do not focus on these models.

Public and private demand shocks cannot be disentangled Let two latent demand components follow independent AR(1) laws,

$$z_t = \rho_z z_{t-1} + \varepsilon_t^z, \qquad g_t = \rho_g g_{t-1} + \varepsilon_t^g,$$

with $|\rho_z|, |\rho_g| < 1$, innovations $\varepsilon_t^z \sim (0, \sigma_z^2)$ and $\varepsilon_t^g \sim (0, \sigma_g^2)$, mutually independent over time and across processes. The observed aggregate is $x_t = z_t + g_t$. The agent only observes $\{x_s\}_{s \leq t}$ and forms the optimal forecast of x_{t+1}. Define the period-t innovation in the observable as

$$u_t \equiv x_t - \mathbb{E}_{t-1}[x_t] = \varepsilon_t^z + \varepsilon_t^g,$$

Because everything is linear and Gaussian, the optimal one-step forecast revision is linear in u_t,

$$\mathbb{E}_t[x_{t+1}] - \mathbb{E}_{t-1}[x_{t+1}] = \beta\, u_t,$$

where $\beta = \mathrm{Cov}(x_{t+1}, u_t) / \mathrm{Var}(u_t)$. Using $x_{t+1} = z_{t+1} + g_{t+1} = \rho_z z_t + \rho_g g_t + \varepsilon_{t+1}^z + \varepsilon_{t+1}^g$ and $u_t = \varepsilon_t^z + \varepsilon_t^g$, independence gives

$$\mathrm{Cov}(x_{t+1}, u_t) = \rho_z \sigma_z^2 + \rho_g \sigma_g^2, \qquad \mathrm{Var}(u_t) = \sigma_z^2 + \sigma_g^2,$$

hence

$$\beta = \frac{\rho_z \sigma_z^2 + \rho_g \sigma_g^2}{\sigma_z^2 + \sigma_g^2}.$$

A simple reason for the linearity of the revision in u_t is that $\mathbb{E}_t[x_{t+1}]$ is the orthogonal projection of x_{t+1} onto the time-t information set. The only new information between $t-1$ and t is the scalar innovation u_t, which spans a one-dimensional subspace orthogonal to information at time $t-1$. By the projection theorem, the increment in the optimal forecast must lie in this span, hence it is proportional to u_t; under joint normality, least-squares and conditional expectations coincide, which pins down the slope as β.

To assess reaction to a period-t shock in g_t, hold $\varepsilon_t^z = 0$ and perturb ε_t^g. The true effect on the next aggregate is $\partial x_{t+1}/\partial \varepsilon_t^g = \rho_g$, while the agent's revision in expected x_{t+1} per unit of u_t is β. The difference $\beta - \rho_g$ determines over- versus underreaction to g-news. Substituting the expression for β yields

$$\beta - \rho_g = \frac{\sigma_z^2}{\sigma_z^2 + \sigma_g^2}(\rho_z - \rho_g).$$

Whenever $\sigma_z^2 > 0$, the sign is governed by the relative persistence of the latent components: if $\rho_z > \rho_g$ the agent overreacts to shocks in g_t because part of the innovation in x_t is (optimally) attributed to the more persistent z_t; if $\rho_z < \rho_g$ the agent underreacts; if $\rho_z = \rho_g$ the reaction is exactly correct.

Now define $\mathrm{FE}_{t+1} \equiv x_{t+1} - \mathbb{E}_t[x_{t+1}]$. Combine

$$x_{t+1} = \mathbb{E}_{t-1}[x_{t+1}] + \rho_z \varepsilon_t^z + \rho_g \varepsilon_t^g + \varepsilon_{t+1}^z + \varepsilon_{t+1}^g$$

with

$$\mathbb{E}_t[x_{t+1}] = \mathbb{E}_{t-1}[x_{t+1}] + \beta(\varepsilon_t^z + \varepsilon_t^g)$$

to obtain

$$\mathrm{FE}_{t+1} = (\rho_z - \beta)\varepsilon_t^z + (\rho_g - \beta)\varepsilon_t^g + \varepsilon_{t+1}^z + \varepsilon_{t+1}^g.$$

This last equation makes it clear why we don't choose this model as our baseline model, in spite of it being able to generate overreaction in public spending shocks. First, it requires $\rho_g \gg \rho_z$ to generate sizable overreaction to g-news (given that $\sigma_z^2/(\sigma_z^2 + \sigma_g^2) < 1$), yet empirical estimates typically do not show such a large persistence gap between aggregate (g) and idiosyncratic (z) components. Second, because the single updating coefficient β must satisfy $\beta > \rho_g$ to produce overreaction to g, the same β necessarily implies $\beta < \rho_z$ whenever $\rho_g \geq \rho_z$, i.e., underreaction to z. This is at odds with much of the empirical literature, which tends to find overreaction to idiosyncratic shocks rather than underreaction. Third, this assumes that firms cannot disentangle public and private demand which is at odds with the data. Finally this framework does not allow us to think about the forecast of z separately and therefore cannot rationalize our results on export forecast.

Underlying quality Let $u_t = \rho_u u_{t-1} + \varepsilon_t^u$ with $|\rho_u| < 1$, and let

$$z_t = u_t + \nu_t^z, \quad \nu_t^z = \rho_z \nu_{t-1}^z + \varepsilon_t^z, \qquad g_t = u_t + \nu_t^g, \quad \nu_t^g = \rho_g \nu_{t-1}^g + \varepsilon_t^g,$$

with mutually independent innovations $\varepsilon_t^u \sim (0, \sigma_u^2)$, $\varepsilon_t^z \sim (0, \sigma_z^2)$, $\varepsilon_t^g \sim (0, \sigma_g^2)$. The target is

$$x_{t+1} = z_{t+1} + g_{t+1} = 2u_{t+1} + \nu_{t+1}^z + \nu_{t+1}^g.$$

Define the time-t observation innovations

$$v_t^z \equiv z_t - \mathbb{E}_{t-1}[z_t] = \varepsilon_t^u + \varepsilon_t^z, \qquad v_t^g \equiv g_t - \mathbb{E}_{t-1}[g_t] = \varepsilon_t^u + \varepsilon_t^g.$$

By linear-Gaussian projection,

$$\mathbb{E}_t[x_{t+1}] - \mathbb{E}_{t-1}[x_{t+1}] = b_z\, v_t^z + b_g\, v_t^g,$$

where $b_z = \mathrm{Cov}(x_{t+1}, v_t^z)$ and $b_g = \mathrm{Cov}(x_{t+1}, v_t^g)$ premultiplied by $\mathrm{Var}(v_t)^{-1}$. Writing

$$\mathrm{Var}(v_t) = \begin{pmatrix} \sigma_u^2 + \sigma_z^2 & \sigma_u^2 \\ \sigma_u^2 & \sigma_u^2 + \sigma_g^2 \end{pmatrix}, \qquad \Delta \equiv (\sigma_u^2 + \sigma_z^2)(\sigma_u^2 + \sigma_g^2) - \sigma_u^4 = \sigma_u^2\sigma_z^2 + \sigma_u^2\sigma_g^2 + \sigma_z^2\sigma_g^2,$$

and

$$\mathrm{Cov}(x_{t+1}, v_t^z) = 2\rho_u\sigma_u^2 + \rho_z\sigma_z^2, \qquad \mathrm{Cov}(x_{t+1}, v_t^g) = 2\rho_u\sigma_u^2 + \rho_g\sigma_g^2,$$

a short calculation yields the closed-form coefficients

$$b_g = \rho_g + \frac{\sigma_u^2\sigma_z^2}{\Delta}\left(2\rho_u - \rho_z - \rho_g\right), \qquad b_z = \rho_z + \frac{\sigma_u^2\sigma_g^2}{\Delta}\left(2\rho_u - \rho_z - \rho_g\right).$$

Hence the loadings on the primitive shocks in the time-t forecast revision are

$$\alpha_u = b_z + b_g, \qquad \alpha_z = b_z, \qquad \alpha_g = b_g,$$

so that the $t+1$ forecast error can be written as

$$\mathrm{FE}_{t+1} \equiv x_{t+1} - \mathbb{E}_t[x_{t+1}] = (2\rho_u - \alpha_u)\varepsilon_t^u + (\rho_z - \alpha_z)\varepsilon_t^z + (\rho_g - \alpha_g)\varepsilon_t^g + 2\varepsilon_{t+1}^u + \varepsilon_{t+1}^z + \varepsilon_{t+1}^g.$$

Overreaction to g-news is the condition $b_g > \rho_g$. From the expression above,

$$b_g - \rho_g = \frac{\sigma_u^2 \sigma_z^2}{\Delta} \left(2\rho_u - \rho_z - \rho_g\right),$$

which is positive if and only if

$$2\rho_u > \rho_z + \rho_g.$$

Under the same condition $b_z > \rho_z$ and, correspondingly,

$$\alpha_u - 2\rho_u = \frac{\sigma_g^2 \sigma_z^2}{\Delta} \left(\rho_g + \rho_z - 2\rho_u\right) < 0,$$

so the forecast overreacts to the measurement-noise shocks $\varepsilon_t^z, \varepsilon_t^g$ while underreacting to the fundamental shock ε_t^u. Intuitively, a positive surprise in g_t raises the inferred u_t; because x_{t+1} depends on u_t through $2\rho_u$, the optimal revision incorporates this channel in addition to the mechanical ρ_g propagation of ν_t^g, making the loading on g-news exceed ρ_g.

