Lecture 1

Fixed Income Derivatives (33601)

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Session Outline

- Interest rates
- Fixed income securities
- Term structure
- Zero coupon bonds
- Fixed coupon bonds
- Bond mathematics
- DepositsTreasury Bills
- Treasury Notes and Bonds
- Federal Funds Deposits
- Reourchase agreements

Interest Rates

What is an interest rate?

- Interest is a rental cost paid by the borrower to the lender in exchange for the use of capital
- An interest rate is the borrowing cost per unit of time. It is typically expressed as a percentage with an annual unit of time, i.e. an annual cost of \$5 per \$100 borrowed would be simply 5%.
- Money is lent for different lengths of time, known as terms or tenors. Different tenors have different interest rates
- Interest rate levels reflect the likelihood that the borrowed money will be paid back
- Depend on creditworthiness of the issuer or borrower
- Take into account any collateralization arrangements
- Levels are also influenced by underlying currency, including impacts due to inflation
- Interest rates fluctuate with time in the market place.

Fixed Income Securities

A fixed income security is an investment providing periodic interest payments and (possibly) the return of principal at maturity. The payment amounts and times are known in advance

- Examples include bonds, notes, and bills
- Investors are lenders
- The issuer is the borrower
- Market price is based on a present value calculation. Discounting is determined using interest rates corresponding to the timing of the future payments and the creditworthiness of the issuer
- Typically negotiable, i.e. traded in the secondary market.

Define the discount factor $P(t, T)$ as the present value at time t of 1 unit of currency payable at the future time T.

