Linear Algebra Workbook

Matrix-vector products

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MULTIPLYING MATRICES BY VECTORS

■ 1. Find the matrix-vector product, $A\vec{x}$.

$$A = \begin{bmatrix} 0 & 2 \\ -1 & 1 \\ 0 & -2 \end{bmatrix}$$

$$\vec{x} = (4, -1)$$

■ 2. Find the matrix-vector product, $\vec{x}A$.

$$A = \begin{bmatrix} 3 & -1 & 0 \\ 0 & 0 & 4 \end{bmatrix}$$

$$\vec{x} = (-2,3)$$

■ 3. Find the matrix-vector product, $A\vec{x}$.

$$A = \begin{bmatrix} 4 & -2 & 1 \\ 0 & 0 & -1 \\ -3 & 1 & 2 \end{bmatrix}$$

$$\vec{x} = (2,0,1)$$

■ 4. Find the matrix-vector product, $\vec{x}A$.

$$A = \begin{bmatrix} 1 & -1 & 0 & -2 \\ -3 & 0 & -2 & 1 \end{bmatrix}$$

$$\vec{x} = (2, -6)$$

5. Find the matrix-vector product, $A\vec{x}$.

$$A = \begin{bmatrix} 4 & 6 \\ -2 & -3 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\vec{x} = (3,3)$$

6. Find the matrix-vector product, $\vec{x}A$.

$$A = \begin{bmatrix} 6 & -4 & -4 \\ 1 & -4 & -4 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\vec{x} = (-3,1,1)$$

THE NULL SPACE AND AX=O

1. Is $\vec{x} = (1,2)$ in the null space of A?

$$A = \begin{bmatrix} 4 & -2 \\ 2 & -1 \end{bmatrix}$$

2. Is $\vec{x} = (5, -8, -9)$ in the null space of A?

$$A = \begin{bmatrix} 6 & 1 & 1 \\ 0 & -2 & 3 \\ -1 & 0 & 4 \end{bmatrix}$$

3. Is $\vec{x} = (1,1,1)$ in the null space of A?

$$A = \begin{bmatrix} 2 & -3 & 1 \\ 1 & 4 & -5 \\ 1 & -6 & 5 \end{bmatrix}$$

4. Is $\vec{x} = (4, -2)$ in the null space of A?

$$A = \begin{bmatrix} 1 & 2 \\ -1 & -2 \\ 2 & 4 \\ -2 & -4 \end{bmatrix}$$

5. Is $\vec{x} = (1,1,2,1)$ in the null space of A?

$$A = \begin{bmatrix} 1 & -7 & 3 & 0 \\ 0 & 1 & -1 & 1 \end{bmatrix}$$

6. Is $\vec{x} = (-1,-3,1)$ in the null space of A?

$$A = \begin{bmatrix} -4 & 3 & 5 \\ 3 & 1 & 6 \\ 0 & -2 & -6 \end{bmatrix}$$

NULL SPACE OF A MATRIX

1. Find the null space of A.

$$A = \begin{bmatrix} 4 & -3 \\ 0 & 4 \end{bmatrix}$$

2. Find the null space of A.

$$A = \begin{bmatrix} -2 & 1 & 1 \\ 5 & 1 & -6 \\ 1 & 4 & -5 \end{bmatrix}$$

3. Find the null space of A.

$$A = \begin{bmatrix} 3 & -1 \\ -3 & 1 \\ 9 & -3 \\ 0 & 0 \end{bmatrix}$$

4. Find the null space of A.

$$A = \begin{bmatrix} -1 & 0 & 6 & 3 \\ 3 & 1 & 1 & 4 \\ 0 & 0 & 4 & 2 \end{bmatrix}$$

5. Find the null space of A.

$$A = \begin{bmatrix} 4 & -2 & 1 & 1 \\ -1 & 0 & 3 & -3 \\ 0 & 0 & -4 & 6 \end{bmatrix}$$

6. Find the null space of A.

$$A = \begin{bmatrix} -2 & 0 & 7 \\ 3 & -1 & 4 \\ 0 & 3 & -2 \\ 1 & 4 & -5 \\ 2 & 2 & 1 \end{bmatrix}$$

THE COLUMN SPACE AND AX=B

1. Find the column space of A.

$$A = \begin{bmatrix} -2 & 1 & 1 \\ 5 & 1 & -6 \\ 1 & 4 & -5 \end{bmatrix}$$

2. Find the column space of A.

$$A = \begin{bmatrix} -1 & 0 & 6 & 3 \\ 3 & 1 & 1 & 4 \\ 0 & 0 & 4 & 2 \end{bmatrix}$$

