CS130 - Transformations

Name: ______________________________ SID: __________________

Identify what each of the following does to a point in homogeneous coordinates. You may choose from:

- uniform scale by a (identify a)
- non-uniform scale by a, b, c (identify a, b, c)
- translation by a, b, c (identify a, b, c)
- rotation by angle θ about axis a, b, c (identify θ, a, b, c)
- reflections (identify the direction about the reflection is occurring)
- a sequence of the above (specify the operations in the order they are applied)

If the transformation cannot be obtained by applying a sequence of the above, explain why.

1. $\begin{pmatrix} 4 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$

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2. $\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$

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3. $\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$

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4. $\begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$

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5. $\begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$

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6. $\begin{pmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$

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7. $\begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$

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8. $\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \end{pmatrix}$

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9. $\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 2 \end{pmatrix}$

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10. $\begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 2 \end{pmatrix}$

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11. $\begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$

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12. $\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$

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13. $\mathbf{u} = \begin{pmatrix} x \\ y \end{pmatrix}, \mathbf{R} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$. Let $\mathbf{v} = \mathbf{R}\mathbf{u}$ be the rotated version of $\mathbf{u}$. Use the dot product $\mathbf{v} \cdot \mathbf{u}$ to show that the angle between $\mathbf{v}$ and $\mathbf{u}$ is θ.

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14. Show that $\mathbf{R}^T\mathbf{R} = \mathbf{I}$ for the 2×2 matrix in the previous problem.

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15. Let $\mathbf{R}$ be a 3×3 matrix with columns $\mathbf{u}$, $\mathbf{v}$, $\mathbf{w}$. Show that $\mathbf{R}^T\mathbf{R} = \mathbf{I}$ is equivalent to $\mathbf{u}$, $\mathbf{v}$, $\mathbf{w}$ being unit vectors and mutually orthogonal.

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16. Let $\mathbf{R}$ be a matrix with $\mathbf{R}^T\mathbf{R} = \mathbf{I}$ and let $\mathbf{y} = \mathbf{R}\mathbf{x}$. Show that $\mathbf{x}$ and $\mathbf{y}$ must have the same length.

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17. Let $\mathbf{R}$ be a matrix with $\mathbf{R}^T\mathbf{R} = \mathbf{I}$ and let $\mathbf{y} = \mathbf{R}\mathbf{x}$ and $\mathbf{v} = \mathbf{R}\mathbf{u}$. Show that $\mathbf{u} \cdot \mathbf{x} = \mathbf{y} \cdot \mathbf{v}$ so that the dot product between vectors is preserved under rotation.

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18. Given the results of the previous two problems, explain why the angle between two vectors must also be preserved under rotation.

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