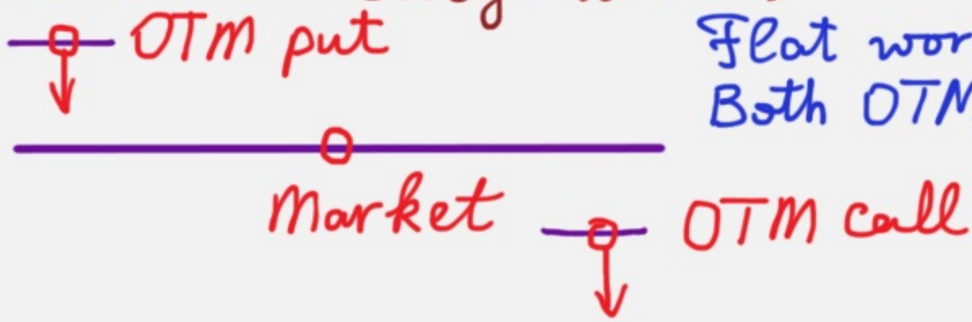
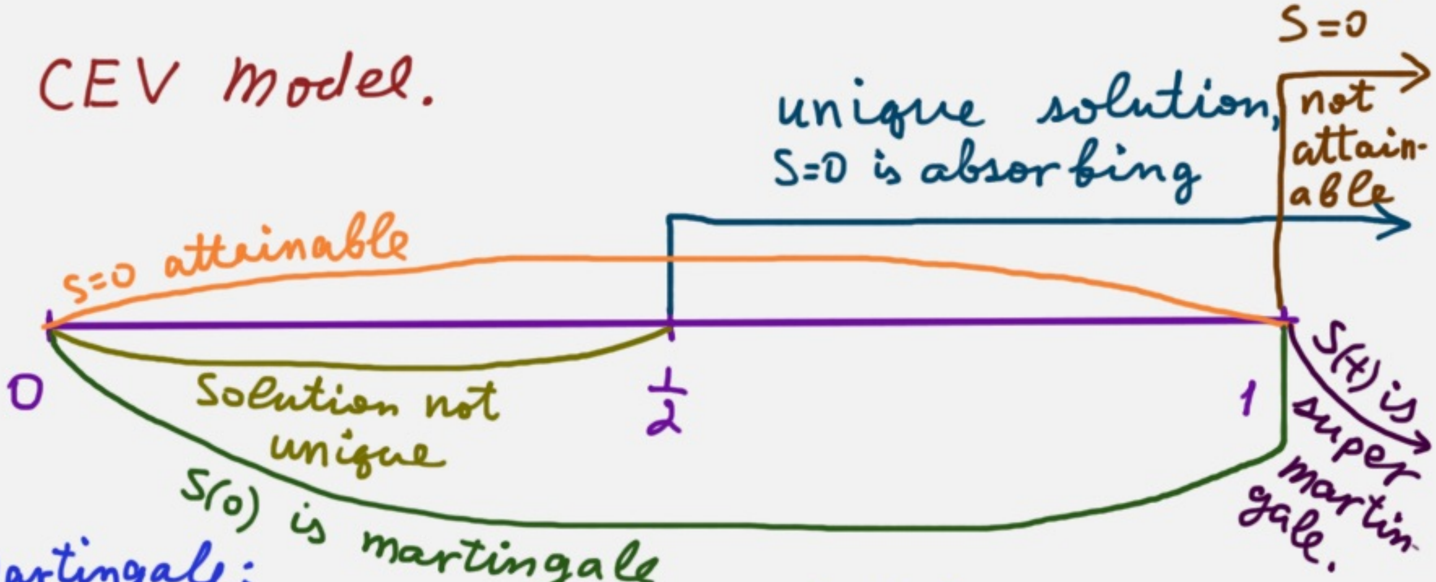
Flat volatility world.



Flat world paradigm: Both OTM volatilities move up or down simultaneously

CEV model.



