Chapter 3:-

Parking Space problem

$$ED = \sum_{i=1}^{10}(11-i)(1-0.15)^{i-1}0.15 + \sum_{i=11}^{\infty}(i-11)0.85^{i-1}0.15$$

Properties of Geometric Series

$$\sum_{i=1}^{\infty} i t^{i-1} = \frac{1}{(1-t)^2} \qquad \sum_{i=0}^{\infty} t^i = \frac{1}{1-t}$$

$t = i - 11$

$$\sum_{t=0}^{\infty} t \, 0.85^{t+10} \, 0.15$$

$$0.15 \times 0.85^{10} \sum_{t=0}^{\infty} t \, 0.85^{t}$$

$$0.15 \times 0.85^{11} \sum_{t=0}^{\infty} t \, 0.85^{t-1} = \frac{0.15 \times 0.85^{11}}{(1-0.85)^2}$$

Q. 15

$$P(x) = \binom{n}{x} p^{x} (1-p)^{n-x}$$

What is $P(X=\frac{n}{2})$?

$$P(X=\frac{n}{2}) = \binom{n}{n/2} p^{n/2} (1-p)^{n/2}$$

For $p = 0.5$

$$= \binom{n}{n/2} 0.5^{n}$$

$$\binom{n}{r} = \frac{n!}{(n-r)!\,r!} \qquad = \frac{n!}{n/2!\; n/2!}\, 0.5^{n}$$

$$= \frac{n!}{(n/2!)^{2}}\, 0.5^{n}$$

Using :-

$$k! \approx \sqrt{2\pi k}\left(\frac{k}{e}\right)^{k}$$

$$= \frac{\sqrt{2\pi n}\left(\frac{n}{e}\right)^{n}}{2\pi \frac{n}{2}\left(\frac{n}{2e}\right)^{n}}\, 0.5^{n}$$

$$\frac{\sqrt{x}}{x} = \frac{1}{\sqrt{x}}$$

$$x^{\frac{1}{2}} \times x^{-1} = x^{-\frac{1}{2}}$$

$$= \frac{2}{\sqrt{2\pi n}} \left(\frac{\cancel{n}}{\cancel{e}} \times \frac{2\cancel{e}}{\cancel{n}}\right)^{n} 0.5^{n}$$

$$= \frac{2}{\sqrt{2\pi n}}\, \cancel{2^{n}\, 0.5^{n}} = \frac{2}{\sqrt{2\pi n}}$$

Q.10 $Z = Y_6 - X_6$

Let X_k denote the number of heads Jack has gotten through his k^{th} toss, and let Y_k be the head count for Jill at that same time, i.e. among only k-2 tosses for her. (So, $Y_1 = Y_2 = 0$.) Let's find the probability that Jill is winning after the 6^{th} toss, i.e. $P(Y_6 > X_6)$.

$$P(X_6 = i) = \binom{6}{i} 0.5^i \, 0.5^{6-i}$$

$$P(Y_6 = i) = \binom{4}{i} 0.5^i \, 0.5^{4-i}$$

$$P(Z = i) = \sum_{k=i}^{4} P(Y_6 = k) P(X_6 = k-i)$$

Work this through or use (3.4 9) & (3.13)

Q.22 Properties of Poisson &.

Maclaurin series

$$P(X = k) = \frac{e^{-\lambda}\lambda^k}{k!}, k = 0, 1, 2, ...$$

$$e^t = \sum_{i=0}^{\infty} \frac{t^i}{i!}$$

$$EX = \lambda$$

$$Var(X) = \lambda$$

$P(M \mid N) \sim$ binomial (N, p)

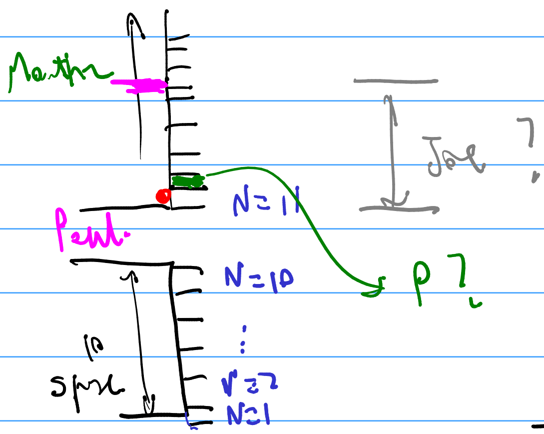
by marginalization Find $P(M)$

$$\lim_{n\to\infty} \frac{2}{\sqrt{2\pi n}} = 0$$

Q.E.D.

3.12.3.2

$$P(B|A) = \frac{P(A \text{ and } B)}{P(A)}$$



$P(A \text{ and } B) = P(A)P(B|A)$

$P(N = 12 \mid N > 10 \text{ and } N < 16)$

$$P(N=12 \;\&\; 10<N<16)$$
$$= P(N=12).$$
$$P(10<N<16) =$$
$$P(11)+P(12)+P(13)+\dots P(15)$$

Solved in notebook