Finally, shocks to the g-series also generate cross–reactions in expectations of z. From the forecast–revision equation

$$\mathbb{E}_t[x_{t+1}] - \mathbb{E}_{t-1}[x_{t+1}] = b_z\, v_t^z + b_g\, v_t^g,$$

the revision to expected z_{t+1} alone can be written as

$$\mathbb{E}_t[z_{t+1}] - \mathbb{E}_{t-1}[z_{t+1}] = c_z\, v_t^z + c_g\, v_t^g,$$

where c_g measures how a surprise in g_t affects the forecast of z_{t+1}.

Because both z_t and g_t load on the same latent factor u_t, a positive innovation in g_t raises the agent's inference about u_t and, therefore, their forecast of z_{t+1}. Formally, applying the same linear–Gaussian projection yields

$$c_g = \frac{\sigma_u^2 \sigma_z^2}{\Delta} \left(\rho_u - \rho_z\right),$$

which is positive whenever $\rho_u > \rho_z$. Thus, even though the shock originates in the idiosyncratic g-noise process, it induces a positive co–movement in expected z_{t+1}. This cross–channel response arises because g_t is an imperfect but correlated signal of the underlying fundamental u_t; a transitory improvement in g_t is partly interpreted as evidence of a stronger u_t, leading the agent to revise upward their expectation of z_{t+1} and, consequently, of the aggregate x_{t+1}.

In this limited information rational-expectations model, the one-step forecast error is an innovation, orthogonal to all information available at the time the forecast is made. This implies that its autocovariance must be zero, since the previous forecast error belongs to the information set. Formally, $\mathrm{Cov}(\mathrm{FE}_t, \mathrm{FE}_{t-1}) = \mathbb{E}[\mathrm{FE}_{t-1}\, \mathbb{E}(\mathrm{FE}_t \mid \mathcal{F}_{t-1})] = 0$. Substituting the shock representation from the model, $\mathrm{FE}_t = (2\rho_u - \alpha_u)\varepsilon^u_{t-1} + (\rho_z - \alpha_z)\varepsilon^z_{t-1} + (\rho_g - \alpha_g)\varepsilon^g_{t-1} + 2\varepsilon^u_t + \varepsilon^z_t + \varepsilon^g_t$, and an analogous expression for FE_{t-1}, only the $(t-1)$ shocks overlap across the two errors. This yields $\mathrm{Cov}(\mathrm{FE}_t, \mathrm{FE}_{t-1}) = 2(2\rho_u - \alpha_u)\sigma_u^2 + (\rho_z - \alpha_z)\sigma_z^2 + (\rho_g - \alpha_g)\sigma_g^2 = 0$, because the projection coefficients b_z and b_g defining $\alpha_u = b_z + b_g$, $\alpha_z = b_z$, and $\alpha_g = b_g$ are chosen to ensure orthogonality. Hence, under rational expectations, forecast errors are serially uncorrelated. This theoretical result contrasts with the data, where forecast errors display positive autocorrelation.

C Appendix to Section 4

C.1 Derivation of profit function

Demand aggregation. From the CES aggregators for Y_t^z and G_t, the associated price indices are

$$P_t^z = \left(\int_0^1 e^{z_{it}} p_{it}^{1-\varepsilon}\, di\right)^{\frac{1}{1-\varepsilon}}, \qquad P_t^G = \left(\int_0^1 e^{g_{it}} p_{it}^{1-\varepsilon}\, di\right)^{\frac{1}{1-\varepsilon}}.$$

Individual demands are therefore

$$y_{it}^z = e^{z_{it}} \left(\frac{p_{it}}{P_t^z}\right)^{-\varepsilon} Y_t^z, \qquad y_{it}^g = e^{g_{it}} \left(\frac{p_{it}}{P_t^G}\right)^{-\varepsilon} G_t,$$

so total demand faced by firm i is isoelastic:

$$y_{it} = y_{it}^z + y_{it}^g = p_{it}^{-\varepsilon}\Big[e^{z_{it}} Y_t^z\, (P_t^z)^{\varepsilon} + e^{g_{it}} G_t\, (P_t^G)^{\varepsilon}\Big] \equiv \Lambda_{it}\, p_{it}^{-\varepsilon}. \tag{C.1}$$

Technology and costs. Let production be Cobb–Douglas with predetermined capital k_{it}:

$$y_{it} = k_{it}^{\alpha} \ell_{it}^{1-\alpha},$$

and let w^r be the real wage. The conditional labor requirement for output y is

$$\ell(y,k) = \left(\frac{y}{k^{\alpha}}\right)^{\frac{1}{1-\alpha}},$$

so the variable cost is

$$VC(y,k) = w^r\, \ell(y,k) = w^r\, k^{-\frac{\alpha}{1-\alpha}}\, y^{\frac{1}{1-\alpha}}.$$

Hence marginal cost is

$$mc_{it}(y) = \frac{\partial VC}{\partial y} = \frac{w^r}{1-\alpha}\, k_{it}^{-\frac{\alpha}{1-\alpha}}\, y^{\frac{\alpha}{1-\alpha}}. \tag{C.2}$$

Optimal pricing (markup condition). Given (C.1), profit as a function of price is

$$\pi_{it}(p) = p\, y_{it}(p) - VC\big(y_{it}(p), k_{it}\big), \qquad y_{it}(p) = \Lambda_{it}\, p^{-\varepsilon}.$$

The first-order condition uses $dy/dp = -\varepsilon y/p$:

$$0 = \frac{d\pi}{dp} = y + \left(p - mc_{it}\right)\frac{dy}{dp} \implies \frac{p_{it} - mc_{it}}{p_{it}} = \frac{1}{\varepsilon} \implies p_{it} = \mu\, mc_{it}, \quad \mu \equiv \frac{\varepsilon}{\varepsilon - 1}.$$

Substituting (C.2) and $y_{it} = \Lambda_{it} p_{it}^{-\varepsilon}$ gives

$$p_{it} = \mu \cdot \frac{w^r}{1-\alpha}\, k_{it}^{-\frac{\alpha}{1-\alpha}} \left(\Lambda_{it}\, p_{it}^{-\varepsilon}\right)^{\frac{\alpha}{1-\alpha}} = \left[\mu \cdot \tfrac{w^r}{1-\alpha}\, k_{it}^{-\frac{\alpha}{1-\alpha}}\right] \Lambda_{it}^{\frac{\alpha}{1-\alpha}}\, p_{it}^{-\frac{\varepsilon\alpha}{1-\alpha}}.$$

Collecting powers of p_{it} yields

$$p_{it}^{1+\frac{\varepsilon\alpha}{1-\alpha}} = \left[\mu \cdot \tfrac{w^r}{1-\alpha}\, k_{it}^{-\frac{\alpha}{1-\alpha}}\right] \Lambda_{it}^{\frac{\alpha}{1-\alpha}}.$$

Let $D \equiv 1 - \alpha + \varepsilon\alpha = 1 + (\varepsilon - 1)\alpha$. Solving for p_{it},

$$p_{it} = \left[\mu \cdot \frac{w^r}{1-\alpha} \cdot k_{it}^{-\frac{\alpha}{1-\alpha}}\right]^{\frac{1-\alpha}{D}} \cdot \Lambda_{it}^{\frac{\alpha}{D}}, \qquad \mu = \frac{\varepsilon}{\varepsilon - 1}. \tag{C.3}$$

Profit in closed form. At the optimum, the Lerner condition implies $p_{it} - mc_{it} = p_{it}/\varepsilon$, so

$$\pi_{it}^* = \frac{p_{it}^*}{\varepsilon}\, y_{it}^* = \frac{1}{\varepsilon}\, \Lambda_{it}\, (p_{it}^*)^{1-\varepsilon}.$$

Using (C.3) and $D = 1 - \alpha + \varepsilon\alpha$,

$$(p_{it}^*)^{1-\varepsilon} = \left[\mu \cdot \frac{w^r}{1-\alpha} \cdot k_{it}^{-\frac{\alpha}{1-\alpha}}\right]^{(1-\varepsilon)\frac{1-\alpha}{D}} \cdot \Lambda_{it}^{(1-\varepsilon)\frac{\alpha}{D}}.$$

Multiplying by the leading Λ_{it} gives the exponent on Λ_{it} as

$$1 + (1-\varepsilon)\frac{\alpha}{D} = \frac{D}{D} + (1-\varepsilon)\frac{\alpha}{D} = \frac{1}{D}.$$

Therefore,

$$\pi_{it}^* = \frac{1}{\varepsilon}\left[\frac{\varepsilon}{\varepsilon-1} \cdot \frac{w^r}{1-\alpha} \cdot k_{it}^{-\frac{\alpha}{1-\alpha}}\right]^{(1-\varepsilon)\,\frac{1-\alpha}{1-\alpha+\varepsilon\alpha}} \cdot \Lambda_{it}^{\frac{1}{1-\alpha+\varepsilon\alpha}} \tag{C.4}$$

with

$$\Lambda_{it} = e^{z_{it}}\, Y_t^z\, (P_t^z)^{\varepsilon} \; + \; e^{g_{it}}\, G_t\, (P_t^G)^{\varepsilon}.$$

Now using the normalization $P_t^z = 1$ we can rewrite Λ_{it} as

$$\Lambda_{it} = \left[e^{z_{it}} + \alpha_{gt}\, e^{g_{it}}\right] Y_t^z, \quad \alpha_{gt} \equiv \frac{G_t\,(P_t^G)^{\varepsilon}}{Y_t^z}$$

which yields the desired expression.

