

# Algebra 2 Workbook

**Exponential and logarithmic functions** 



## WHAT IS A LOGARITHM?

■ 1. How would you read the logarithmic equation out loud?

$$\log_7 57 = y$$

■ 2. Rewrite the equation in logarithmic form.

$$64^{\frac{1}{2}} = 8$$

■ 3. Rewrite the equation in exponential form.

$$\log_u \frac{19}{20} = v$$

 $\blacksquare$  4. Use the general log rule to solve for x.

$$\log_5(125) = x$$

■ 5. Use the general log rule to solve the logarithm.

$$\log_8 64 = x$$

■ 6. Write the logari	ithm that answers this que	estion: To what power do you
have to raise 289 in		



## COMMON BASES AND RESTRICTED VALUES

■ 1. Is there something wrong with the logarithm? If so, what is it?

$$\log_{-5}(8)$$

■ 2. Is there something wrong with the logarithm? If so, what is it?

$$\log_5(-8)$$

■ 3. Is the following statement true or false? Why?

$$\log 5 = \log_{10} 5$$

- 4. How else can you write  $\log_e 7$ ?
- $\blacksquare$  5. Solve the logarithm for x.

$$log(10,000) = x$$

 $\blacksquare$  6. Solve the logarithm for x.

	. 5.		
ln	$(e^3)$	=	$\boldsymbol{\mathcal{X}}$



## **EVALUATING LOGS**

■ 1. Find the exact value of the logarithm.

$$\log_{\frac{1}{3}}\frac{1}{27}$$

■ 2. Find the exact value of the logarithm.

$$\log_9 \frac{1}{81}$$

■ 3. Find the value given by the log.

■ 4. Find the exact value of the logarithm.

$$log_{343} 7$$

■ 5. Find the exact value of the logarithm.

$$\log_{\frac{1}{4}}(1,024)$$



■ 6. Find the exact value of the logarithm.

$$\log_{64} \frac{1}{4}$$



## LAWS OF LOGARITHMS

■ 1. Write the expression as a single logarithm. Solve if possible.

$$\log_2 2 + \log_2 4$$

■ 2. Write the expression as a single logarithm. Solve if possible.

$$\log_3 216 - \log_3 24$$

■ 3. Write the expression as a single logarithm. Solve if possible.

$$\log_4 10 - 3\log_4 2$$

■ 4. Write the expression as a single logarithm. Solve if possible.

$$2\log_7 4 + 3\log_7 5$$

■ 5. Simplify the logarithmic expression.

$$\log_2\left(\frac{x^4y^2}{z^3}\right)$$

■ 6. Simplify the logarithmic expression.

$$\log_9\left(\frac{9x^3}{2y^4z}\right)$$



# LAWS OF NATURAL LOGS

■ 1. Condense the expression into a single logarithm.

$$2 \ln 2 - 3 \ln 4$$

■ 2. Condense the expression into a single logarithm.

$$\frac{5 \ln 2}{4}$$

■ 3. Condense the expression into a single logarithm.

$$3(\ln 2x - \ln 5) + (\ln 4 - \ln 3y)$$

■ 4. Expand the logarithm.

$$\ln\left(3\cdot5\right)^4$$

■ 5. Expand the logarithm.

$$\ln\left(\frac{4}{5}\right)^2$$

6	Exp	and	the	loga	rithm.

 $\ln(3xy^2)$ 



# **SOLVING LOGARITHMIC EQUATIONS**

■ 1. Solve the equation.

$$2\log_b x = \log_b 49$$

■ 2. Solve the equation.

$$\log_{12} x = \frac{3}{2} \log_{12} 16$$

■ 3. Solve the equation.

$$\log_5(x-1) + \log_5(x+5) = \log_5 7$$

■ 4. Solve the equation.

$$\log_9(-4x) - \log_9 12 = \log_9 5$$

■ 5. Solve the equation.

$$\log_a 2 + \log_a 4 = \log_a (x+2)$$

# ■ 6. Solve the equation.

$$\log_4(x+5) - \log_4(x-2) = \log_4 3$$



## **CHANGE OF BASE**

■ 1. Find the value of the expression to four decimal places.

$$log_3 6$$

■ 2. Find the value of the expression to four decimal places.

$$\frac{\log 25}{\log 5} - \frac{\log 25}{\log 125}$$

■ 3. Find the approximate value to the expression to the nearest ten thousandth.

$$\frac{\log 18}{\log 2} - \frac{\log 9}{\log 2}$$

■ 4. Write the log expression in terms of natural logs.

$$log_46.7$$

■ 5. Find the exact value of the logarithmic expression.