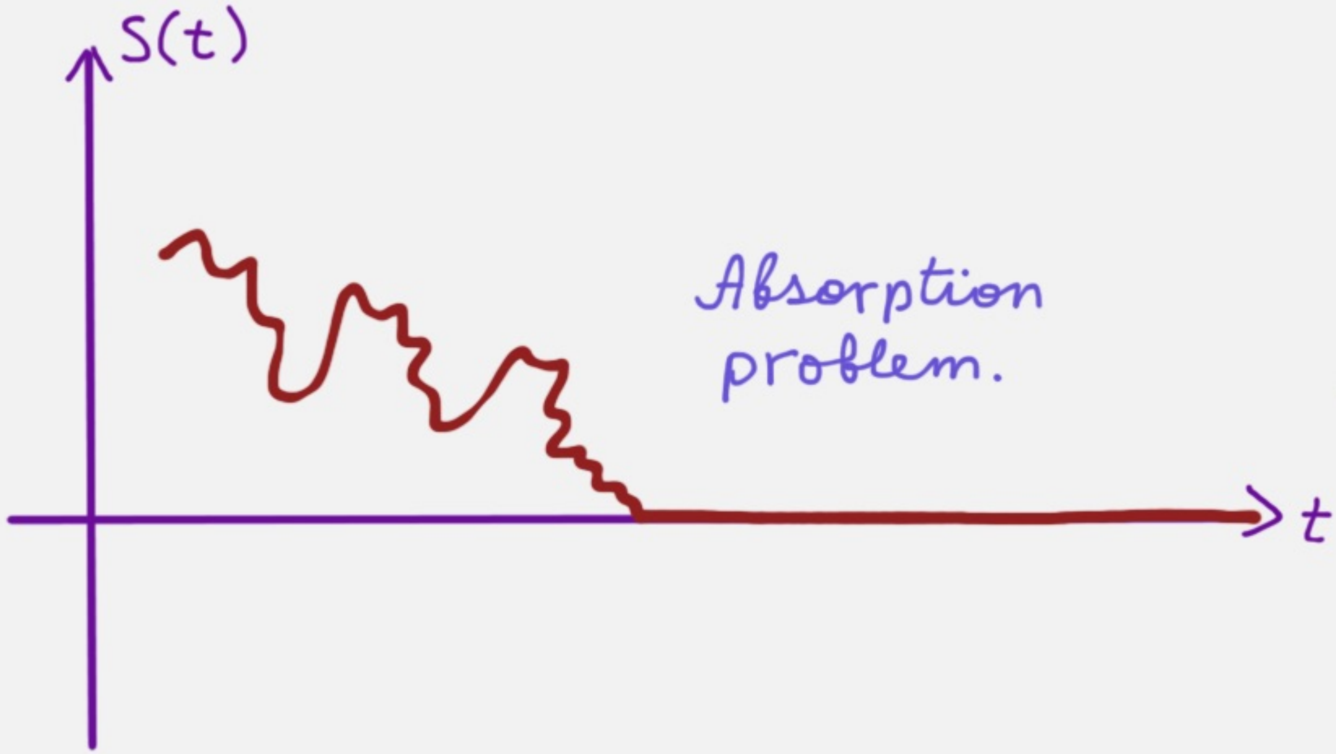
Martingale:

$(\Omega, \mathcal{F}, P)$, $\mathcal{F}_t \subset \mathcal{F}$, $E[|Y_t|] < \infty$. $Y_t = E[Y_s | \mathcal{F}_t]$

Super martingale: $Y_t \geq E[Y_s | \mathcal{F}_t]$

Regularization.

$\varphi(S) = S^p$. $\sigma = S^{p-1}$. $S = 0 \Rightarrow \sigma = 0$.



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To avoid absorption at $S=0$ in case of CEV with $p<1$ instead of $\varphi(x)$ use regularized version

$$\varphi(x)=x\min(\varepsilon^{p-1},x^{p-1}),\ \varepsilon>0$$

for some small ε.

1. $x>\varepsilon \Rightarrow \frac{1}{x^{1-p}}<\frac{1}{\varepsilon^{1-p}}$, $1-p>0$.

   $\min(\varepsilon^{p-1},x^{p-1})=x^{p-1}$. $\varphi(x)=x^p$. $\sigma=S^{p-1}$.

2. $x\le\varepsilon \Rightarrow \frac{1}{\varepsilon^{p-1}}\le\frac{1}{x^{p-1}}$, $1-p>0$.

   $\min(\varepsilon^{p-1},x^{p-1})=\varepsilon^{p-1}$. $\varphi(x)=x\varepsilon^{p-1}$. $\sigma=\varepsilon^{p-1}$.

$$\boxed{\sigma\neq 0}$$