C.2 Proof of Propositions 2 and 3

Consider the HJB equation

$$rV(k, \mathcal{S}, z) \;=\; \max_{\iota} \pi(z,k) - \Phi(\iota,k) - \Gamma(d) + (\iota - \delta k)\, V_k + \mathcal{Z}V + \mathcal{S}V_z$$

The first order condition with respect to investment is:

$$V_k = \Phi_\iota(\iota,k)\left(1 + \Gamma'(\mathrm{d})\right) \tag{C.5}$$

And the envelope condition for capital

$$(r+\delta)V_k = (\pi_k(z,k) - \Phi_k(\iota,k))\left(1 - \Gamma'(\mathrm{d})\right) + \dot{k}V_{kk} + \mathcal{Z}V_k + \mathcal{S}V_{sk}$$

As standard, we define marginal q as the perceived marginal value of capital, i.e., $\tilde{q} \equiv V_k$. Then we can rewrite the equation above as

$$(r+\delta)\tilde{q}_t = (\pi_k(z_t,k_t) - \Phi_k(\iota_t,k_t))\left(1 + \Gamma'(\mathrm{d}_t)\right) + \mathbb{E}_t^{\chi}\left(d\tilde{q}_t/dt\right) \tag{C.6}$$

Where

$$\mathbb{E}^{\chi}\left(d\tilde{q}_t/dt\right) \equiv \dot{k}V_{kk} + \mathcal{Z}V_k + \mathcal{S}V_{sk}$$

Then (19) simply follows from (C.6) by decomposing $\mathbb{E}_t^{\chi}\left(d\tilde{q}_t/dt\right) = \mathbb{E}_t\left(d\tilde{q}_t/dt\right) + \left(\mathbb{E}_t^{\chi} - \mathbb{E}_t\right)\left(d\tilde{q}_t/dt\right)$.

To obtain (20) we apply the Feynman-Kac formula to (C.6) above to get

$$\tilde{q}_t = \int_0^{\infty} e^{-(r+\delta)s}\, \mathbb{E}_t^{\chi}\left(\pi_k(z_{t+s},k_{t+s}) - \Phi_k(\iota_{t+s},k_{t+s})\right)\left(1 + \Gamma'(\mathrm{d}_{t+s})\right) ds$$

Finally, defining $q_t \equiv \int_0^{\infty} e^{-(r+\delta)s}\, \mathbb{E}_t\left(\pi_k(z_{t+s},k_{t+s}) - \Phi_k(\iota_{t+s},k_{t+s})\right)\left(1 + \Gamma'(\mathrm{d}_{t+s})\right) ds$ and $\mathscr{F}_t \equiv \tilde{q}_t - q_t$ and substituting into (C.5) gives (20).

C.3 Derivation of the mapping between sentiment and forecast errors

Given the true law of motion for demand (12), expected $z_{t+\Delta}$ in a Δ time interval after period t according to rational expectations is

$$\mathbb{E}_t \, z_{t+\Delta} = z_t e^{-\mu\Delta} \tag{C.7}$$

The expected $z_{t+\Delta}$ for a firm with sentiment $\mathcal{S}_t$ is the solution to the following ODE:

$$\frac{dz_t}{dt} + \mu z_t = \chi \mathcal{S}_t \tag{C.8}$$

Using the integrating factor $e^{\mu t}$ we have that

$$\mathbb{E}_t^{\chi} \, z_{t+\Delta} = z_t e^{-\mu\Delta} + \frac{\mathcal{S}_t}{\mu}\left(1 - e^{-\mu\Delta}\right) \tag{C.9}$$

Because z_t is equal to the log of the demand shifter, it follows that the expected forecast error over an interval Δ for a firm with sentiment $\mathcal{S}$ is

$$\mathcal{E}_{t+\Delta}\left(\mathcal{S}\right) \equiv \mathbb{E}_t \, z_{t+\Delta} - \mathbb{E}_t^{\chi} \, z_{t+\Delta} = -\frac{\mathcal{S}}{\mu}\left(1 - e^{-\mu\Delta}\right) \tag{C.10}$$

The same goes for public demand g.

D Appendix to Section 5

D.1 Unions

We show that the sticky-wage union problem delivers the (non-linear) wage New-Keynesian Phillips Curve

$$\epsilon\, v'(L_t)L_t - (\epsilon-1)\, u'(C_t)\, w_t\, L_t + \psi\big(\dot{\pi}^w_t - \rho\, \pi^w_t\big) = 0.$$

Consider a unit continuum of unions $k \in [0,1]$. Each union sets a nominal wage path W_{kt} (equivalently, nominal wage inflation $\pi^w_{kt} \equiv \dot{W}_{kt}/W_{kt}$) to maximise the lifetime utility of its members,

$$\max_{\{\pi^w_{kt}\}} \mathbb{E}_0 \int_0^\infty e^{-\rho t}\Big[u(C_t) - v(L_{kt}) - \tfrac{\psi}{2}(\pi^w_{kt})^2 L_t\Big]\, dt,$$

subject to the wage law of motion $\dot{W}_{kt} = \pi^w_{kt} W_{kt}$ and the labor-demand schedule that comes from CES aggregation,

$$L_{kt} = \big(W_{kt}/W_t\big)^{-\epsilon} L_t, \qquad L_t = \Big[\int_0^1 L_{kt}^{\frac{\epsilon-1}{\epsilon}}\, dk\Big]^{\frac{\epsilon}{\epsilon-1}}.$$

Taking W_t and L_t as given, the current-value Hamiltonian is

$$\mathcal{H}_{kt} = u(C_t) - v(L_{kt}) - \frac{\psi}{2}(\pi^w_{kt})^2 L_t + \lambda_{kt}\big(\pi^w_{kt} W_{kt} - \dot{W}_{kt}\big),$$

where λ_{kt} is the co-state variable on W_{kt}.

The first-order condition with respect to π^w_{kt} gives

$$0 = -\psi\, \pi^w_{kt} L_t + \lambda_{kt} W_{kt}. \tag{D.1}$$

The co-state equation is

$$\dot{\lambda}_{kt} - \rho \lambda_{kt} = v'(L_{kt})\frac{\partial L_{kt}}{\partial W_{kt}} = -\epsilon\, v'(L_{kt})\frac{L_{kt}}{W_{kt}}. \tag{D.2}$$

In a symmetric equilibrium $W_{kt} = W_t$, $L_{kt} = L_t$, $\pi^w_{kt} = \pi^w_t$, $\lambda_{kt} = \lambda_t$. Because each household owns the unions, λ_t equals the nominal marginal utility of income: $\lambda_t = W_t\, u'(C_t)$.

Using λ_t from (1) and substituting into (2) yields

$$-\psi\, \dot{\pi}^w_t L_t + \rho\psi\, \pi^w_t L_t = -\epsilon\, v'(L_t)L_t + (\epsilon-1)\, u'(C_t) W_t L_t.$$

Finally, dividing by the price level P_t to express real wages $w_t \equiv W_t/P_t$ and rearranging terms gives the desired Phillips Curve:

$$\epsilon\, v'(L_t)L_t - (\epsilon-1)\, u'(C_t)\, w_t\, L_t + \psi\big(\dot{\pi}_t^w - \rho\, \pi_t^w\big) = 0.$$

D.2 Calibration of private and public demand processes

The idiosyncratic components of public and private demand are modeled as mean-reverting processes driven by aggregate shocks. In continuous time, these evolve according to

$$\begin{aligned} dg_t &= -\mu_g g_t\, dt + dG_t, \\ dz_t &= -\mu_z z_t\, dt + dZ_t, \end{aligned}$$

where μ_g and μ_z denote the mean-reversion rates of the public and private demand shocks, respectively, and G_t and Z_t represent the corresponding aggregate components. To estimate these parameters, we use firm-level data on public procurement and private sales. For each firm i, we estimate the following autoregressive specifications in discrete time:

$$\log(Proc_{it}) = \alpha_i^g + \rho_g \log(Proc_{it-1}) + u_{it}^g, \tag{D.3}$$

$$\log(Sales_{it}) = \alpha_i^z + \rho_z \log(Sales_{it-1}) + u_{it}^z, \tag{D.4}$$

where ρ_g and ρ_z capture persistence, and u_{it}^g, u_{it}^z are innovation terms with variances σ_g^2 and σ_z^2. The estimated coefficients $\widehat{\rho}_g, \widehat{\rho}_z, \widehat{\sigma}_g^2$, and $\widehat{\sigma}_z^2$ are then converted to their continuous-time counterparts. To construct an yearly measure of public *sales*, we use a subset of our procurement data for which we can observe the timing of individual payments of procurement contracts, which allows us to estimate an autoregressive process for public revenues. We then construct our measure of private sales residually, by taking the difference between total sales, as reported from the income statement, and payments for the public sector received over that year.[43]

The mapping between discrete- and continuous-time parameters follows the standard relationship between an AR(1) process and an Ornstein–Uhlenbeck process. For each vari-

[43] We use payments rather than awards of public contracts because the distribution of public procurement contracts *awards* tends to be lumpy, as many contracts are spread over multiple years, which gives rise to unreliable estimates of the autoregressive coefficient and would also imply that our residual measure of private sales would be negative for many observations.

able $x \in \{g, z\}$, we compute

$$\mu_x = -\log(\widehat{\rho}_x),$$
$$\sigma_x = \sqrt{\frac{\widehat{\sigma}_x^2}{2\mu_x(1-\widehat{\rho}_x^2)}}.$$

These transformations yield continuous-time parameters (μ_z, σ_z) and (μ_g, σ_g) consistent with the estimated persistence and volatility in the data.

Table D.1: Results from (D.3) and (D.4)

	Private	Public
$\log Sales_{it-1}$	0.519***	
	(0.006)	
$\log Proc_{it-1}$		0.454***
		(0.004)
Observations	32551	60393
R-squared	0.36	0.20
Within R-squared	0.28	0.13
Firm FEs	✓	✓

Note: estimates for $\widehat{\rho}_g, \widehat{\rho}_z$ *from* (D.3) *and* (D.4).

