Linear Algebra Workbook

Transposes

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TRANSPOSES AND THEIR DETERMINANTS

1. Find the transpose A^T.

$$A = [5 \quad 6 \quad 0 \quad 7 \quad 5 \quad -7]$$

2. Find the transpose A^T.

$$A = \begin{bmatrix} 7 & 9 & -6 \\ 0 & -1 & 9 \end{bmatrix}$$

3. Find the transpose A^T.

$$A = \begin{bmatrix} -4 & -7 \\ 5 & 1 \\ 7 & -2 \\ 4 & -2 \end{bmatrix}$$

4. Find the determinant of the transpose of A.

$$A = \begin{bmatrix} 5 & 3 & 6 & -1 \\ 9 & 0 & 1 & -2 \\ 8 & -2 & -4 & 8 \\ 5 & 4 & 9 & 7 \end{bmatrix}$$

5. Find the determinant of the transpose of A.

$$A = \begin{bmatrix} -9 & -3 & -1 \\ -4 & 7 & 3 \\ -4 & 8 & 7 \end{bmatrix}$$

6. Find the determinant of the transpose of A.

$$A = \begin{bmatrix} -8 & 6 & 8 \\ 3 & -9 & -1 \\ 4 & -9 & 9 \end{bmatrix}$$

TRANSPOSES OF PRODUCTS, SUMS, AND INVERSES

1. Find $(AB)^T$.

$$A = \begin{bmatrix} -1 & 2 \\ 2 & 3 \end{bmatrix}$$

$$B = \begin{bmatrix} -3 & -2 \\ 1 & 2 \end{bmatrix}$$

2. Find $(AB)^T$.

$$A = \begin{bmatrix} -1 & 2 & -2 \\ 2 & 3 & 1 \\ 3 & -3 & 1 \end{bmatrix}$$

$$B = \begin{bmatrix} 2 & -4 & 1 \\ 0 & -3 & -2 \\ -1 & 1 & 2 \end{bmatrix}$$

3. Find $(X + Y)^T$.

$$X = \begin{bmatrix} 4 & 1 \\ -2 & 0 \end{bmatrix}$$

$$Y = \begin{bmatrix} -3 & 2 \\ 0 & -1 \end{bmatrix}$$

4. Find $(X+Y)^T$.

$$X=\begin{bmatrix}2 & 0 & 3\\ 4 & 1 & -1\\ -2 & 0 & 3\end{bmatrix}$$

$$Y=\begin{bmatrix}-1 & 2 & -3\\ 0 & -1 & 2\\ 4 & -1 & 0\end{bmatrix}$$

5. Find $(X^T)^{-1}$.

$$X=\begin{bmatrix}1 & -2\\ 2 & 3\end{bmatrix}$$

6. Find $(A^T)^{-1}$.

$$A=\begin{bmatrix}4 & 1 & -3\\ 1 & 2 & 1\\ 0 & -1 & 4\end{bmatrix}$$

NULL AND COLUMN SPACES OF THE TRANSPOSE

1. Find the null and column spaces of the transpose M^T, identify their spaces $\mathbb{R}^i$, and name the dimension of the subspaces.

$$M = \begin{bmatrix} -1 & 0 \\ 2 & 4 \\ -2 & -2 \\ 0 & 4 \end{bmatrix}$$

2. Find the row space and left null space of A, and the dimensions of those spaces.

$$A = \begin{bmatrix} 1 & 2 \\ 0 & 1 \\ -4 & 0 \end{bmatrix}$$

3. Find the row space and left null space of B, and the dimensions of those spaces.

$$B = \begin{bmatrix} 2 & 3 & 1 & 0 \\ 1 & -2 & -1 & 4 \\ 0 & 0 & 2 & -2 \end{bmatrix}$$

4. Find the row space and left null space of C, and the dimensions of those spaces.

$$C = \begin{bmatrix} -1 & 2 & 0 \\ 1 & 4 & 3 \\ 0 & 0 & 3 \end{bmatrix}$$

5. Find the row space and left null space of A, and the dimensions of those spaces.

$$A = \begin{bmatrix} 1 & 3 \\ -3 & 1 \\ 0 & -2 \end{bmatrix}$$

6. Find the null and column subspaces of the transpose M^T, identify their spaces $\mathbb{R}^i$, and name the dimension of the subspaces of M^T.

$$M = \begin{bmatrix} 2 & 4 \\ 1 & 0 \\ -1 & -1 \\ 0 & 3 \end{bmatrix}$$

THE PRODUCT OF A MATRIX AND ITS TRANSPOSE

1. Is A^TA invertible?

$$A = \begin{bmatrix} 1 & -2 \\ 0 & 2 \\ 3 & 0 \end{bmatrix}$$

2. Is A^TA invertible?

$$A = \begin{bmatrix} -12 & 6 \\ 8 & -4 \end{bmatrix}$$

3. Is A^TA invertible?

$$A = \begin{bmatrix} 1 & 1 & -2 \\ 0 & 3 & 2 \\ 1 & 0 & -2 \end{bmatrix}$$

4. Is A^TA invertible?

$$A = \begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & 3 \end{bmatrix}$$

5. Is A^TA invertible?

$$A = \begin{bmatrix} 4 & -2 \\ -6 & 3 \end{bmatrix}$$

6. Is A^TA invertible?

$$A = \begin{bmatrix} -1 & 0 & 2 \\ 0 & 3 & 3 \end{bmatrix}$$

A=LU FACTORIZATION

1. Rewrite the matrix A in factored LU form.

$$A = \begin{bmatrix} 2 & 4 \\ 12 & 21 \end{bmatrix}$$

2. Rewrite the matrix A in factored LU form.

$$A = \begin{bmatrix} -1 & 0 & 3 \\ -5 & 2 & 18 \\ -5 & -8 & 5 \end{bmatrix}$$

3. Rewrite the matrix A in factored LDU form, where D is the diagonal matrix that factors the pivots out of U.

$$A = \begin{bmatrix} 2 & 0 & 2 \\ 0 & 1 & 1 \\ -6 & 3 & 0 \end{bmatrix}$$

4. Rewrite the matrix A in factored LDU form, where D is the diagonal matrix that factors the pivots out of U.

$$A = \begin{bmatrix} -4 & 8 & 0 & 4 \\ 0 & 1 & 1 & 2 \\ 8 & -12 & 7 & -3 \\ -4 & 16 & 5 & 27 \end{bmatrix}$$

5. Rewrite the matrix A in factored LDU form, where D is the diagonal matrix that factors the pivots out of U.

$$A = \begin{bmatrix} 5 & -5 & -10 & 5 \\ 20 & -17 & -31 & 8 \\ 20 & -20 & -38 & 26 \\ -15 & 6 & 3 & 25 \end{bmatrix}$$

6. Rewrite the matrix A in factored LDU form, where D is the diagonal matrix that factors the pivots out of U.

$$A = \begin{bmatrix} 4 & 4 & 4 & 4 \\ 8 & 11 & 11 & 11 \\ 8 & 8 & 10 & 10 \\ 0 & 15 & 27 & 28 \end{bmatrix}$$


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