3. Find a basis for the column space of A.

$$A = \begin{bmatrix} 4 & -3 \\ 0 & 4 \end{bmatrix}$$

4. Find a basis for the column space of A.

$$A = \begin{bmatrix} -1 & 0 & 6 & 3 \\ 3 & 1 & 1 & 4 \\ 0 & 0 & 4 & 2 \end{bmatrix}$$

5. Find a basis for the column space of A.

$$A = \begin{bmatrix} 5 & -2 & 6 \\ -3 & 1 & 0 \\ 0 & -1 & -4 \\ 8 & 2 & 2 \end{bmatrix}$$

6. Find a basis for the column space of A.

$$A = \begin{bmatrix} 2 & -4 & 3 & -6 \\ 1 & -2 & 0 & 0 \\ 4 & -8 & 5 & -10 \end{bmatrix}$$

SOLVING AX=B

1. Find the general solution to $A\vec{x} = \vec{b}$.

$$A = \begin{bmatrix} 2 & -4 & 3 & -6 \\ 1 & -2 & 0 & 0 \\ 4 & -8 & 5 & -10 \end{bmatrix} \text{ with } \vec{b} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

2. Find the general solution to $A\vec{x} = \vec{b}$.

$$A = \begin{bmatrix} 3 & 6 \\ 6 & 12 \\ 1 & 1 \\ 2 & 2 \end{bmatrix} \text{ with } \vec{b} = \begin{bmatrix} 1 \\ 2 \\ 1 \\ 2 \end{bmatrix}$$

3. Find the general solution to $A\vec{x} = \vec{b}$.

$$A = \begin{bmatrix} 1 & -5 & 3 \\ -1 & 4 & 0 \\ 3 & -16 & 12 \end{bmatrix} \text{ with } \vec{b} = \begin{bmatrix} 1 \\ -1 \\ 3 \end{bmatrix}$$

4. Find the general solution to $A\vec{x} = \vec{b}$.

$$A = \begin{bmatrix} -2 & 10 & -6 & 2 \\ 1 & -5 & 3 & -1 \end{bmatrix} \text{ with } \vec{b} = \begin{bmatrix} -2 \\ 1 \end{bmatrix}$$

5. Find the general solution to $A\vec{x} = \vec{b}$.

$$A = \begin{bmatrix} 2 & 0 & 0 & 12 \\ -1 & 2 & -1 & 4 \\ 5 & -6 & 3 & 0 \end{bmatrix} \text{ with } \vec{b} = \begin{bmatrix} 1 \\ 1 \\ -2 \end{bmatrix}$$

6. Find the general solution to $A\vec{x} = \vec{b}$.

$$A = \begin{bmatrix} 1 & 0 & 3 & -5 \\ 4 & -2 & 2 & 0 \\ -1 & 2 & -1 & 1 \\ 3 & 2 & -1 & 5 \end{bmatrix} \text{ with } \vec{b} = \begin{bmatrix} 2 \\ 1 \\ 1 \\ 3 \end{bmatrix}$$

DIMENSIONALITY, NULLITY, AND RANK

1. Find the nullity of A.

$$A = \begin{bmatrix} 1 & -3 & 2 & -1 \\ 3 & -7 & 0 & 1 \end{bmatrix}$$

2. Find the rank of X.

$$X = \begin{bmatrix} -2 & 3 & 1 \\ -1 & 0 & -4 \\ 2 & -4 & 3 \end{bmatrix}$$

3. Find the nullity and the rank of A.

$$A = \begin{bmatrix} -1 & -3 & 2 & 4 & -2 \\ -3 & -5 & -2 & 1 & 4 \\ 0 & 4 & -8 & -11 & 10 \\ 1 & 3 & -2 & -4 & 5 \end{bmatrix}$$

4. Find the nullity of M.

$$M = \begin{bmatrix} -4 & 2 & -2 & 1 \\ -1 & 0 & -3 & 2 \\ 3 & -2 & 5 & 0 \end{bmatrix}$$

5. Find the rank of M.

$$M = \begin{bmatrix} -2 & 0 & -5 & 6 & 2 \\ 1 & -1 & 3 & 0 & 5 \\ 0 & -2 & 1 & 6 & 12 \end{bmatrix}$$

6. Find the nullity and the rank of M.

$$M = \begin{bmatrix} -1 & 2 & 0 & 3 \\ -2 & 0 & -1 & 2 \\ 3 & -2 & 0 & -4 \\ 1 & -4 & 2 & 0 \end{bmatrix}$$


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