D.3 Calibration of psychology parameters

Numerical identification strategy We calibrate the XDE parameters in the stationary model. To calibrate the two unknown psychology parameters χ and κ, we establish the following mapping between the model parameters and our empirical estimates

$$\mathcal{F} : (\chi, \kappa) \longmapsto (\beta, \rho),$$

where (β, ρ) are the coefficients that summarize, respectively, how forecast errors respond to realized sales shocks (which we map to our IV regression in Table 1) and the autocorrelation of forecast error. Given empirical targets $(\widehat{\beta}, \widehat{\rho})$, our strategy is to choose (χ, κ) so that the model-implied pair $F(\chi, \kappa)$ coincides with these targets.

Mapping model parameters to empirical targets. The mapping $\mathcal{F}$ is evaluated in the following steps:

1. *Choose a candidate parameter vector.* Select a guess (χ, κ)

2. *Simulate the state vector.* Over a monthly grid with step size $\Delta \equiv 1/12$, draw i.i.d. innovations $\varepsilon_t^g, \varepsilon_t^z \sim \mathcal{N}(0,1)$ and simulate for $t = 0, \Delta, \ldots, N-\Delta$:

$$\begin{aligned} g_{t+\Delta} &= g_t - \mu_g\, g_t \Delta + \sigma_g \sqrt{\Delta}\, \varepsilon_t^g, \\ z_{t+\Delta} &= z_t - \mu_z\, z_t \Delta + \sigma_z \sqrt{\Delta}\, \varepsilon_t^z, \\ \mathcal{S}_{t+\Delta} &= \mathcal{S}_t - \kappa\, \mathcal{S}_t \Delta + \sigma_z \sqrt{\Delta}\, \varepsilon_t^z + \sigma_g \sqrt{\Delta}\, \varepsilon_t^g. \end{aligned}$$

3. *Construct forecast errors.* At horizon $\tau \geq 0$ the perceived fundamentals are

$$\begin{aligned} \widetilde{z}_\tau &= z_0\, e^{-\mu_z \tau} + \frac{\chi_z\, \mathcal{S}_0}{\mu_z} \big(1 - e^{-\mu_z \tau}\big), \\ \widetilde{g}_\tau &= g_0\, e^{-\mu_g \tau} + \frac{\chi_g\, \mathcal{S}_0}{\mu_g} \big(1 - e^{-\mu_g \tau}\big), \end{aligned}$$

   whereas the *true* trajectories follow pure exponential decay, $z_\tau = z_0 e^{-\mu_z \tau}$ and $g_\tau = g_0 e^{-\mu_g \tau}$. Firm sales as a function of the two demand components are

$$Y(z, g) \;\propto\; \Big(e^z Y^p + G\,(P^G)^\varepsilon e^g\Big)^{\frac{1-\alpha}{1-\alpha+\alpha\varepsilon}}.$$

   The proportionality factor depends on model parameters and steady-state values that will be irrelevant for this exercise. Define the forecast-error series

$$\mathcal{E}_{t+\tau} \;=\; \frac{Y(g_{t+\tau}, z_{t+\tau})}{Y(\widetilde{g}_{t+\tau}, \widetilde{z}_{t+\tau})} \;-\; 1.$$

4. *Compute annual growth shocks.* Annual percentage growth is measured by

$$\Delta y_t \;=\; \frac{Y(z_t, g_t)}{Y\big(z_{t-12} e^{-12\mu_z}, g_{t-12} e^{-12\mu_g}\big)} - 1$$

5. *Estimate the empirical moments.* Run ordinary-least-squares regressions

$$\begin{aligned} \mathcal{E}_t &= \beta\, \Delta y_t + u_t, \\ \mathcal{E}_t &= \rho\, \mathcal{E}_{t-1} + \nu_t, \end{aligned}$$

Figure D.1: Identification of χ and κ. The solid (dashed) curve is the locus L_β (L_ρ) defined in (D.5). Their unique, transversal intersection delivers the identified parameter vector.

and to get the model estimates$(\widetilde{\beta}, \widetilde{\rho})$ and set $\mathcal{F}(\chi, \kappa) \equiv (\widetilde{\beta}, \widetilde{\rho})$

Given the mapping $\mathcal{F}$, the structural parameters are obtained by solving the nonlinear system

$$\mathcal{F}(\chi, \kappa) \; = \; (\widehat{\beta}, \widehat{\rho}).$$

In practice we apply a damped Newton–Raphson algorithm, which converges rapidly.

Identification diagnostics. To verify that the structural parameters are well identified, we study the zero–level sets of the mapping $\mathcal{F} = (\mathcal{F}_1, \mathcal{F}_2)$. Define the two loci

$$L_\beta \; = \; \{(\chi, \kappa) : \mathcal{F}_1(\chi, \kappa) = \widehat{\beta}\}, \qquad L_\rho \; = \; \{(\chi, \kappa) : \mathcal{F}_2(\chi, \kappa) = \widehat{\rho}\}, \tag{D.5}$$

where $\widehat{\beta}$ and $\widehat{\rho}$ are the empirical targets. The identified set is the intersection $L_\beta \cap L_\rho$.

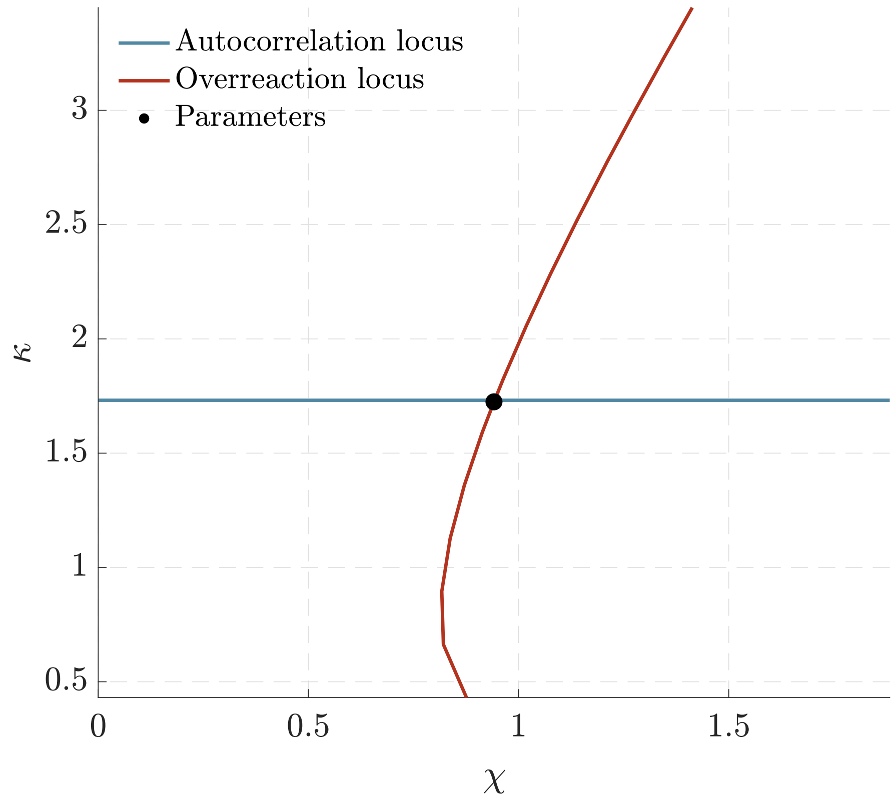
Figure D.2 plots the two curves. Two features confirm well-posed identification:

1. *Transversal intersection.* The curves cross with distinct slopes, so the Jacobian $D\mathcal{F}(\chi, \kappa)$ is nonsingular at the solution. Small perturbations of the target moments therefore lead to proportionally small changes in the estimated parameters, indicating local identification.

2. *Uniqueness in the economically relevant region.* Within a calibrated grid that contains reasonable parameter values, the curves intersect exactly once. Hence the mapping yields a single admissible parameter pair, establishing global identification over the domain explored.

Together, these properties indicate that the estimation problem is both well-posed and numerically stable: (χ, κ) are pinned down by $(\widehat{\beta}, \widehat{\rho})$ without ambiguity.

Figure D.2: Identification of XDE parameters



Note: The red line represents the set L_β and the blue line represents the set L_ρ in (D.5).

D.4 Algorithm

1. **Form an initial guess for the following sequences**

$$\left\{ w_t,\ r_t,\ Y_t^{p,\text{in}},\ P_t^{g,\text{in}} \right\}_{t=0}^{T}.$$

These vectors are the four blocks of the Newton search vector $\mathbf{x} \in \mathbb{R}^{4N}$ in the code.

2. **Solve firm block**

- Solve the Hamilton–Jacobi–Bellman equation (27) given the current guess for aggregate sequences to obtain policy functions $\iota_t(\boldsymbol{s}), \ell_t(\boldsymbol{s}), d_t(\boldsymbol{s})$.
- Update the sequence of distributions of firms $\{\mu_t(\boldsymbol{s})\}_t$ via the Kolmogorov-Forward equation (28).
- Aggregate to get $Y_t^{p,\text{out}}, I_t^{\text{gross}}, L_t^D, D_t^{\text{net}}, P_t^{g,\text{out}}$, and capital K_t.

3. **Solve government block** Impose the period-by-period balanced-budget condition

$$0 = r_t B_t + G_t - T_t$$

i.e. equation (30). Together with the guessed path for r_t and the computed G_t, this pins down the tax path $\{T_t\}$.

4. **Solve household block** Solve the representative household's problem (29), using the dividend flow D_t^{net}, taxes T_t, labor L_t^D, the wage rate w_t and interest rate path r_t. This yields consumption C_t, bond holdings A_t, and the bond flow $\dot{A}_t$.

5. **Solve union block** Backward-integrate the nonlinear Phillips curve

$$\epsilon\, v'(L_t)L_t - (\epsilon - 1)u'(C_t)w_t L_t + \psi\big(\dot{\pi}_t^w - \rho\, \pi_t^w\big) = 0,$$

This provides wage inflation π_t^w, price inflation π_t, nominal interest i_t, labor supply L_t^S, and the real wage w_t.