 $\frac{\ln(16,807)}{\ln(7)}$ 

■ 6. Use logarithms to solve the equation to the nearest ten thousandth.

$$8 \cdot 6^{3x} = 4,104$$



## **GRAPHING EXPONENTIAL FUNCTIONS**

■ 1. Will the function have a vertical or horizontal asymptote? Where is it located? What is the end behavior of the function?

$$y = \left(\frac{1}{3}\right)^{x-2} + 3$$

■ 2. Will the function have a vertical or horizontal asymptote? Where is it located? What is the end behavior of the function?

$$x = -4^{y+3} - 2$$

■ 3. Graph the exponential function.

$$f(x) = \left(\frac{4}{5}\right)^x$$

 $\blacksquare$  4. What is the *x*-intercept of the function?

$$x = \left(\frac{11}{8}\right)^y - 4$$

■ 5. Sketch the graph of the function by finding the y-intercept and figuring out the function's end behavior.

$$y = -2^{x+1} - 5$$

 $\blacksquare$  6. Sketch the graph of the function using the x-intercept and the end behavior.

$$x = \left(\frac{1}{4}\right)^{y+2} - 2$$



### GRAPHING TRANSFORMATIONS OF EXPONENTIAL FUNCTIONS

■ 1. Sketch the graph of the function.

$$y = 3^{x-2} - 3$$

■ 2. Sketch the graph of the function.

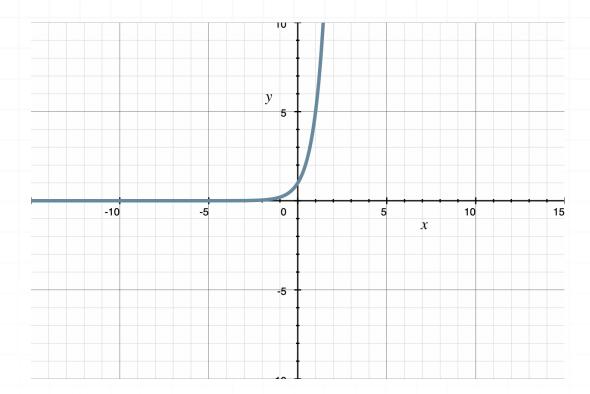
$$y = -2(2^{x+3}) + 1$$

■ 3. Graph the exponential function.

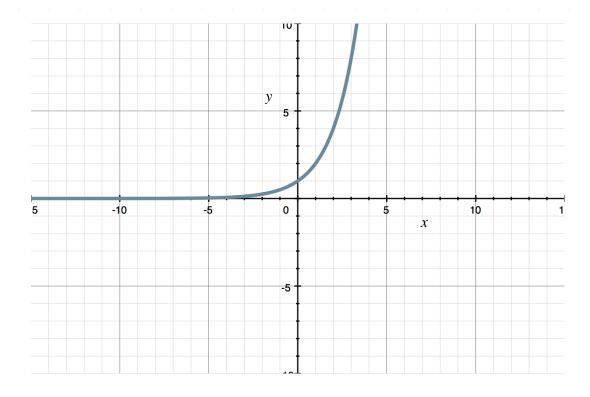
$$f(x) = \left(\frac{1}{2}\right)^{-x-2} - 5$$

- 4. Write the equation for the function that results from applying the list of transformations below to the parent function  $y = 6^x$ . The list of transformations is not necessarily given in the order in which they should be applied.
  - Vertically stretch by a factor of 2
  - Vertical shift 4 units up and horizontal shift 3 units to the right
  - Reflection across the x-axis

- Reflection across the y-axis
- 5. Given the graph of  $y = 5^x$ , use transformations to graph  $y = -5^{x+1} 2$ .



■ 6. Given the graph of  $y = 2^x$ , use transformations to graph  $y = 2^{x-1} + 4$ .



## THE GENERAL LOG RULE

■ 1. Write the inverse of the log function.

$$\log y = 2x$$

■ 2. Write the inverse of the log function.

$$ln y = x$$

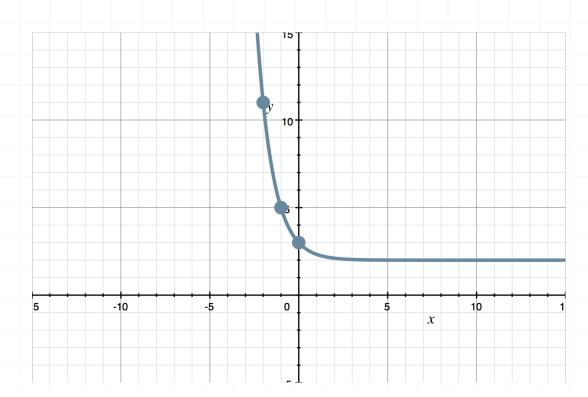
■ 3. The table shows points that satisfy an exponential function. Write a set of four points that will satisfy its inverse.

