$$S_s = \sum_{i=1}^{\infty} t^i$$

a) $$S_s = t + t^2 + t^3 + t^4 + t^5 + t^6 \dots + t^s$$

$$S_s = t(1 + t + t^2 + \dots + t^{s-1})$$

$$S_s = t(1 + S_{s-1})$$

$$S_s = t + t\, S_{s-1}$$

$$= t + t\, S_s - t\, t^s$$

$$S_s = \frac{t - t^{s+1}}{1-t}$$

$$S_{rs} = t^r + t^{r+1} + \dots + t^s$$

$$= t^r(1 + t^1 + \dots + t^{s-r})$$

$$= t^r(1 + S_{s-r})$$

$$= t^r\left(\frac{1 - \cancel{t} + \cancel{t} - t^{s-r+1}}{1-t}\right) = \boxed{t^r\frac{(1 - t^{s-r+1})}{1-t}}$$

b) $$\lim_{s\to\infty} S_s = \lim_{s\to\infty} \frac{t - t^{s+1}}{1-t} = \frac{t}{1-t}$$

$$\sum_{i=0}^{\infty} t^i = \frac{t}{1-t} + 1 = \frac{1}{1-t}$$

c) $$\sum_{i=1}^{\infty} i t^i = ?$$

$$\frac{\partial S_\infty}{\partial t} = \sum_{i=1}^{\infty} i\, t^{i-1} = \frac{1}{1-t} + \frac{t}{(1-t)^2}$$

$$= \frac{1}{(1-t)^2}$$

$$EW = \sum_{i=1}^{\infty} i(1-p)^{i-1}p = \frac{1}{p}$$ Following the result in b

$$E(W^2) = \sum_{i=1}^{\infty} i^2 (1-p)^{i-1} p$$

We now try to obtain this by differentiating EW

$$\frac{\partial EW}{\partial p} = \sum_{i=1}^{\infty} i(1-p)^{i-1} - \sum_{i=1}^{\infty} i(i-1)(1-p)^{i-2} p$$

$$= \frac{1}{p^2} - \sum_{i=1}^{\infty} i^2 (1-p)^{i-2} p + \sum_{i=1}^{\infty} i(1-p)^{i-2} p$$

$$= \frac{1}{p^2} - \frac{E(W^2)}{1-p} + \frac{1}{p(1-p)} = -\frac{1}{p^2}$$

Differentiating the RHS of EW

$$\frac{E(W^2)}{1-p} = \frac{2}{p^2} + \frac{1}{p(1-p)}$$

$$= \frac{2-2p+p}{p^2(1-p)}$$

$$E(W^2) = \frac{2-p}{p^2}$$

$$Var(W) = E(W^2) - (EW)^2 = \frac{2-p}{p^2} - \frac{1}{p^2} = \boxed{\frac{1-p}{p^2}}$$

Q.E.D.