6. **Market-clearing**

$$\begin{aligned} \text{Goods:} \quad & Y_t = C_t + I_t + P_t^g\, G_t, \\ \text{Labor:} \quad & L_t^S = L_t^D, \\ \text{Assets:} \quad & B_t = A_t, \end{aligned}$$

where I_t is the integral of investment and adjustment costs defined in Section 4.1. Compute the resulting excess-demand vectors.

7. **Update the guess and iterate.** Feed the excess demands into a solver; iterate on initial guess the norm of all residuals is below a chosen tolerance.

D.5 Aggregate response in forecast errors

The goal of this section is to establish a mapping from the distribution of idiosyncratic states to the average forecast error on total revenue at a given horizon τ

$$\mu(\mathcal{S}, g, z) \to \mathcal{E}_\tau$$

where the average forecast error is defined as the average across all states.

To construct this mapping we must first compute the expected forecast error for a firm with states $(\mathcal{S}, g, z)$. Such a firm expects the states to evolve according to

$$\dot{z}_t = -\mu_z z_t + \chi_z \mathcal{S}_t, \quad \dot{g}_t = -\mu_g g_t + \chi_g \mathcal{S}_t$$

when in fact the true evolution is given by

$$\dot{z}_t = -\mu_z z_t, \quad \dot{g}_t = -\mu_g g_t$$

Starting at $t = 0$, denote the forecast horizon by $\tau \geq 0$. Under the firm's subjective law of motion, the linear ODEs integrate to

$$\begin{aligned}
\widetilde{z}_\tau &= z_0 \, e^{-\mu_z \tau} + \chi_z \int_0^\tau e^{-\mu_z(\tau - s)} \, \mathcal{S}_0 \, ds, \\
\widetilde{g}_\tau &= g_0 \, e^{-\mu_g \tau} + \chi_g \int_0^\tau e^{-\mu_g(\tau - s)} \, \mathcal{S}_0 \, ds.
\end{aligned}$$

Which implies

$$\begin{aligned}
\widetilde{z}_\tau &= z_0 \, e^{-\mu_z \tau} + \frac{\chi_z \, \mathcal{S}_0}{\mu_z} \big(1 - e^{-\mu_z \tau}\big), \\
\widetilde{g}_\tau &= g_0 \, e^{-\mu_g \tau} + \frac{\chi_g \, \mathcal{S}_0}{\mu_g} \big(1 - e^{-\mu_g \tau}\big).
\end{aligned}$$

however in reality, the actual trajectories are purely exponential:

$$\begin{aligned}
z_\tau &= z_0 \, e^{-\mu_z \tau}, \\
g_\tau &= g_0 \, e^{-\mu_g \tau}
\end{aligned}$$

This allows us to construct the forecast error for a given firm starting with $(\mathcal{S}_0, g_0, z_0)$. The firm's total sales are a function of future idiosyncratic demands z_τ and g_τ as well as some aggregate variables $\boldsymbol{X}_\tau$: $Y_\tau = Y(\widetilde{g}_\tau, \widetilde{z}_\tau; \boldsymbol{X}_\tau)$. Therefore we can define the forecast error for $(\mathcal{S}_0, g_0, z_0)$ as

$$\mathcal{E}_\tau(\mathcal{S}, g, z) = \log Y(g_\tau, z_\tau; \boldsymbol{X}_\tau) - \log Y(\widetilde{g}_\tau, \widetilde{z}_\tau; \boldsymbol{X}_\tau)$$

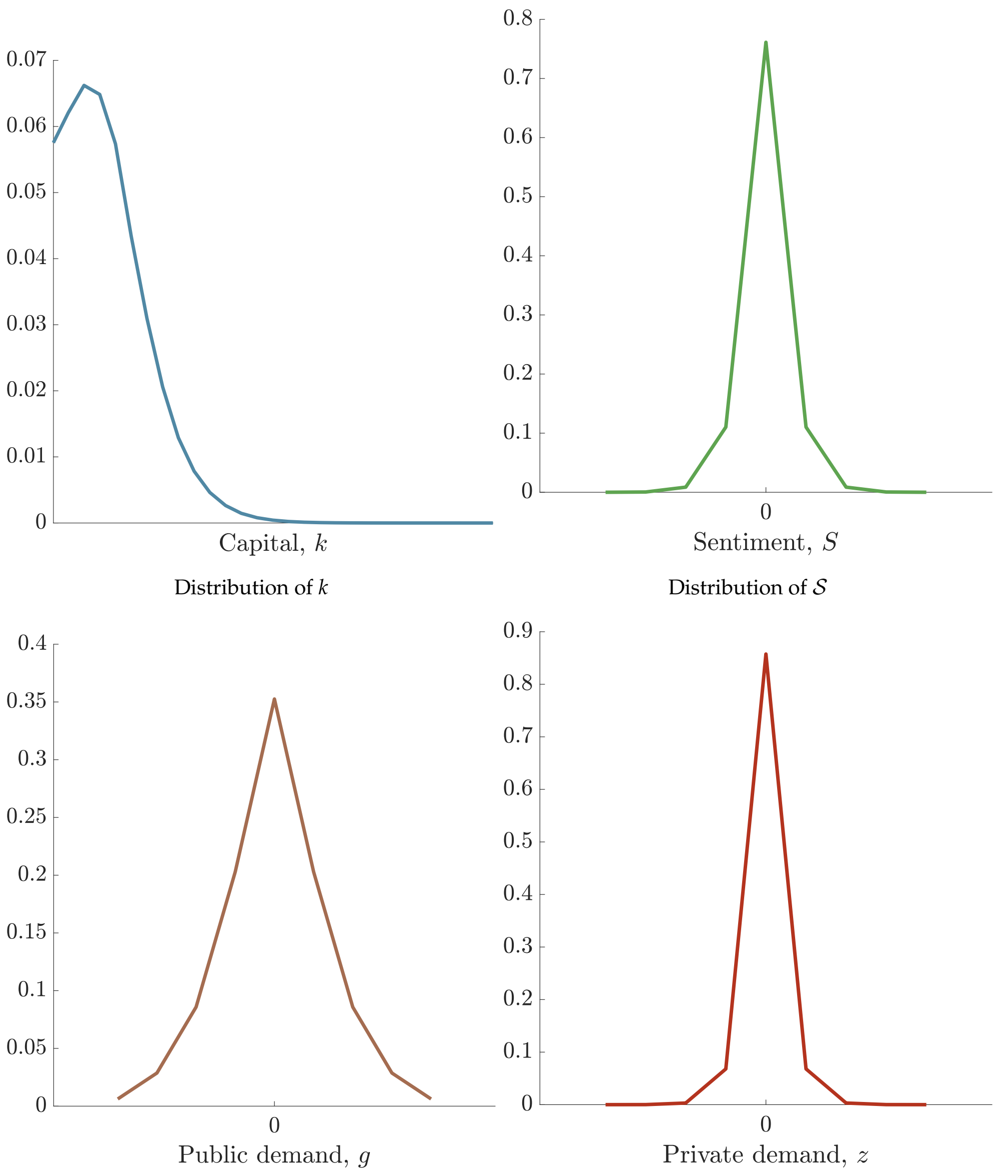
Integrating over the distribution gives the final mapping:

$$\mathcal{E}_\tau(\mu) = \int \mathcal{E}_\tau(\mathcal{S}, g, z) d\mu(\mathcal{S}, g, z)$$

D.6 Additional model figures

Figure D.3 plots the distribution of capital, sentiment, idiosyncratic public and private demand in the stationary equilibrium of our model. Figure D.4 plots the steady state investment policy functions for different levels of sentiment. Figure D.5 plots the distribution of the yearly growth rates of capital in the stationary equilibrium. Figure D.6 plots IRFs to the government spending shock considered in Figure 9 for additional variables. Figure D.7 leverages the decomposition in (37) to decompose the investment response to the government spending shock considered in Figure 9. Figure D.8 replicates Figure 11 in the model where the sentiment channel of fiscal policy is turned off, i.e., government spending does not affect sentiment. Figure D.12 plots the IRFs to the government spending shock considered in Figure 9 in a calibration of our model without financial frictions, while Figure D.10 plots the IRFs to the government spending shock considered in Figure 9 in a Real Business Cycle version of our model, i.e., a calibration with fully flexible wages. Finally, Figure D.13 plots the GIRFs for the experiment analyzed in Figure 11 for additional model variables.

