`topk` documentation

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1 Introduction

Twentyonepk (`topk`) is a likelihood python code created to be run with the code for bayesian analysis Cobaya (Torrado and Lewis (2021)). The code is written in python 3.9 and requires installations of the python package *scipy* and the Cobaya package (see Cobaya's documentation for details on the installation). It is designed for 21cm power spectra analysis and 21cm-galaxy cross correlation power spectra analysis (also referred to as *cross*). The likelihood is designed to be interfaced with Cobaya, therefore it can be used with both theory codes supported by Cobaya, namely CAMB (Lewis et al. (1999); Howlett et al. (2012)) and CLASS (Blas et al. (2011)), as well as their modifications, such as EFTCAMB (Hu et al. (2014); Raveri et al. (2014)), MGCAMB (Zhao et al. (2009); Hojjati et al. (2011); Zucca et al. (2019); Wang et al. (2023)), hi_class (Zumalacárregui et al. (2017); Bellini et al. (2020)). In the following, we first describe the theory implemented in the code, we then describe the input options of the code and finally we present a few examples of runs made with this code.

! Note

`topk` was developed for the following publications: Berti et al. (2022), Berti et al. (2023), Berti et al. (2024).

`topk` is free to use. If you use it, please cite at least:
M. Berti, M. Spinelli, M. Viel, *Multipole expansion for 21cm Intensity Mapping power spectrum: forecasted cosmological parameters estimation for the SKA Observatory*, Monthly Notices of the Royal Astronomical Society, 521 (2023) 3, arXiv:2209.07595.

2 Cosmology

2.1 The linear 21cm power spectrum

In the code, the most basic model of the 21cm power spectrum that can be used is

$$P_{21}(z,k,\mu) = \bar{T}_b^2(z)\left[\left(b_{\rm HI}(z) + f(z)\mu^2\right)^2 P_{\rm m}(z,k)\right]\tilde{B}^2(z,k,\mu), \tag{1}$$

where $\bar{T}_b$ is the mean brightness temperature of neutral hydrogen (HI), $b_{\rm HI}$ is the HI bias, f is the growth rate, $\mu = \hat{k}\cdot\hat{z}$ is the cosine of the angle between the wave number and the line-of-sight, $P_{\rm m}(z,k)$ is the matter power spectrum. $\tilde{B}(z,k,\mu)$ is an additional term which is introduced to account for the effect of a Gaussian telescope beam as a suppression on scales smaller than the full width at half maximum of the telescope's beam, and it can be written in terms of the physical dimension of the beam, $R_{\rm beam}$, as

$$\tilde{B}(z,k,\mu) = \exp\left[\frac{-k^2 R_{\rm beam}^2(z)(1-\mu^2)}{2}\right]. \tag{2}$$

This in turn can be computed as follows

$$R_{\rm beam} = \frac{r(z)\theta_{\rm FWHM}}{2\sqrt{2\ln 2}}, \tag{3}$$

where the full-width at half maximum $\theta_{\rm FWHM}$ is computed from the physical dimension of the telescope dish, $R_{\rm dish}$ as

$$\theta_{\rm FWHM} = \frac{1.22\lambda_{21}}{R_{\rm dish}}(1+z). \tag{4}$$

Here, the only quantity that needs to be specified by input when running the code is the physical dimension of the dish, $R_{\rm dish}$. If this is not specified, the default value used is $R_{\rm dish} = 15\,{\rm m}$.

Mean Brightness Temperature Model

The code supports 2 different models for the HI brightness temperature

- **T_b model from Battye et al. (2013)**
  Using this model, the brightness temperature is computed as

$$\bar{T}_b(z) = 44\,\mu{\rm K}\left(\frac{\Omega_{\rm HI}(z)h}{2.45\times10^{-4}}\right)\frac{(1+z)^2\,H_0}{H(z)} \tag{5}$$

  where $\Omega_{\rm HI}$ is the HI density parameter and in the code is computed following the results of Crighton et al. (2015) as $\Omega_{\rm HI} = 4\times10^{-4}(1+z)^{0.6}$. This model is the default option in the code.

