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 - ;
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 - ().
- Python pygame.

2

2.1 RK4 (- 4-)

:

$$\begin{aligned}k_1 &= f(\mathbf{y}(t)), \\k_2 &= f\left(\mathbf{y}(t) + \frac{\Delta t}{2} k_1\right), \\k_3 &= f\left(\mathbf{y}(t) + \frac{\Delta t}{2} k_2\right), \\k_4 &= f\left(\mathbf{y}(t) + \Delta t k_3\right),\end{aligned}$$

:

$$\mathbf{y}(t + \Delta t) \approx \mathbf{y}(t) + \frac{\Delta t}{6} (k_1 + 2k_2 + 2k_3 + k_4).$$

$\mathbf{y}(t) = [x, y, v_x, v_y]$ — .

2.2 Dormand–Prince (Dopri5)

Dormand–Prince 7 k_1, \dots, k_7 , .

:

$$\mathbf{y}(t + \Delta t) \approx \mathbf{y}(t) + \Delta t (b_1 k_1 + b_2 k_2 + b_3 k_3 + b_4 k_4 + b_5 k_5 + b_6 k_6 + b_7 k_7),$$

$a_{ij} \quad b_i$ (5-).

3

3.1

$$\begin{aligned}\frac{dx}{dt} &= v_x, & \frac{dy}{dt} &= v_y, \\ \frac{dv_x}{dt} &= a_{\text{drag},x}, & \frac{dv_y}{dt} &= -g + a_{\text{drag},y}.\end{aligned}$$

3.2

$$\mathbf{F}_{\text{drag}} = -\frac{1}{2} C_d \rho A |\mathbf{v}_{\text{rel}}| \hat{\mathbf{v}}_{\text{rel}},$$

- C_d — ,
- ρ — ,
- $A = \pi r^2$ — (r),
- $\mathbf{v}_{\text{rel}} = \mathbf{v} - \mathbf{v}_{\text{wind}}$ — ,
- $|\mathbf{v}_{\text{rel}}|$ — ,
- $\hat{\mathbf{v}}_{\text{rel}}$ — \mathbf{v}_{rel} .

$$\mathbf{a}_{\text{drag}} = -\frac{1}{2m} C_d \rho A |\mathbf{v}_{\text{rel}}| \mathbf{v}_{\text{rel}}.$$

3.3

$$g \quad , \quad v_y \quad -g.$$

4

- . RK4 Dormand–Prince , -
odeint solve_ivp.
- . (, , , , , ,
) params, .
- . ($y \leq 0$) ,
.
- . pygame , RK4 Dormand–
Prince, () .

5

- Python 3.x
- pygame
- numpy

1. `pip install pygame numpy`

`python your_program.py`

2. `python your_program.py`

`your_program.py` —

`your_program.py` —

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(RK4 Dormand–Prince)

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