Figure D.3: Stationary marginal distributions of idiosyncratic states



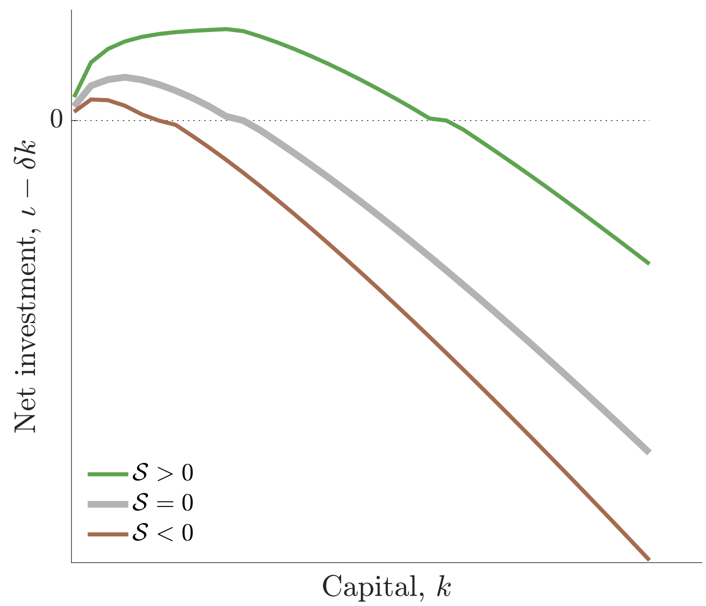
Distribution of k

Distribution of $\mathcal{S}$

Distribution of g

Distribution of z

Figure D.4: Stationary Policy Functions for Investment



(a) Net investment policy function

Figure D.5: Distribution of growth rates of capital

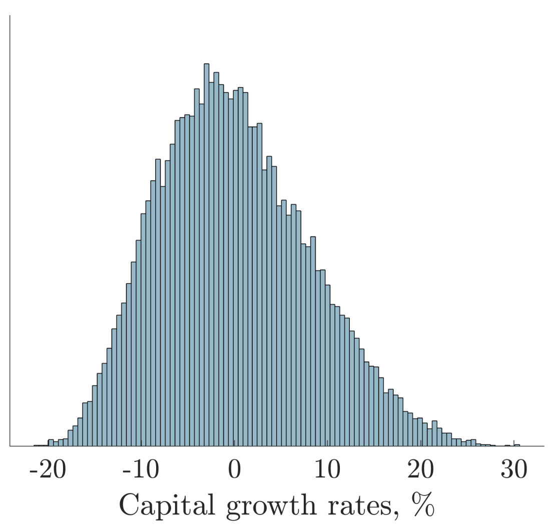
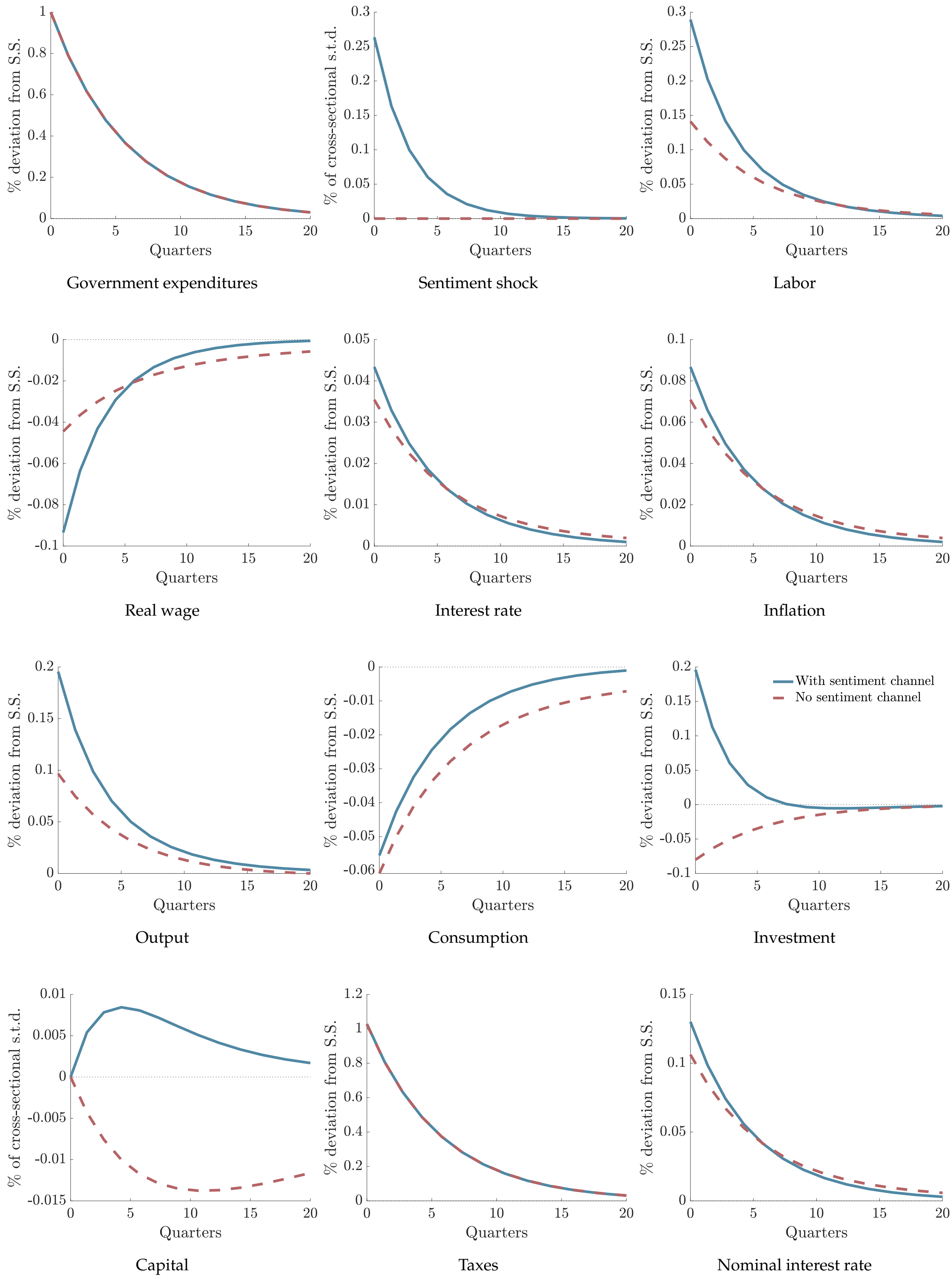


Figure D.6: The sentiment channel of fiscal policy



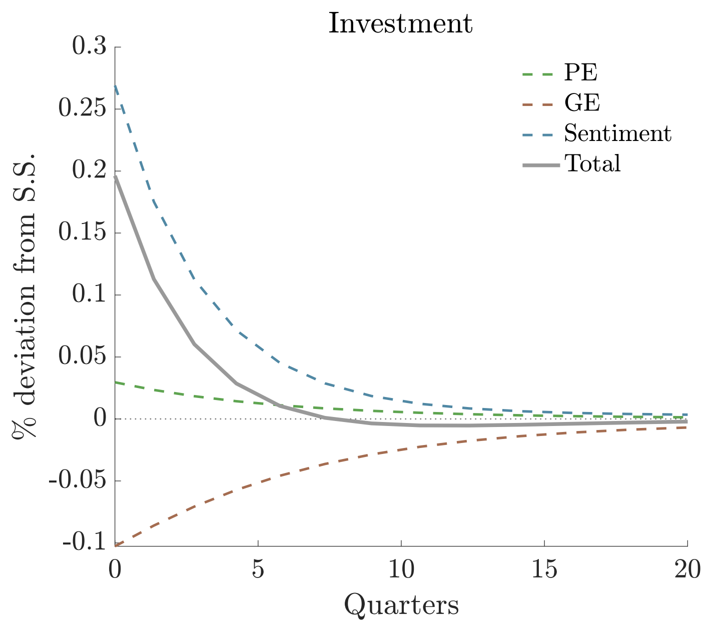


Figure D.7: Decomposition of the investment response

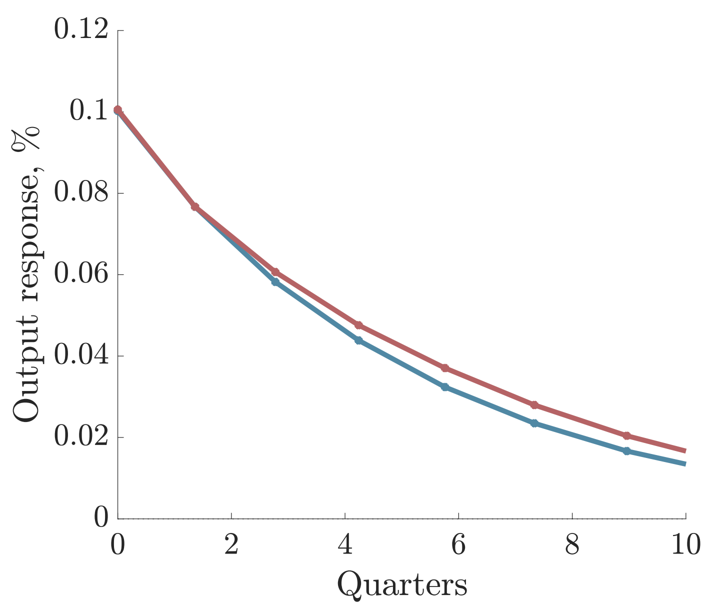


Figure D.8: Marginal effect of government expenditures without the sentiment channel (cost-push vs credit-crunch)