- **T_b model from Furlanetto et al. (2006)**
  With this model the brightness temperature is

$$\bar{T}_b(z) = 23.88\left(\frac{\Omega_b h^2}{0.02}\right)\sqrt{\frac{0.15}{\Omega_m h^2}\frac{(1+z)}{10}}\frac{\Omega_{\rm HI}}{0.74\Omega_b}\,{\rm mK}, \tag{6}$$

  where Ω_b is the density parameter of baryons.

HI Bias Model

The code also supports 2 models for the HI bias

- **$b_{\rm HI}$ model from Villaescusa-Navarro et al. (2018)**
  The HI bias is computed by interpolating the following points

z	0	1	2	3	4	5
$b_{\rm HI}$	0.84	1.49	2.03	2.56	2.82	3.18

  This is also the default option of the code.

- **$b_{\rm HI}$ model from Casas et al. (2023)**
  In this model, the HI bias is parametrized as

$$b_{\rm HI}(z) = 0.3(1+z)+0.6. \tag{7}$$

Nuisance parameters

Alternatively to the models above, the brightness temperature and the HI bias can be treated as nuisance parameters by sampling the two combinations $\bar{T}_b b_{\rm HI}\sigma_8(z)$ and $\bar{T}_b f\sigma_8(z)$ for each redshift bin. However, when dealing with a high number of redshift bins, using nuisances adds many parameters to be sampled over. A more convenient approach is that of fitting the two parameters in redshift with a cubic polynomial as

$$\bar{T}_b b_{\rm HI}\sigma_8(z),\ \bar{T}_b f\sigma_8(z) = az^3+bz^2+cz+d, \tag{8}$$

so that only the 8 coefficients of the two cubic polynomials are added as parameters to be sampled. One can also choose to fit the parameters with a quadratic polynomial, which requires less parameters to be sampled over. In this case,

$$\bar{T}_b b_{\rm HI}\sigma_8(z),\ \bar{T}_b f\sigma_8(z) = az^2+bz+c. \tag{9}$$

This approach is case-sensitive, one should always check what is best for their specific case.

2.2 Non-linear 21cm power spectrum

To account for non-linear scales, one needs to compute the non-linear matter power spectrum, which is done using CAMB or CLASS and turning on the appropriate flag (see the flags section below). Also at non-linear scales we have to take into account the shot-noise. Thus, the power spectrum when including the non-linearities is modeled as

$$P_{21}^{NL}(z,k,\mu) = \bar{T}_b^2(z)\left[\left(b_{\rm HI}(z) + f(z)\mu^2\right)^2 P_{\rm m}^{NL}(z,k)\right]\tilde{B}^2(z,k,\mu) + P_{\rm SN}(z), \tag{10}$$

where in this parametrization $P_{\rm SN}$ is in units of $\mathrm{mK}^2(h^{-1}\,\mathrm{Mpc})^3$. In the code, the shot-noise is computed by multiplying T_b^2 by the value of the $(h^{-1}\,\mathrm{Mpc})^3$-units shot-noise computed from interpolating the following points from Villaescusa-Navarro et al. (2018)

z	0	1	2	3	4	5
$P_{SN}(h^{-1}\,\mathrm{Mpc})^3$	104	124	65	39	14	7

The shot-noise can also be treated as a nuisance parameter as we described above for the other nuisance parameters. Alternatively, the shot-noise, in units of $\mathrm{mK}^2(h^{-1}\,\mathrm{Mpc})^3$ can also be passed as input, instead of using the interpolation or the nuisance parameter.

2.3 The cross-correlation power spectrum

The cross-correlation power spectrum between 21cm and galaxy clustering is modeled as follows

$$P_{21,g}(z,k,\mu) = r\,\tilde{B}(z,k,\mu)\,\bar{T}_b(z)\left(b_{\rm HI}(z) + f(z)\mu^2\right)\left(b_g(z) + f(z)\mu^2\right)P_{\rm m}(z,k,\mu). \tag{11}$$

where $b_g(z)$ is the galaxy bias, r is the cross parameter and all other quantities have already been defined in the previous section. r by default is set to $r = 1$ unless differently specified by input.

