# \newkeytheorem and \newkeytheoremstyle test

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November 5, 2024

### 1 Test of standard theorem styles

Ahlfors' Lemma gives the principal criterion for obtaining lower bounds on the Kobayashi metric.

**Ahlfors' Lemma.** Let  $ds^2 = h(z)|dz|^2$  be a Hermitian pseudo-metric on  $\mathbf{D}_r$ ,  $h \in C^2(\mathbf{D}_r)$ , with  $\omega$  the associated (1,1)-form. If  $\mathrm{Ric}\,\omega \geq \omega$  on  $\mathbf{D}_r$ , then  $\omega \leq \omega_r$  on all of  $\mathbf{D}_r$  (or equivalently,  $ds^2 \leq ds_r^2$ ).

**Lemma 1.1** (negatively curved families). Let  $\{ds_1^2, \ldots, ds_k^2\}$  be a negatively curved family of metrics on  $\mathbf{D}_r$ , with associated forms  $\omega^1, \ldots, \omega^k$ . Then  $\omega^i \leq \omega_r$  for all i.

Then our main theorem:

**Theorem 1.2.** Let  $d_{\max}$  and  $d_{\min}$  be the maximum, resp. minimum distance between any two adjacent vertices of a quadrilateral Q. Let  $\sigma$  be the diagonal pigspan of a pig P with four legs. Then P is capable of standing on the corners of Q iff

$$\sigma \ge \sqrt{d_{\text{max}}^2 + d_{\text{min}}^2}. (1)$$

**Corollary 1.3.** Admitting reflection and rotation, a three-legged pig P is capable of standing on the corners of a triangle T iff (1) holds.

Remark. As two-legged pigs generally fall over, the case of a polygon of order 2 is uninteresting.

## 2 Custom theorem styles

Exercise 1: Generalize Theorem 1.2 to three and four dimensions.

*Note* 1: This is a test of the custom theorem style 'note'. It is supposed to have variant fonts and other differences.

#### B-Theorem 1.

Test of the 'linebreak' style of theorem heading.

This is a test of a citing theorem to cite a theorem from some other source.

**Theorem 3.6 in [1].** No hyperlinking available here yet ... but that's not a bad idea for the future.

## 3 The proof environment

<i>Proof.</i> Here is a test of the proof environment.	
Proof of Theorem 1.2. And another test.	
Proof (necessity). And another.	
Proof (sufficiency). And another, ending with a display:	
1 + 1 = 2.	

## 4 Test of number-swapping

This is a repeat of the first section but with numbers in theorem heads swapped to the left.

Ahlfors' Lemma gives the principal criterion for obtaining lower bounds on the Kobayashi metric.

**Ahlfors' Lemma.** Let  $ds^2 = h(z)|dz|^2$  be a Hermitian pseudo-metric on  $\mathbf{D}_r$ ,  $h \in C^2(\mathbf{D}_r)$ , with  $\omega$  the associated (1,1)-form. If  $\mathrm{Ric}\,\omega \geq \omega$  on  $\mathbf{D}_r$ , then  $\omega \leq \omega_r$  on all of  $\mathbf{D}_r$  (or equivalently,  $ds^2 \leq ds_r^2$ ).

**4.1 Lemma** (negatively curved families). Let  $\{ds_1^2, \ldots, ds_k^2\}$  be a negatively curved family of metrics on  $\mathbf{D}_r$ , with associated forms  $\omega^1, \ldots, \omega^k$ . Then  $\omega^i \leq \omega_r$  for all

Then our main theorem:

**4.2 Theorem.** Let  $d_{\text{max}}$  and  $d_{\text{min}}$  be the maximum, resp. minimum distance between any two adjacent vertices of a quadrilateral Q. Let  $\sigma$  be the diagonal pigspan of a pig P with four legs. Then P is capable of standing on the corners of Q iff

$$\sigma \ge \sqrt{d_{\text{max}}^2 + d_{\text{min}}^2}. (2)$$

**4.3 Corollary.** Admitting reflection and rotation, a three-legged pig P is capable of standing on the corners of a triangle T iff (2) holds.

#### References

[1] Dummy entry.