

# \newkeytheorem and \newkeytheoremstyle test

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January 7, 2025

## 1 Test of standard theorem styles

Ahlfors' Lemma gives the principal criterion for obtaining lower bounds on the Kobayashi metric.

**Ahlfors' Lemma.** *Let  $ds^2 = h(z)|dz|^2$  be a Hermitian pseudo-metric on  $\mathbf{D}_r$ ,  $h \in C^2(\mathbf{D}_r)$ , with  $\omega$  the associated  $(1,1)$ -form. If  $\text{Ric } \omega \geq \omega$  on  $\mathbf{D}_r$ , then  $\omega \leq \omega_r$  on all of  $\mathbf{D}_r$  (or equivalently,  $ds^2 \leq ds_r^2$ ).*

**Lemma 1.1** (negatively curved families). *Let  $\{ds_1^2, \dots, ds_k^2\}$  be a negatively curved family of metrics on  $\mathbf{D}_r$ , with associated forms  $\omega^1, \dots, \omega^k$ . Then  $\omega^i \leq \omega_r$  for all  $i$ .*

Then our main theorem:

**Theorem 1.2.** *Let  $d_{\max}$  and  $d_{\min}$  be the maximum, resp. minimum distance between any two adjacent vertices of a quadrilateral  $Q$ . Let  $\sigma$  be the diagonal pigspan of a pig  $P$  with four legs. Then  $P$  is capable of standing on the corners of  $Q$  iff*

$$\sigma \geq \sqrt{d_{\max}^2 + d_{\min}^2}. \tag{1}$$

**Corollary 1.3.** *Admitting reflection and rotation, a three-legged pig  $P$  is capable of standing on the corners of a triangle  $T$  iff (1) holds.*

*Remark.* As two-legged pigs generally fall over, the case of a polygon of order 2 is uninteresting.

## 2 Custom theorem styles

**Exercise 1:** Generalize Theorem 1.2 to three and four dimensions.

*Note 1:* This is a test of the custom theorem style ‘note’. It is supposed to have variant fonts and other differences.

**B-Theorem 1.***Test of the ‘linebreak’ style of theorem heading.*

This is a test of a citing theorem to cite a theorem from some other source.

**Theorem 3.6 in [1].** *No hyperlinking available here yet ... but that’s not a bad idea for the future.*

### 3 The proof environment

*Proof.* Here is a test of the proof environment. □

*Proof of Theorem 1.2.* And another test. □

*Proof (necessity).* And another. □

*Proof (sufficiency).* And another, ending with a display:

$$1 + 1 = 2. \quad \square$$

### 4 Test of number-swapping

This is a repeat of the first section but with numbers in theorem heads swapped to the left.

Ahlfors’ Lemma gives the principal criterion for obtaining lower bounds on the Kobayashi metric.

**Ahlfors’ Lemma.** *Let  $ds^2 = h(z)|dz|^2$  be a Hermitian pseudo-metric on  $\mathbf{D}_r$ ,  $h \in C^2(\mathbf{D}_r)$ , with  $\omega$  the associated  $(1, 1)$ -form. If  $\text{Ric } \omega \geq \omega$  on  $\mathbf{D}_r$ , then  $\omega \leq \omega_r$  on all of  $\mathbf{D}_r$  (or equivalently,  $ds^2 \leq ds_r^2$ ).*

**4.1 Lemma** (negatively curved families). *Let  $\{ds_1^2, \dots, ds_k^2\}$  be a negatively curved family of metrics on  $\mathbf{D}_r$ , with associated forms  $\omega^1, \dots, \omega^k$ . Then  $\omega^i \leq \omega_r$  for all  $i$ .*

Then our main theorem:

**4.2 Theorem.** *Let  $d_{\max}$  and  $d_{\min}$  be the maximum, resp. minimum distance between any two adjacent vertices of a quadrilateral  $Q$ . Let  $\sigma$  be the diagonal pigspan of a pig  $P$  with four legs. Then  $P$  is capable of standing on the corners of  $Q$  iff*

$$\sigma \geq \sqrt{d_{\max}^2 + d_{\min}^2}. \quad (2)$$

**4.3 Corollary.** *Admitting reflection and rotation, a three-legged pig  $P$  is capable of standing on the corners of a triangle  $T$  iff (2) holds.*

### References

- [1] Dummy entry.