Galaxy bias

The code offers two options for the galaxy bias. If no option is specified, the galaxy bias is computed from the formula

$$b_g(z) = \sqrt{1+z}. \tag{12}$$

The second option is that of passing the galaxy bias for each redshift bin as input parameters.

Nuisance parameters and fitting

Similarly to the 21cm power spectrum, one can decide to use the following combinations of parameters as nuisance parameters by activating the *nuisances_cross* flag. The three combinations for the cross power spectrum are $\sqrt{rT_b}b_{\rm HI}\sigma_8(z)$, $\sqrt{rT_b}b_{\rm g}\sigma_8(z)$, $\sqrt{rT_b}f\sigma_8(z)$. As before, these can also be fitted with a cubic polynomial.

2.4 Additional modeling options

Through the flags provided by the code, the model of the 21cm power spectrum can be refined, adding the effects described in the following.

AP effect

The Alcock-Paczynski effect (Alcock and Paczynski (1979)) takes into account the distorsions due to differences between the chosen fiducial cosmology and the real cosmology. Thus, one can introduce the so-called AP parameters

$$\alpha_\perp(z) = \frac{D_A(z)}{D_A^{\rm fid}(z)} \qquad \text{and} \qquad \alpha_\parallel(z) = \frac{H^{\rm fid}(z)}{H(z)}. \tag{13}$$

Here, $D_A^{\rm fid}(z)$ and $H^{\rm fid}(z)$ are the fiducial values of the angular diameter distance and the Hubble parameter. The AP parameters modify the power spectrum and the wave vectors. The wave vectors along and transverse to the line-of-sight are distorted as

$$q = \frac{k}{\alpha_\perp}\sqrt{1 + \mu^2\left(\frac{\alpha_\perp^2}{\alpha_\parallel^2} - 1\right)} \tag{14}$$

and

$$\nu = \frac{\alpha_\perp \mu}{\alpha_\parallel \sqrt{1+\mu^2\left(\frac{\alpha_\perp^2}{\alpha_\parallel^2}-1\right)}} \tag{15}$$

Therefore, adding the AP effect, the 21cm power spectrum becomes

$$P_{21}^{AP}(z,k,\mu) = \frac{1}{\alpha_\perp^2 \alpha_\parallel} P_{21}(z,q,\nu), \tag{16}$$

where P_{21} is the linear or non-linear power spectrum.

Fingers-of-God effect

The Fingers-of-God (FoG) effect is a non-linear effect which depends on the velocity dispersion of the tracer object, σ_v. Including the FoG effect, the power spectrum becomes

$$P_{21}^{FoG}(z,k,\mu) = \frac{1}{1+(k\mu\sigma_v/H_0)^2} P_{21}(z,k,\mu). \tag{17}$$

The velocity dispersion σ_v can be given as input to the code. If it isn't specified, the default value is $\sigma_v = 200\,\mathrm{km/h}$.

3 Likelihood

The code then computes the log-likelihood, assuming gaussianly distributed data

$$-\ln\left[\mathcal{L}\right] = \sum_z \frac{1}{2} \Delta\Theta(z)^{\mathrm{T}} C^{-1} \Delta\Theta(z), \tag{18}$$

where $\Delta\Theta(z) = \Theta^{\mathrm{th}}(z) - \Theta^{\mathrm{obs}}(z)$ is the difference between the vector Θ predicted by the theory and the data. The vector Θ is defined as $\Theta(z) = \Big(P(z,k_1),\, \dots\,, P(z,k_N)\Big)$, where P can be either power spectrum. C is the covariance matrix, which in general is a non-diagonal matrix. However, if the *off_diag_cov* flag is turned off, then the off diagonal terms are set to zero, and the covariance matrix becomes $C = \mathrm{diag}(\sigma^2(z,k_1),\, \dots\,, \sigma^2(z,k_N))$.

