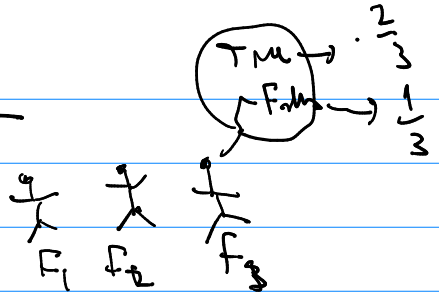
Quiz



F_T^i = Friend i is telling the truth
F_F^i = Friend i is not telling the truth

Meta Info

R = It is raining in London
S = It is sunny in London
fR_i = Friend i says it will rain
FS_i = Friend i says it is sunny

Specific to our problem

Let probability enter:-

$$P(F_T^i) = 1 - P(F_F^i) \quad \forall i$$

$$P(R) = 1 - P(S)$$

What do I want to know?

$$P(R \mid FS_1, FS_2, FS_3)$$

What do I know

$$P(R) = 0.2$$

$$P(FS_i \mid R) = \frac{1}{3} \quad \forall i \quad \leftrightarrow \quad P(FR_i \mid S) = \frac{1}{3}$$

$$P(FS_i \mid S) = \frac{2}{3} \quad \forall i \quad \leftrightarrow \quad P(FR_i \mid R) = \frac{2}{3}$$

$P(A|B) = \frac{P(B|A)P(A)}{P(B)}$

What do I want to know?

$$P(R \mid FS_1\ FS_2\ FS_3) = \frac{P(FS_1 FS_2 FS_3 \mid R)\, P(R)}{P(FS_1 FS_2 FS_3)}$$

What do I know

$P(R) = 0.2$

$P(FS_i \mid R) = \frac{1}{3}$ $\forall i$ $\quad P(FR_i \mid S) = \frac{1}{3}$

$P(FS_i \mid S) = \frac{2}{3}$ $\forall i$ $\quad P(FR_i \mid R) = \frac{2}{3}$

$$P(FS_1 FS_2 FS_3 \mid R) = P(FS_1 \mid R)\, P(FS_2 \mid R)\, P(FS_3 \mid R)$$
$$= \left(\frac{1}{3}\right)^3 = \frac{1}{27}$$

$$P(FS_1 FS_2 FS_3) = P(FS_1 FS_2 FS_3 \mid R)\, P(R) + P(FS_1 FS_2 FS_3 \mid S)\, P(S)$$

$P(FS_1 FS_2 FS_3 R) = \frac{1}{27 \times 5}$

$$= \frac{1}{27} \times \frac{1}{5} + \frac{8}{27} \cdot \frac{4}{5}$$

$$= \frac{1}{5}\left(\frac{1}{27} + \frac{32}{27}\right) = \frac{1}{5}\left(\frac{33}{27}\right)$$

$$= \frac{1}{5} \times \frac{11}{9} = \frac{11}{45}$$

$$P(R \mid FS_1\ FS_2\ FS_3) = \frac{\frac{9}{27} \times \frac{9}{5}}{11/45}$$

$$= \frac{1}{3} \times \frac{45}{11} = \frac{1}{33} \approx 3\%$$

REVISION

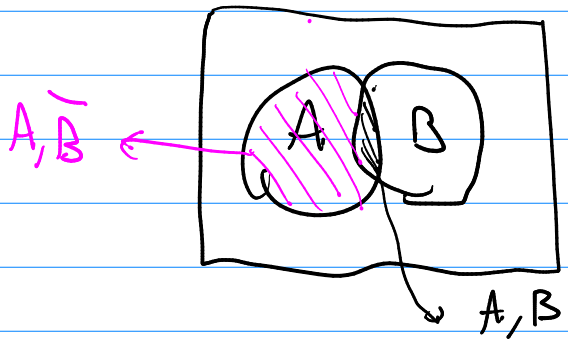
If I Know $P(A|B)$ and $P(A|\bar{B})$ $P(B) + P(\bar{B}) = 1$

what is $P(A) =$

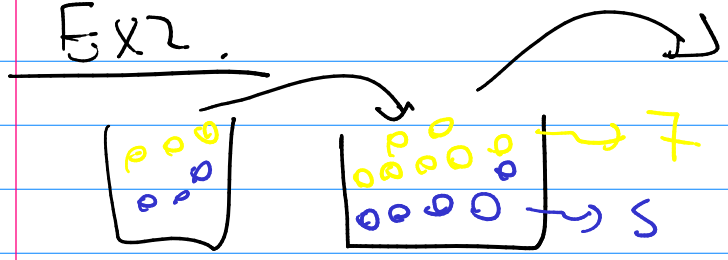
$$P(A) = P(A, B) + P(A, \bar{B})$$

AND AND

MARGINALISATION



Ex2.



2B = URN II

draw in

blue

$P(2B)$

$P(2B \mid 1B) = \frac{6}{13}$

$P(2B \mid 1Y) = \frac{5}{13}$

$P(2B, 1B) + P(2B, 1Y) = P(2B)$

$P(2B \mid 1B)P(1B) + P(2B \mid 1Y)P(1Y)$

$= \left(\frac{6}{13} + \frac{5}{13}\right)\frac{1}{2} = \frac{11}{26}$