x	1	2	3	4
y=a <sup>x</sup>	1.5	2.25	3.375	5.0625

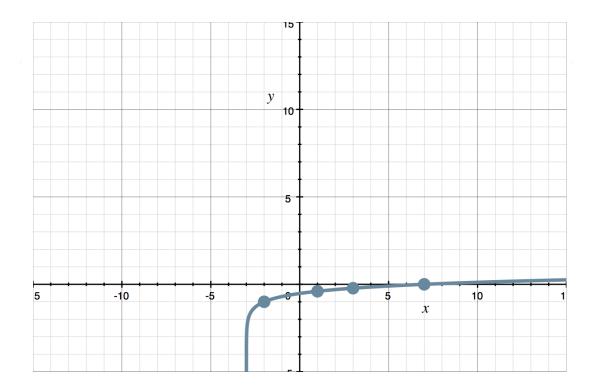
■ 4. The table shows points that satisfy a logarithmic function. Write a set of four points that will satisfy its inverse.

X	10	100	1,000	10,000
y=log(x)	1	2	3	4

■ 5. The graph shown passes through (-2,11), (-1,5), and (0,3). Sketch the graph of its inverse.



■ 6. The graph shown passes through (-2, -1), (1, -0.3979), (3, -0.2218), and (7,0). Sketch the graph of its inverse.



### **GRAPHING LOG FUNCTIONS**

■ 1. Will the function have a vertical or horizontal asymptote? Where is it located?

$$y = \log_2(x - 1)$$

- 2. Graph the function  $y = \log_6(x)$ .
- 3. Sketch the graph of the function using intercepts and end behavior.

$$y = 5\log_2(x+4)$$

- 4. Graph the function  $y = \ln(x)$ .
- 5. Sketch the graph of the function by making a table of values.

$$y = \log_5 x$$

■ 6. Sketch the graph of the function using intercepts and end behavior.

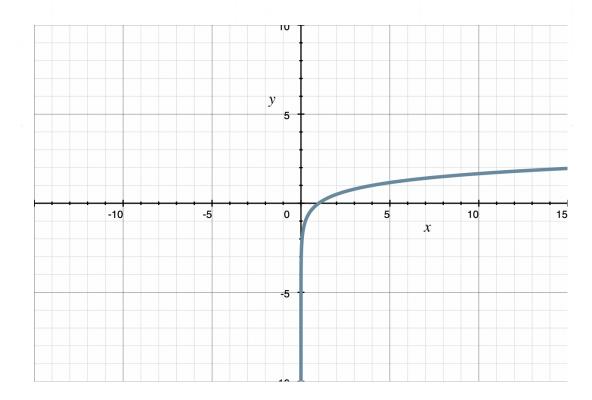
$$x = 2\log_3(y+1)$$

## **GRAPHING TRANSFORMATIONS OF LOG FUNCTIONS**

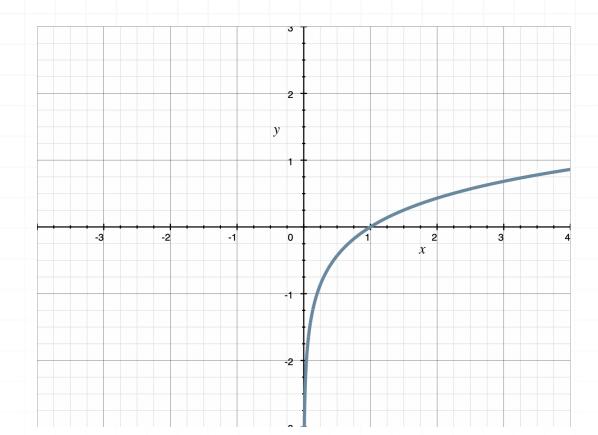
- 1. Use transformations to sketch the graph of  $y = -3 \log_7(x-2) + 4$ .
- 2. Will the function have a vertical or horizontal asymptote? Where is it located?

$$y = \log_4(2x) + \log_4(3)$$

■ 3. Given the graph of  $y = \log_4(x)$ , use transformations to sketch the graph of  $y = \log_4(x-2) - 3$ .



■ 4. Given the graph of  $y = \log_5(x)$ , use transformations to sketch the graph of  $y = -\log_5(x+1) - 2$ .



- 5. Use transformations to sketch the graph of  $y = \log_4(x + 8)$ .
- 6. Use transformations to sketch the graph of  $y = \log(5x) 3$ .