4 Input options

All the options will be listed and briefly explained in what follows. Input options are divided into ***global options***, ***21cm options*** and ***cross options***. All options can be specified in the likelihood block in the YAML file used to run Cobaya. If not, the code will automatically use the defaults. Note also that some options may be conflicting with one another: in cases in which two or more options are conflicting the code will automatically either revert one of the options back to default and run, or give an error and stop. Such cases are reported in the options sections below.

4.1 Global input options

Here we report options that affect both 21cm and cross-correlation power spectrum runs.

- `path`
  This string specifies the path to the data folder for the run. If not specified, it automatically uses `packages_path`, which is the default path where Cobaya installs external packages. For more detailed information, see Cobaya's documentation.

- `FoG`
  This flag turns on/off the Fingers-of-God effect.

- `sigma_v`
  This specifies the value of the velocity dispersion σ_v for the computation of the FoG effect. If not specified, this is set to $\sigma_v = 200$ km/h by default.

- `AP_effect`
  This flag turns on/off the Alcock-Paczynski effect. See below for how to set the fiducial values for the cosmological parameters.

- `H0_fid`, `ombh2_fid`, `omch2_fid`, `tau_fid`, `mnu_fid`
  These determine the fiducial values of the cosmological parameters needed for the computation of the AP effect. If they are not specified, the default options are: $H_0 = 67.32$ km/s/Mpc, $\Omega_b h^2 = 0.022383$, $\Omega_c h^2 = 0.12011$, $\tau = 0.0543$, $m_\nu = 0.06$ eV.

- `D`
  This specifies the value of the physical dimension of the telescope's dish (in meters). If not specified, the default value of $D = 15\,\mathrm{m}$ is used.

- `T_b _model`
  This string specifies the model for the brightness temperature T_b. The available models (showcased in the Cosmology section above) are called `'Battye_13'` (Battye et al. (2013)) and `'Furlanetto_06` (Furlanetto et al. (2006)). If no option is specified, `'Battye_13'` is set as default.

- `b_HI_model`
  This string specifies the model for the HI bias. Options are `'Navarro_18'` (Villaescusa-Navarro et al. (2018)) and `'Casas_23'` (Casas et al. (2023)). If not specified, `'Navarro_18'` is set by default.

- `nonlinear_matter`
  This option turns on/off non-linearities in the matter power spectrum.

- `nuisances_fitted`
  This flag turns on/off the fitting of the nuisance parameters. If True, the nuisance parameters are fitted with a cubic polynomial and the coefficients are sampled over. Thus this is the treatment of nuisance parameters:

  1. `nuisances_21`/`nuisances_cross` is True and `nuisances_fitted` is False
     In this case, the nuisances are not fitted, therefore the nuisance parameters are sampled for each redshift bin. For 21cm nuisances, the names of the input parameter expected from the code are `Tbsigma8_i` and `Tfsigma8_i` where i runs over the number of redshift bins. For the cross-correlation, they are `rTbsigma8_i`, `rTbgsigma8_i` and `rTfsigma8_i`.
  2. `nuisances_21`/`nuisances_cross` is True and `nuisances_fitted` is True
     In this case the nuisance parameters are fitted in redshift with a cubic polynomial. The name of the input parameters sampled are then the following: for 21cm, `a_21_1`, `b_21_1`, `c_21_1`, `d_21_1`, `a_21_2`, `b_21_2`, `c_21_2`, `d_21_2`. For the cross they are `a_cross_1`, `b_cross_1`, `c_cross_1`, `d_cross_1`, `a_cross_2`, `b_cross_2`, `c_cross_2`, `d_cross_2`, `a_cross_3`, `b_cross_3`, `c_cross_3`, `d_cross_3`.

- `nuis_quadratic_fit`
  This flag allows to fit the nuisance parameters with a quadratic polynomial instead of the default cubic one, therefore reducing the number of extra parameters.

