# \newkeytheorem and \newkeytheoremstyle test

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## 1 Test of standard theorem styles

Ahlfors' Lemma gives the principal criterion for obtaining lower bounds on the Kobayashi metric.

**Ahlfors' Lemma.** *Let $ds^2 = h(z)|dz|^2$ be a Hermitian pseudo-metric on $\mathbf{D}_r$, $h \in C^2(\mathbf{D}_r)$, with $\omega$ the associated $(1,1)$-form. If $\operatorname{Ric}\omega \geq \omega$ on $\mathbf{D}_r$, then $\omega \leq \omega_r$ on all of $\mathbf{D}_r$ (or equivalently, $ds^2 \leq ds_r^2$).*

**Lemma 1.1** (negatively curved families)**.** *Let $\{ds_1^2, \ldots, ds_k^2\}$ be a negatively curved family of metrics on $\mathbf{D}_r$, with associated forms $\omega^1$, ..., $\omega^k$. Then $\omega^i \leq \omega_r$ for all i.*

Then our main theorem:

**Theorem 1.2.** *Let $d_{\max}$ and $d_{\min}$ be the maximum, resp. minimum distance between any two adjacent vertices of a quadrilateral $Q$. Let $\sigma$ be the diagonal pigspan of a pig $P$ with four legs. Then $P$ is capable of standing on the corners of $Q$ iff*

$$\sigma \geq \sqrt{d_{\max}^2 + d_{\min}^2}. \tag{1}$$

**Corollary 1.3.** *Admitting reflection and rotation, a three-legged pig $P$ is capable of standing on the corners of a triangle $T$ iff (1) holds.*

*Remark.* As two-legged pigs generally fall over, the case of a polygon of order 2 is uninteresting.

## 2 Custom theorem styles

**Exercise 1:** Generalize Theorem 1.2 to three and four dimensions.

*Note* 1*:* This is a test of the custom theorem style 'note'. It is supposed to have variant fonts and other differences.

**B-Theorem 1.**
*Test of the 'linebreak' style of theorem heading.*

This is a test of a citing theorem to cite a theorem from some other source.

**Theorem 3.6 in [1].** *No hyperlinking available here yet … but that's not a bad idea for the future.*

# 3    The proof environment

*Proof.* Here is a test of the proof environment. □

*Proof of Theorem 1.2.* And another test. □

*Proof (necessity).* And another. □

*Proof (sufficiency).* And another, ending with a display:

$$1 + 1 = 2 \,.$$
□

# 4    Test of number-swapping

This is a repeat of the first section but with numbers in theorem heads swapped to the left.

Ahlfors' Lemma gives the principal criterion for obtaining lower bounds on the Kobayashi metric.

**Ahlfors' Lemma.** *Let $ds^2 = h(z)|dz|^2$ be a Hermitian pseudo-metric on $\mathbf{D}_r$, $h \in C^2(\mathbf{D}_r)$, with $\omega$ the associated $(1,1)$-form. If $\mathrm{Ric}\,\omega \geq \omega$ on $\mathbf{D}_r$, then $\omega \leq \omega_r$ on all of $\mathbf{D}_r$ (or equivalently, $ds^2 \leq ds_r^2$).*

**4.1 Lemma** (negatively curved families). *Let $\{ds_1^2, \ldots, ds_k^2\}$ be a negatively curved family of metrics on $\mathbf{D}_r$, with associated forms $\omega^1$, …, $\omega^k$. Then $\omega^i \leq \omega_r$ for all $i$.*

Then our main theorem:

**4.2 Theorem.** *Let $d_{\max}$ and $d_{\min}$ be the maximum, resp. minimum distance between any two adjacent vertices of a quadrilateral $Q$. Let $\sigma$ be the diagonal pigspan of a pig $P$ with four legs. Then $P$ is capable of standing on the corners of $Q$ iff*

$$\sigma \geq \sqrt{d_{\max}^2 + d_{\min}^2}. \tag{2}$$

**4.3 Corollary.** *Admitting reflection and rotation, a three-legged pig $P$ is capable of standing on the corners of a triangle $T$ iff (2) holds.*

# References

[1] Dummy entry.