

a)

$$S_s = \sum_{i=1}^s t^i$$

$$S_s = t + t^2 + t^3 + t^4 + t^5 + t^6 \dots + t^s$$

$$S_s = t(1 + t + t^2 + \dots + t^{s-1})$$

$$S_s = t(1 + S_{s-1})$$

$$S_s = t + t S_{s-1}$$

$$= t + t S_s - t t^s$$

$$S_s = \frac{t - t^{s+1}}{1-t}$$

$$S_{rs} = t^r + t^{r+1} + \dots + t^s$$

$$= t^r (1 + t^1 + \dots + t^{s-r})$$

$$= t^r (1 + S_{s-r})$$

$$= t^r \left( \frac{1 - t + t - t^{s-r+1}}{1-t} \right) = t^r \frac{(1 - t^{s-r+1})}{1-t}$$

b)

$$\lim_{s \rightarrow \infty} S_s = \lim_{s \rightarrow \infty} \frac{t - t^{s+1}}{1-t} = \frac{t}{1-t}$$

$$\sum_{i=0}^{\infty} t^i = \frac{t}{1-t} + 1 = \frac{1}{1-t}$$

c)

$$\sum_{i=1}^{\infty} i t^i = ?$$

$$\frac{\partial S_{\infty}}{\partial t} = \sum_{i=1}^{\infty} i t^{i-1} = \frac{1}{1-t} + \frac{t}{(1-t)^2}$$

$$= \frac{1}{(1-t)^2}$$

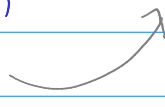
$$EW = \sum_{i=1}^{\infty} i (1-p)^{i-1} p = \frac{1}{p}$$

Following the result in 6

$$E(W^2) = \sum_{i=1}^{\infty} i^2 (1-p)^{i-1} p$$

We now try to obtain this by differentiating  $EW$

$$\begin{aligned} \frac{\partial EW}{\partial p} &= \sum_{i=1}^{\infty} i (1-p)^{i-1} - \sum_{i=1}^{\infty} i (i-1) (1-p)^{i-2} p \\ &= \frac{1}{p^2} - \sum_{i=1}^{\infty} i^2 (1-p)^{i-2} p + \sum_{i=1}^{\infty} i (1-p)^{i-2} p \\ &= \frac{1}{p^2} - \frac{E(W^2)}{1-p} + \frac{1}{p(1-p)} = -\frac{1}{p^2} \end{aligned}$$

Differentiating the RHS of  $EW$  

$$\frac{E(W^2)}{1-p} = \frac{2}{p^2} + \frac{1}{p(1-p)}$$

$$= \frac{2 - 2p + p}{p^2(1-p)}$$

$$E(W^2) = \frac{2-p}{p^2}$$

$$\text{Var}(W) = E(W^2) - (EW)^2 = \frac{2-p}{p^2} - \frac{1}{p^2} = \boxed{\frac{1-p}{p^2}}$$

Q.E.D.