- `off_diag_cov`
  This flag turns on/off the off-diagonal terms in the covariance matrix to include correlation between multipoles.

4.2 21cm input options

- `topk_0` , `topk_2` , `topk_4`
  These flags turn on/off the 21cm power spectrum multipoles: the monopole, quadrupole and hexadecapole respectively.

- `zs_21`
  This list specifies the 21cm redshift bins. This option must be specified in the input.

- `nuisances_21`
  This flag turns on/off nuisances for the 21cm power spectrum.

- `shot_noise`
  This flag turns on/off the shot-noise.

- `SN_as_nuis`, `SN_nuis_fitted`, `shot_noise _from _input`
  These three flags determine how the shot-noise (if turned on) is treated:

  1. `All three are False`
     In this case, if the shot-noise flag is turned on, the shot-noise is computed as in Villaescusa-Navarro+2018.
  2. `shot_noise_from _input` is True and others are False
     In this case, the shot-noise (in units of $\mathrm{mK}^2\,(h^{-1}\,\mathrm{Mpc})^3$) is read from input and ***`not sampled over`***. Therefore the code expects input parameters named `shot_noise_input_i` where again i runs over the number of redshift bins.
  3. `SN_as_nuis` is True, others are False
     In this case, the shot-noise is treated as a nuisance parameter. Therefore, the code expects input parameters (as many as the number of redshift bins) named `shot_noise_i` where i is the index that runs over the number of redshift bins (starting from 1).
  4. `SN_as_nuis` and `SN_nuis_fitted` are True, the other is False
     In this case, the shot-noise is treated as a nuisance parameter by fitting in redshift with a cubic polynomial. This means that the code expects as input the 4 coefficients of the cubic polynomial, named $a_{\rm sn}$, $b_{\rm sn}$, $c_{\rm sn}$, $d_{\rm sn}$. Here, $a_{\rm sn}$ is the coefficient of the highest power (so z^3), $b_{\rm sn}$ of the second-highest power and so on.

4.3 Cross-correlation input options

- `cross_0`, `cross_2`
  These flags turn on/off the cross-correlation power spectrum monopole and quadrupole respectively.

- `zs_cross`
  This list specifies the cross-correlation redshift bins. The redshift bins must be specified in the input if any cross-correlation multipole is turned on.

- `nuisances_cross`
  This flag turns on/off the nuisances for the cross-correlation power spectra.

- `r_cross`
  This specifies the value of the r cross parameter, if not specified, $r = 1$ by default.

- `galaxy_bias_from_input`
  This flag turns on/off the possibility of passing the galaxy bias from input. If turned on, the code expects as input parameters named `galaxy_bias_i` where i runs over the number of redshift bins.

5 Examples

5.1 21cm

`topk`

Here we show a few examples of `topk` runs with Cobaya with different options. Firstly, we start by showing the results of three runs with `topk` alone as a likelihood. We use mock data sets constructed to mimic SKA observations, following Bacon et al. (2020). See Berti et al. (2023) and the `topk`'s GitHub repository for examples on how to construct mock data sets. We then use the python package *GetDist* Lewis (2019) to analyze and plot the chains resulting from the MCMC analysis. We run `topk` with 21cm monopole and quadrupole with the following options:

1. $P_0 + P_2$:
   we use default options for the brightness temperature and the HI bias. We turn on the AP effect and the off-diagonal terms for the covariance matrix. Non-linearities in the matter power spectrum and the shot-noise are turned off.

2. $P_0 + P_2 +$ nuisances:
   we turn on the nuisance parameters for the 21cm power spectrum and we fit them with a cubic polynomial. The AP effect and the off diagonal terms in the covariance matrix are turned on. Shot-noise and non-linear matter power spectrum are turned off.

3. $P_0^{\rm NL} + P_2^{\rm NL} +$ nuisances:
   non-linearities for the matter power spectrum are turned on, as well as the shot-noise which is treated as a nuisance parameter. 21cm nuisances, AP effect and off diagonal terms for the covariance are turned on.
   Results for these three runs are showed in Figure 1.