Figure D.9: The sentiment channel of fiscal policy without financial frictions

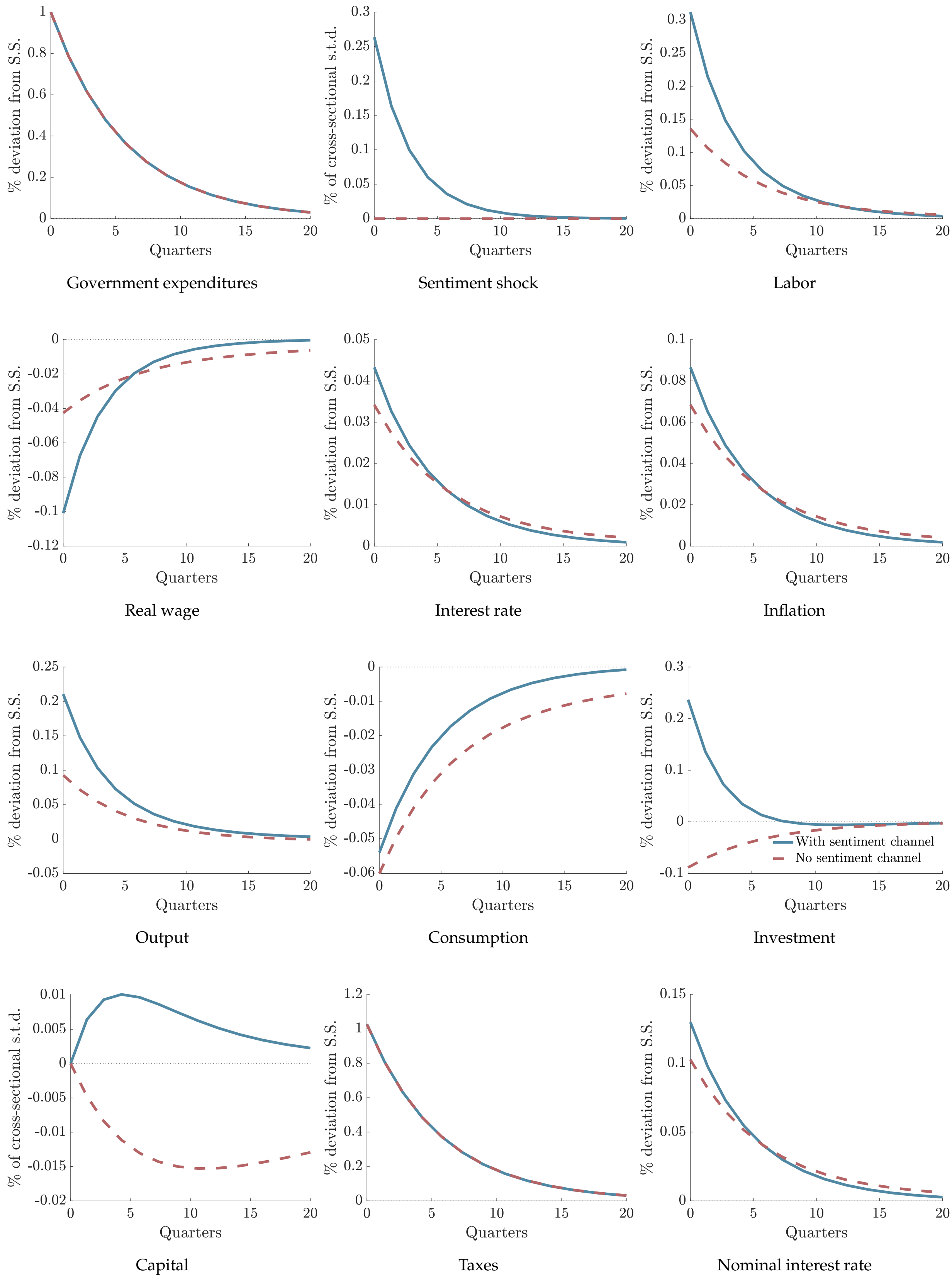


Figure D.10: The sentiment channel of fiscal policy without wage stickiness

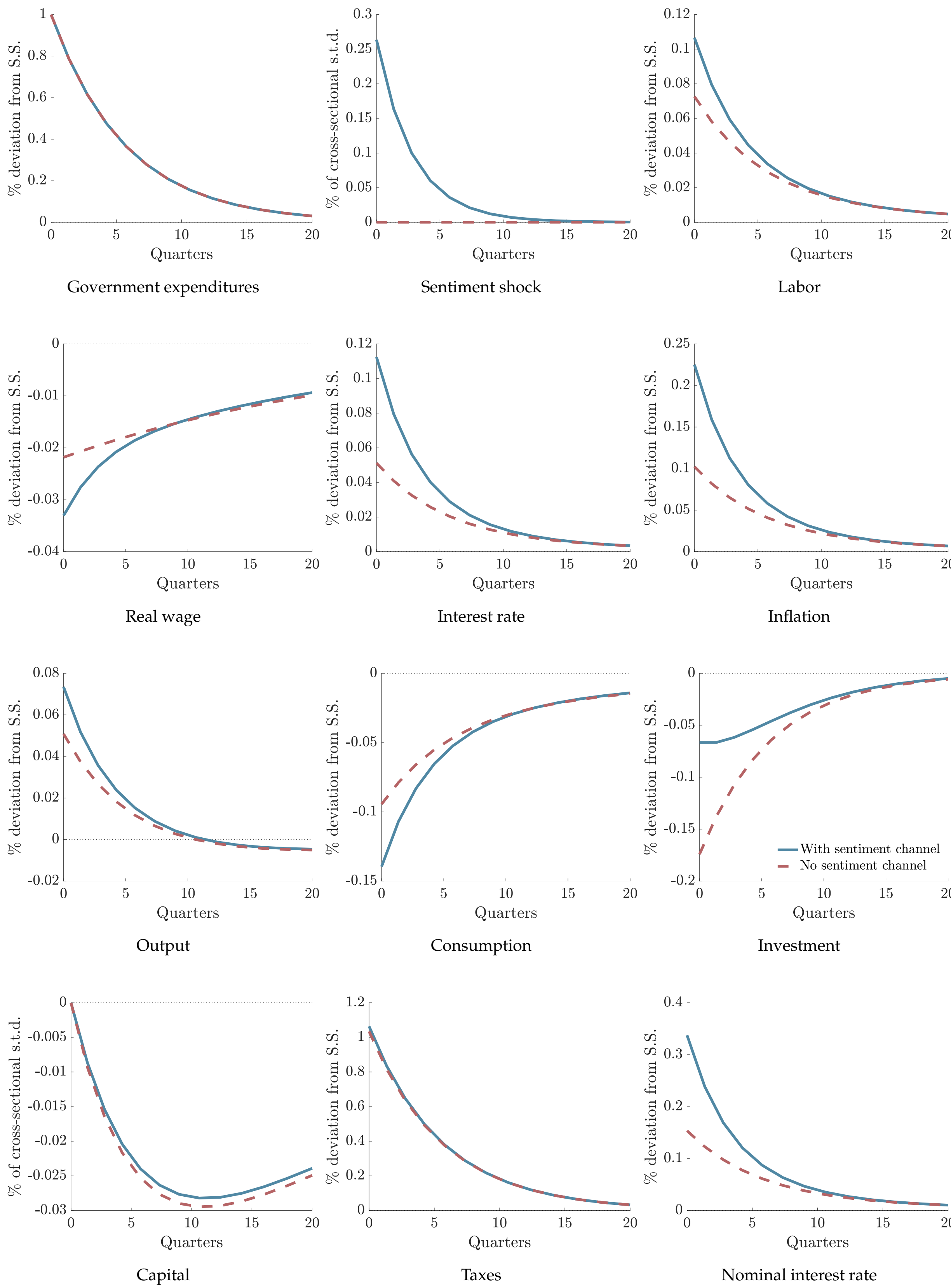


Figure D.11: The sentiment channel of fiscal policy with dovish central bank ($\phi_\pi = 1.2$)

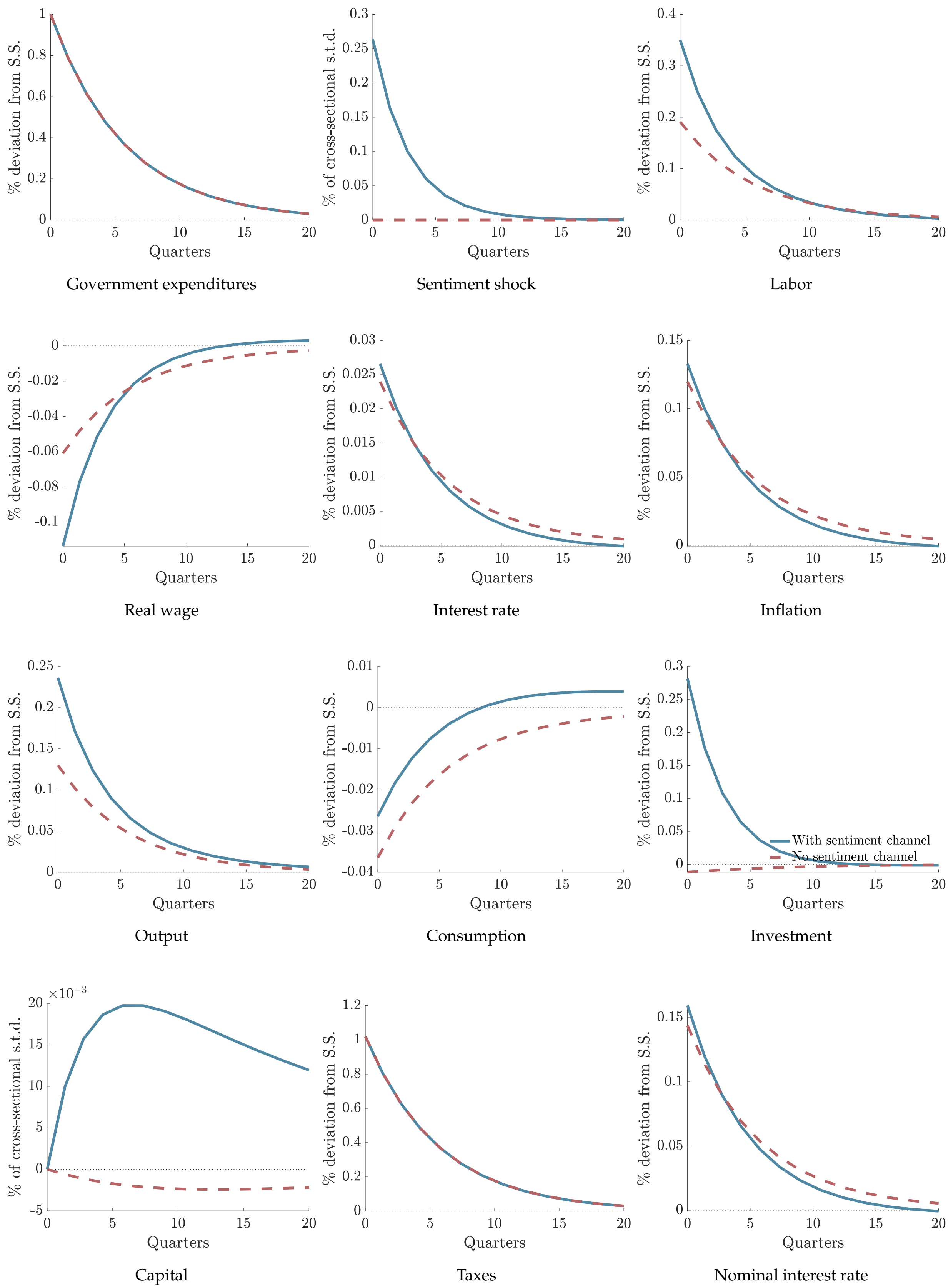


Figure D.12: The sentiment channel of fiscal policy with higher adjustment costs ($\phi' = 4\phi$)

