1. non-uniform scale by 4, 3, 2

2. uniform scale by 0

3. uniform scale by 1 / identity matrix

4. reflection of x, relative to yz plane

5.
$$\begin{matrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{matrix}$$

~~cannot be obtained. The 1s from the top left can only be obtained from a z-axis rotation which would be -sin sin~~

reflect about the plane y=x

6. rotate about the z axis, $\theta = 90$, $\overset{y,\ x,\ z}{0, 0, 1}$

7. translation by 1, 0, 0

8. translation by 1, 0, 0, 0

9. not possible, bottom right value must be 1

10. translation of 1 on the x-axis (1,0,0)
and a uniform scaling of 2

11. 90 degree rotation along x, translation of (0,0,0.5),
scaling of (1,1,2) along z axis

12. cannot be obtained, bottom right cannot be 0
if the rest of a matrix matches an identity matrix

13. $v = Ru = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$

$= \begin{pmatrix} x\cos\theta - y\sin\theta \\ x\sin\theta + y\cos\theta \end{pmatrix}$

$v \cdot u = \begin{pmatrix} x\cos\theta - y\sin\theta \\ x\sin\theta + y\cos\theta \end{pmatrix} \cdot \begin{pmatrix} x \\ y \end{pmatrix}$

$= x(x\cos\theta - y\sin\theta) + y(x\sin\theta + y\cos\theta)$

$= x^2\cos\theta - \cancel{xy\sin\theta} + \cancel{yx\sin\theta} + y^2\cos\theta$

$= \cos\theta(x^2+y^2)$

angle $= |u||v|\cos\theta$

$= \sqrt{x^2+y^2}\left(\sqrt{(x\cos\theta - y\sin\theta)^2 + (x\sin\theta + y\cos\theta)^2}\right)\cos\theta$

$= \left(\sqrt{x^2+y^2}\right)\left(\sqrt{x^2+y^2}\right)\cos\theta$

$= (x^2+y^2)\cos\theta$

$v \cdot u =$ angle ✓

14.

$$R = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \qquad R^T = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$$

$$R \cdot R^T = \begin{pmatrix} \cos\theta \cdot \cos\theta + (-\sin\theta)(-\sin\theta) & \cos\theta \sin\theta + (-\sin\theta)(\cos\theta) \\ \sin\theta \cdot \cos\theta + \cos\theta(-\sin\theta) & \sin\theta \sin\theta + \cos\theta(\cos\theta) \end{pmatrix}$$

$$= \begin{pmatrix} \cos^2(\theta) + \sin^2(\theta) & 0 \\ 0 & \sin^2(\theta) + \cos^2(\theta) \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \text{identity matrix}$$

15.

$$R = \begin{array}{c} \begin{matrix} u & v & w \end{matrix} \\ \begin{pmatrix} u_1 & v_1 & w_1 \\ u_2 & v_2 & w_2 \\ u_3 & v_3 & w_3 \end{pmatrix} \end{array} \qquad R^T = \begin{pmatrix} u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \\ w_1 & w_2 & w_3 \end{pmatrix}$$

$$R \cdot RT = \begin{pmatrix} u_1 \cdot u_1 + u_2 u_2 + u_3 \cdot u_3 & u_1 \cdot u_2 + v_1 \cdot v_2 + w_1 \cdot w_2 & u_1 \cdot u_3 + v_1 \cdot v_3 + w_1 \cdot w_3 \\ u_2 \cdot u_1 + v_2 \cdot v_1 + w_2 \cdot w_1 & u_2 u_2 + v_2 \cdot v_2 + w_2 \cdot w_2 & u_2 \cdot u_3 + v_2 w_3 + w_2 \cdot w_3 \\ u_3 \cdot u_1 + v_2 \cdot v_1 + v_3 \cdot w_1 & u_3 \cdot u_2 + v_3 \cdot v_2 + w_3 \cdot w_2 & u_3 \cdot v_3 + v_3 w_3 + w_3 \cdot w_3 \end{pmatrix}$$

~~$R \cdot R^T$~~ because they are unit vectors $u_1 \cdot u_1 + u_2^2 + u_3^2 = 1$

so $R \cdot R^T$ $\begin{pmatrix} 1 & & \\ & 1 & \\ & & 1 \end{pmatrix}$

because they are mutually orthogonal

$$R \cdot R^T = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = I$$

16. $R^TR=I$ $y=Rx$ x and y must have same length

$R^T=R^{-1}$

R is orthogonal so for any vectors x and y $|Rx|=|x|$ and $|Ry|=|y|$.

$y\cdot y = (Rx)^T(Rx)$

$= x^TR^TRx$

$y\cdot y = x^Tx$

$|y\cdot y| = |x^T\cdot x|$, $= |y|=|x|$

So x and y have the same length

17. $R^TR=I$ $y=Rx$ $v=Ru$

$u\cdot x = y\cdot v$

R is orthogonal, $R^T=R^{-1}$

$u\cdot x = (Ru)^T(Rx) = u^TR^TRx$

$= u^TR^Ty$ $R^T=R^{-1}$ so $u^TR^{-1} = uR = v$

$= v^Ty$

$u\cdot x = v\cdot y$

18. $x\cdot y = |x|\cdot|y|\cos\theta$ From 16 - $|x|=|y|$

$x\cdot y = |x|\cdot|x|\cos\theta$ and $|y|\cdot|y|\cos\theta$

So $x\cdot y = x^R\cdot y^R$ - rotated vectors

So the angle is preserved