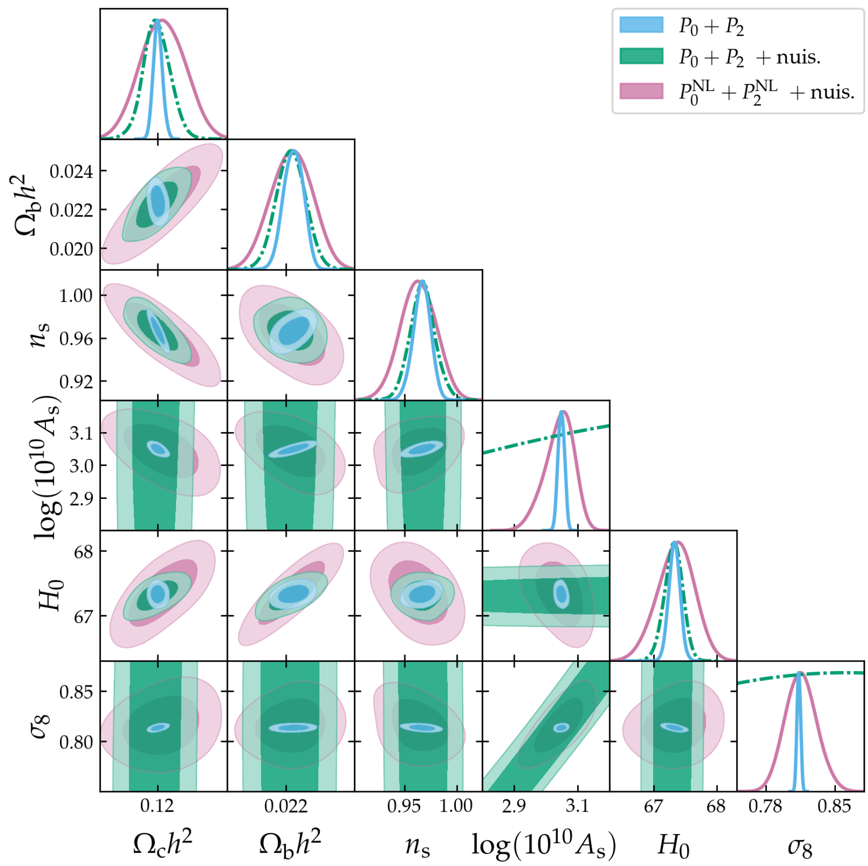

Figure 1: Joint constraints (68% and 95% confidence regions) and marginalised posterior distributions on a subset of cosmological parameters. The label "$P_0 + P_2$" stands for the 21cm monopole and quadrupole, the superscript "NL" stands for the non-linear power spectrum and the label "nuis." indicates that the nuisance parameters are varied.

`topk` + Planck

We also add Planck 2018 (Aghanim et al. (2020b,a)) likelihoods to `topk` and run the same three runs described above. The CMB likelihoods that we use are the low-ℓ TT and EE likelihoods, the high-ℓ `Plik` TTTEEE likelihood and the lensing likelihood. The results of these runs, as well as the results of Planck alone, are showed in Figure 2.

