MSFM, Fixed Income Derivatives (33601)

Homework Assignment on Statistical Model

This assignment helps understanding construction of the statistical model of interest rates and its properties

This assignment is individual

1. Prepare the data (5%)

Fund the project data are in the file `StatisticalModelData2014.csv`.

The format of the data is shown below:

```
##           USGG3M USGG6M USGG2YR USGG3YR USGG5YR USGG10YR USGG30YR
## 1/5/1981   13.52  13.09  12.289   12.28  12.294   12.152   11.672
## 1/6/1981   13.58  13.16  12.429   12.31  12.214   12.112   11.672
## 1/7/1981   14.50  13.90  12.929   12.78  12.614   12.382   11.892
## 1/8/1981   14.76  14.00  13.099   12.95  12.684   12.352   11.912
## 1/9/1981   15.20  14.30  13.539   13.28  12.884   12.572   12.132
## 1/12/1981  15.22  14.23  13.179   12.94  12.714   12.452   12.082
```

2. Estimate the 3-factor model using PCA (15%)

2.1. Define factor and factor loadings

2.2. Calculate relative importance of factors (see slide 17)

2.3. Plot and interpret the shapes of factor loadings (see slide 18)

3. Calculate historical volatilities and correlation coefficients of factors (15%)

3.1. Use the whole period of history to calculate $Var[\Delta f_i(t)], Cor[\Delta f_i(t), \Delta f_j(t)], i = 1, 2, 3$, where $\Delta f_i(t)$ is a one-day increment of the factor (see slides 20,27)

3.2. Calculate the same varibles using a rolling window approximately 1 month

4. Find historical estimates of volatilities of the first 3 factors corresponding to the last month of the observed period (10%)

5. Calculate time series of each of 7 rates predicted by the model (5%)

6. Fit parametric forms from slide 32 to each of the first 3 vectors of factor loadings (10%)

Include parameters a, b in the report.

Hint. You can use the following parameter values as initial guesses.

```
Loading.1
```

```
##        a      b
## 1  0.320  0.070
## 2  0.006  0.285
## 3 36.550 -0.292
```

```
Loading.2
```

```
##       a      b
## 1 0.650 -1.130
## 2 0.004  0.539
```

```
Loading.3
```

```
##           a      b
## 1 4.200e-01 -1.920
## 2 5.000e-08  0.620
## 3 5.000e-01 -0.410
## 4 2.876e+00  3.035
```

Hint. Restrict optimization to searching $a > 0$.

7. Calculate time series of instantaneous forward rates with maturity 5 years and discount bonds with maturity 4.5 years for the whole period of observation. Calculate the histograms of one-day increments (20%)

8. Calculate correlations between the short rate and instantaneous forward rates (20%)

8.1. See slides 34-36.

8.2. Calculate

$$Cor(\Delta F(t,0), \Delta F(t,\tau))$$

as function of τ for increasing $\tau > 0$.

8.3. Repeat the calculations using one-, two- and three-factor models.

8.4. Plot the graph of correlations.