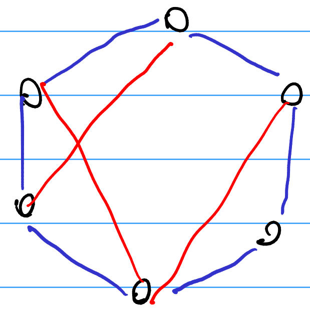
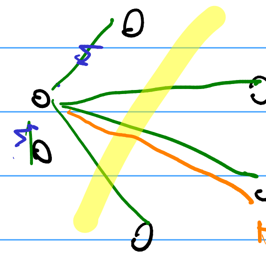
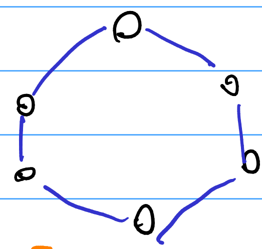
— original
— shortcuts



$6-3=3$



$$\boxed{\binom{n}{2}-n}$$



number of possible additional connection from a node to any other node

$$p_M(r)=\frac{\binom{n-3}{r-2}\binom{n^2/2-3n/2-(n-3)}{k-(r-2)}}{\binom{n^2/2-3n/2}{k}}=\frac{\binom{n-3}{r-2}\binom{n^2/2-5n/2+3}{k-(r-2)}}{\binom{n^2/2-3n/2}{k}}$$

extra connections

number of possible shortcuts cut

$$E(W^2) = p \sum_{i=1}^{\infty} i^2 (1-p)^{i-1}$$

We know that a geometric series looks like

$$a + ar + ar^2 + \dots = \sum_{i=0}^{\infty} a r^i$$

$$EW = p\boxed{\sum_{i=1}^{\infty} i(1-p)^{i-1}}^{\frac{1}{p^2}} = \frac{1}{p}$$

$$\frac{\partial EW}{\partial i} = p\left(\sum_{i=1}^{\infty} (1-p)^{i-1} + (i-1)\, i\, (1-p)^{i-2}\right)$$

$$p\left(\sum_{i=1}^{\infty} (1-p)^{i-1} + \sum_{i=1}^{\infty} i^2 (1-p)^{i-2} - \sum_{i=1}^{\infty} i\,(1-p)^{i-2}\right)$$

$$\frac{1}{p}$$

$$\frac{1}{1-p}\sum i\,(1-p)^{i-1}$$

$$\frac{1}{p(1-p)}E(W^2)$$

$$\frac{1}{(1-p)\,p^2}$$

$$= 1 + \frac{1}{(1-p)}E(W^2) - \frac{1}{(1-p)\,p} = 0$$

$$E(W^2) = \left(\frac{1}{(1-p)p} - 1\right)(1-p)$$

$$\frac{1}{p} - (1-p)$$

$$= \frac{1}{p} - 1 - p$$

$$= \frac{1-(1-p)p}{\cancel{(1-p)}\,p} \times \cancel{(1-p)} =$$

$$\cancel{VAR = \frac{1}{p} - 1 - p - \frac{1}{p^2} = \frac{p}{p^2} - \frac{p^2}{p^2} - \frac{p^3}{p^2}}$$

WRONG

We did a mistake. we differentiated with respect to i. Instead we should differentiate with respect to p.

$$EW = p\,\overset{\frac{1}{p^2}}{\boxed{\sum_{i=1}^{\infty} i(1-p)^{i-1}}} = \frac{1}{p}$$

$$\frac{\partial EW}{\partial p} = \underbrace{\sum_{i=1}^{\infty} i(1-p)^{i-1}}_{\frac{1}{p^2}} + p\sum_{i=1}^{\infty} i\,(i-1)\,(1-p)^{i-2}$$

$$+\,p\underbrace{\sum_{i=1}^{\infty} i^2 (1-p)^{i-2}}_{\frac{E(W^2)}{(1-p)}} - p\underbrace{\sum_{i=1} i\,(1-p)^{i-2}}_{\frac{1}{(1-p)p}}$$

$$= \frac{1}{p^2} + \frac{E(W^2)}{(1-p)} - \frac{1}{(1-p)p} = -\frac{1}{p^2}$$

$$\frac{E(W^2)}{(1-p)} = \frac{1}{(1-p)p} - \frac{2}{p^2} = \frac{p^2 - 2(1-p)p}{(1-p)p^3}$$

$$E(W^2) = \frac{1}{p} - \frac{2(1-p)}{p^2} \qquad E(W^2) = \frac{p - 2(1-p)}{p^2}$$

$$E(W^2) = \frac{1}{p} - \frac{2(1-p)}{p^2}$$

$$= \frac{p - 2 + 2p}{p^2} = \frac{3p - 2}{p^2}$$

$$VAR = E(W^2) - (EW)^2$$

$$= \frac{3p - 2}{p^2} - \frac{1}{p^2} =$$