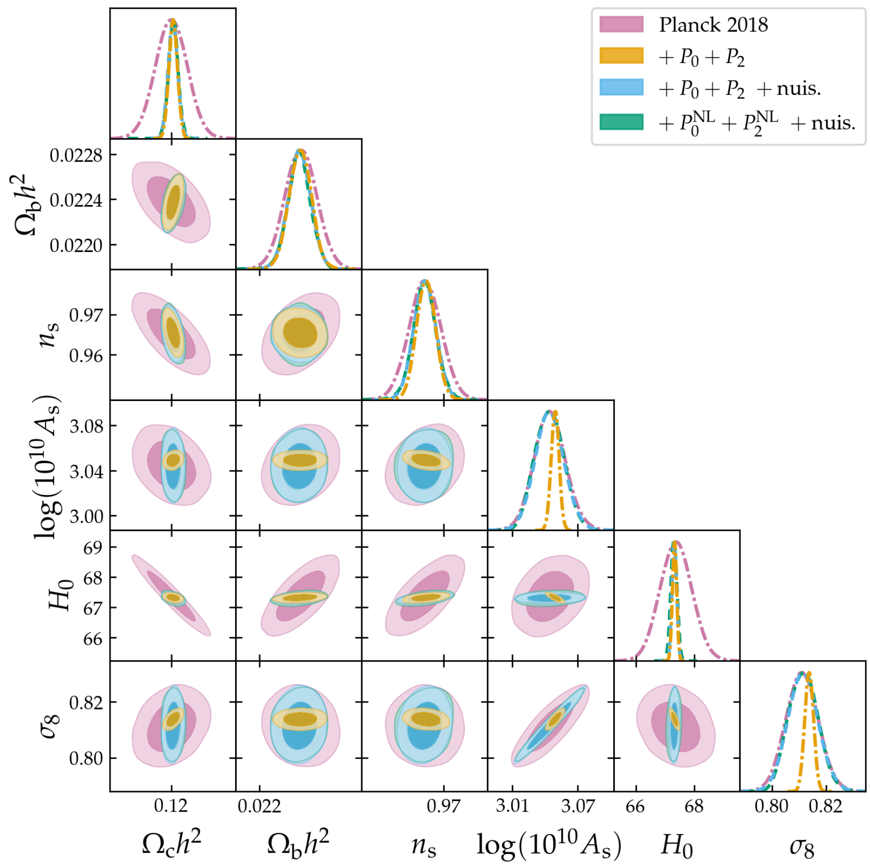

Figure 2: Joint constraints (68% and 95% confidence regions) and marginalised posterior distributions on a subset of cosmological parameters. The label "Planck 2018" stands for low-ℓ TT and EE, high-ℓ `Plik` TTTEEE and lensing. The label "$P_0 + P_2$" stands for the 21cm monopole and quadrupole, the superscript "NL" stands for the non-linear power spectrum and the label "nuis." indicates that the nuisance parameters are varied.

5.2 Cross

`topk`

We now show two runs of the 21cm-galaxy cross-correlation power spectrum. We firstly run these two runs with `topk` alone, using mock data constructed to mimic an SKAO x DESI analysis (see Berti et al. (2024) and the `topk`'s GitHub repository for more details). These two runs have the cross monopole as the only observable turned on, and the following options:

1. $P_{21,\mathrm{g}}$:
   the AP effect is turned on, as well as the off diagonal terms for the covariance matrix. Default options are used for the brightness temperature and the HI bias. Non-linearities in the matter power spectrum are turned on. The galaxy bias in passed from input.

2. $P_{21,\mathrm{g}}$ + nuisances:
   we now turn on the nuisance parameters for the cross power spectrum and fit them with a cubic polynomail. All other options are the same as the first run.

The results of these runs are reported in Figure 3.

`topk` + Planck

As we did for the 21cm runs, we also add Planck likelihoods to `topk` and redo the same two runs. These results are reported together with the result of running Planck alone in Figure 4.