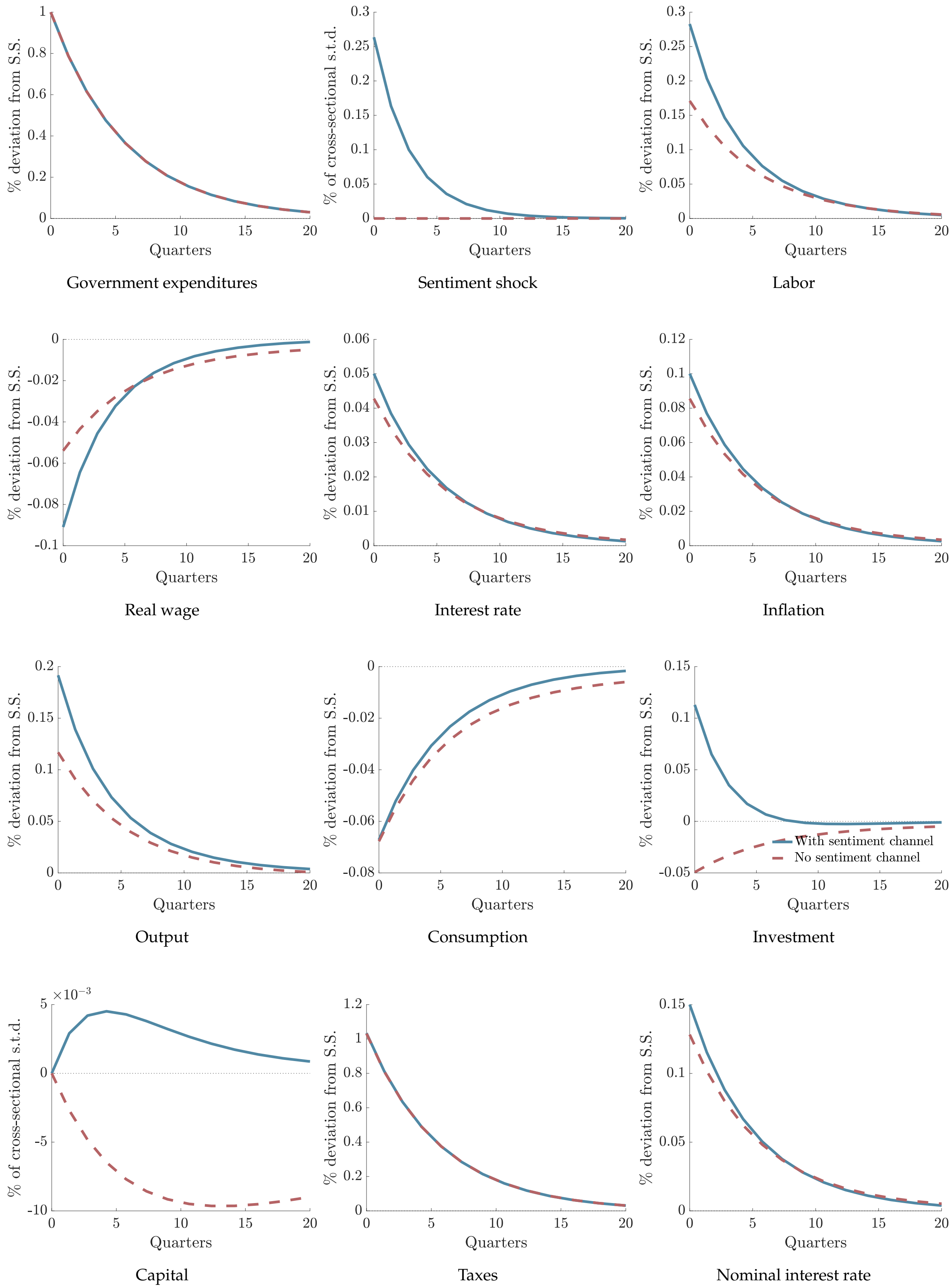
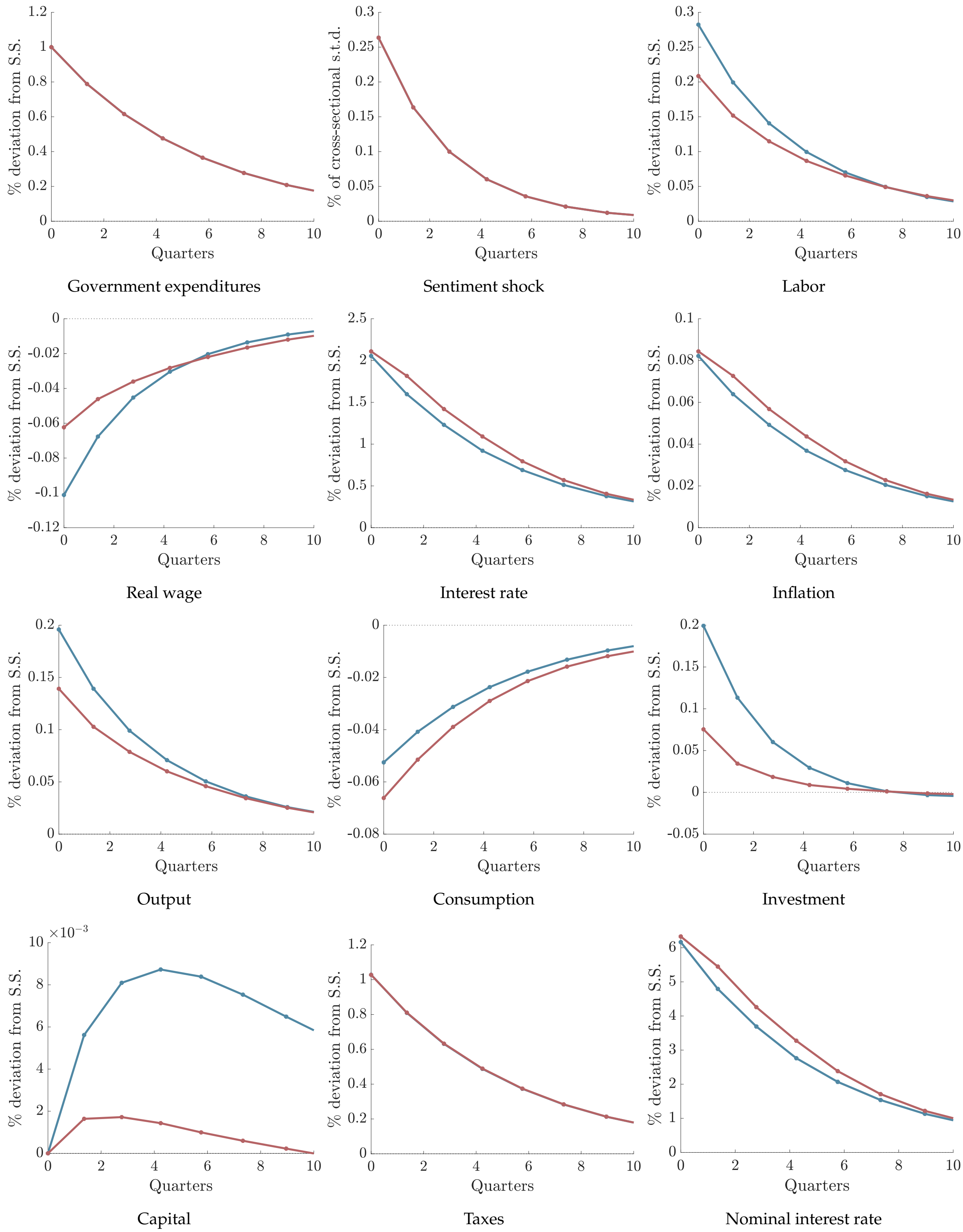


Figure D.13: Generalized IRFs, credit crunch and cost-push shock



D.7 Details on Figure 10

We now describe more in detail how we construct the confidence bands for the multiplier estimates in Figure 10. We focus on the three methodologies replicated in Ramey (2016): the Blanchard and Perotti (2002) timing restriction, the Ramey (2011b) news shock and the defence news shock by Ben Zeev and Pappa (2017). Following Ramey (2019), we plug the shocks in a structural VAR containing real government spending per capita, real GDP per capita, real federal tax revenues per capita, the inflation rate and the 3-month treasury bill rate. Data are at quarterly frequency and we include four lags of each variable. The estimation sample is 1939q1-2015q1, as in Ramey (2019). We then estimate the response of GDP and government expenditures to each shock for up to 16 quarters ahead and compute the 4-year cumulative multiplier using the standard formula as the ratio of the present discounted value of the output response over the present discounted value of the government spending response.[44] We compute 68% bootstrap confidence bands for the multiplier based on 1,000 draws.

[44]We follow Ramey (2019) and use 1.009 as the gross quarterly discount rate.