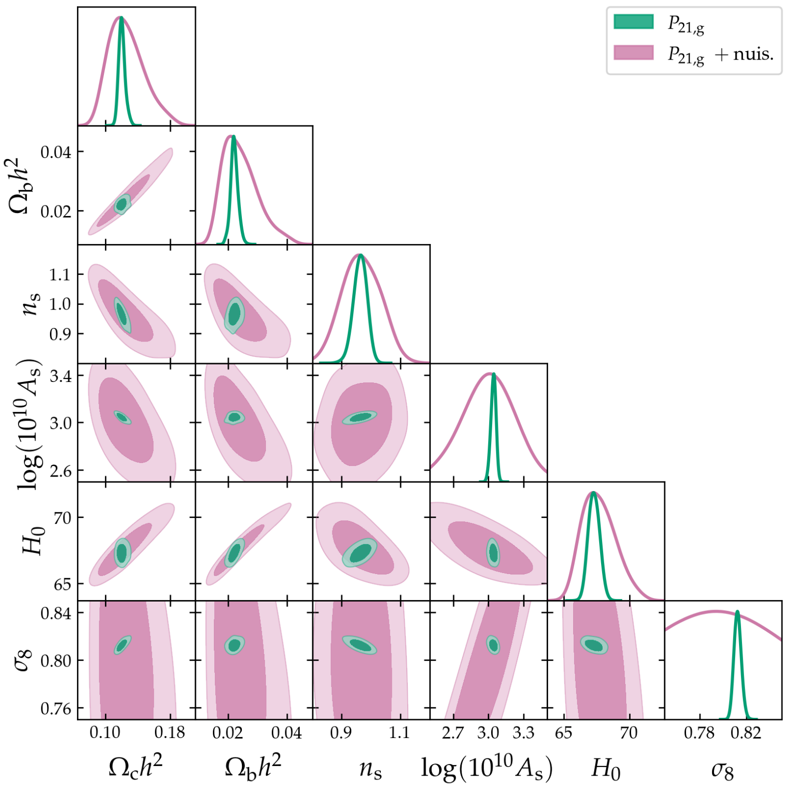

Figure 3: Joint constraints (68% and 95% confidence regions) and marginalised posterior distributions on a subset of cosmological parameters. The label "$P_{21,g}$" stands for the cross-correlation power spectrum and the label "nuis." indicates that the nuisance parameters are varied.

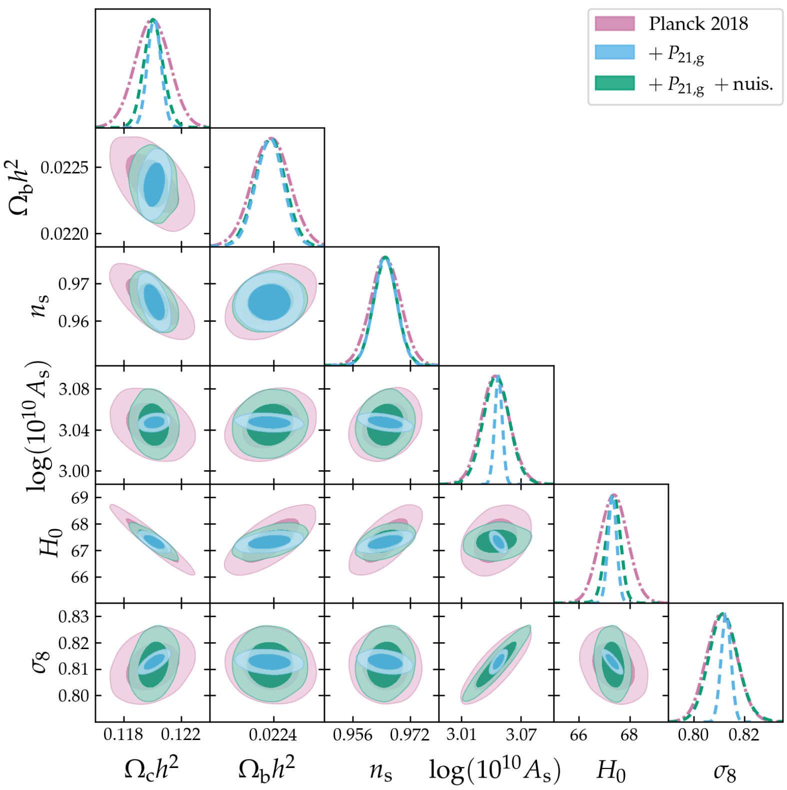

Figure 4: Joint constraints (68% and 95% confidence regions) and marginalised posterior distributions on a subset of cosmological parameters. The label "Planck 2018" stands for low-ℓ TT and EE, high-ℓ `Plik` TTTEEE and lensing. The label "$P_{21,g}$" stands for the cross-correlation power spectrum and the label "nuis." indicates that the nuisance parameters are varied.

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