Definition

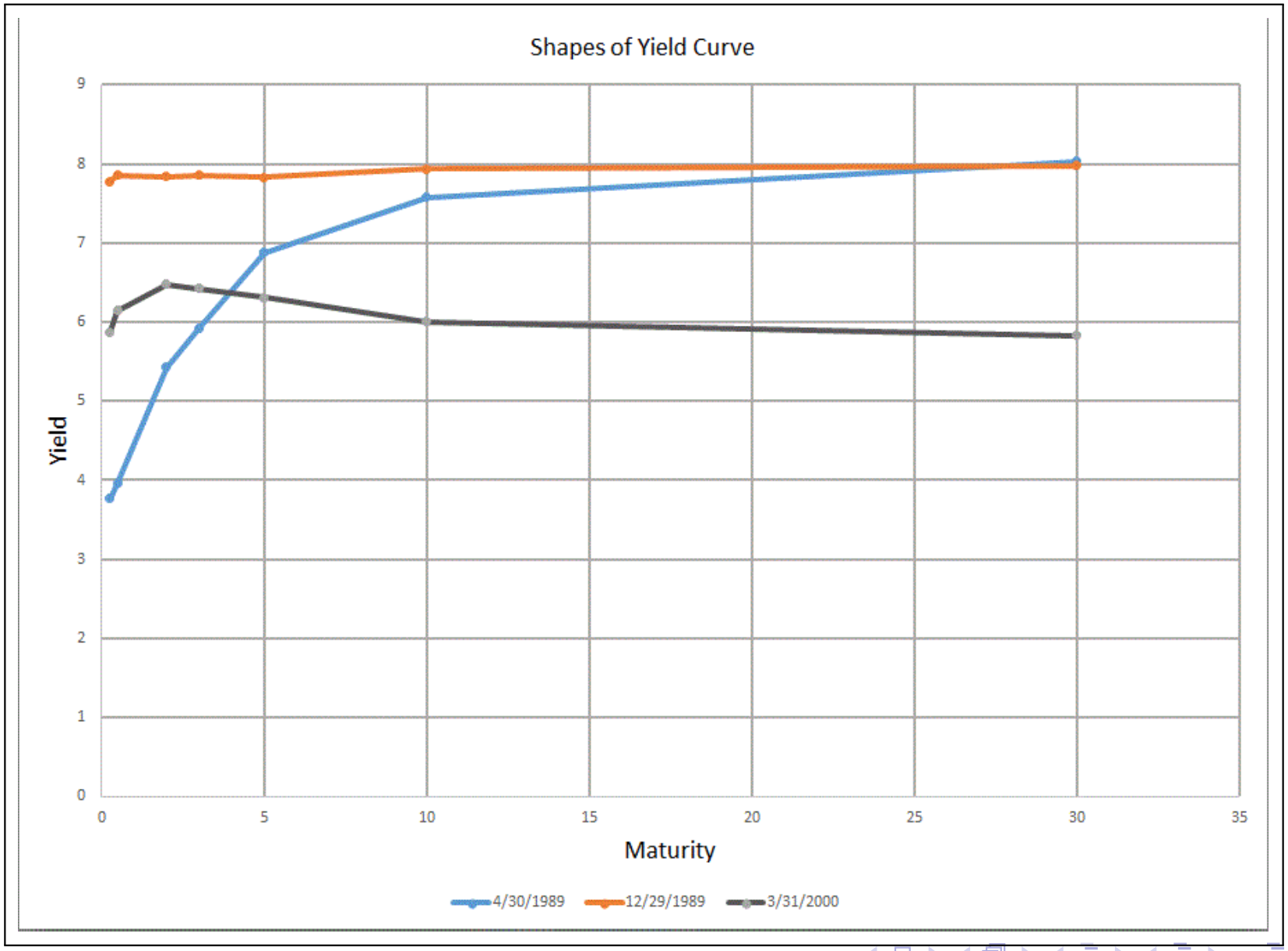
The mapping

$$T \mapsto P(t, T), T \geq t$$

is called the **time t discount curve**.

The discount curve is tied to the creditworthiness of the issuer or borrower, and any collateralization arrangements.
There are multiple discount curves in the marketplace reflecting various credit qualities and currencies.

Shape of Term Structure



Uses of Term Curve

- Comparison of bond yields and maturities
- As a leading indicator of economic conditions
- As a benchmark for pricing fixed income securities
- To create a strategy for portfolio:
- Bullet strategy: maturities concentrated in one maturity, e.g. 10 years;
- Barbell strategy: maturities concentrated in 2 extreme areas, e.g. 5 years and 20 years; outperforms during flattening
- Ladder strategy: equal amounts of securities maturing every year; used to match liability stream
- riding the yield curve, or rolling down the yield curve: in upward sloping yield curve, as a bond approaches maturity ("rolls down the yield curve" it is valued at lower yields and higher prices; a bond is held for a period of time as it appreciates in price and is sold before maturity to realize the gain

Zero Coupon Bonds

- A zero coupon or discount bond pays a fixed amount at a single time in the future
- The payment amount N is called the notional amount, principal amount, face value, or par value
- The payment time T is called the maturity date
- Present value

$$N \cdot P(t, T)$$

is a discount to the principal amount.

Fixed Coupon Bonds

- A coupon bond provides fixed interest payments on a regular schedule and a principal payment at maturity
- It can be regarded as a portfolio of zero coupon bonds
- Ergo its time t present value

$$N\left[c\sum_{i=1}^{n}\tau_i P(t,t_i)+P(t,T)\right],$$

where N is the notional amount, T is the maturity time, c is the fixed coupon rate, $t_1 < t_2 < \ldots < t_n = T$ are the coupon payment times, and $\{\tau_i\}_{i=1}^{n}$ are the coupon day counts (payment period lengths)

Bond Mathematics

Accrued Interest and Clean and Full Prices

You need to understand market quoting conventions to know how a coupon bond is priced

- Accrued interest is a calculation that accounts for coupon income which has accrued between coupon payment dates. Its value grows linearly until the coupon is paid, at which point the accrued interest resets back to zero
- Market quotes are given as clean prices excluding accrued interest
- The invoice or full price equals the clean price plus accrued interest. It represents the acutal price paid for the bond at settlement.

Example

Treasury bond **5 3/8s of 2/15/2031**
Trade Date[a]: November 4, 2002
Settlement Date[b]: November 6, 2002
Bond Price[c]: 104-13+

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[a] Also known as the spot date

[b] Treasury market convention is one business day after spot. However, in 2002 it was two business days after spot.

[c] The Treasury bond quoting convention is in full points and 1/32 of a point, with 100 points equal to the face value. The quote 104-13+ corresponds to a price of \$104.421875 per \$100 face value. I.e., the "13+" means 13.5/32.

- The accrued interest is

$$AI = \frac{5.375}{2} \times \frac{83}{184} = 1.212296.$$

- The full price is $FP = 104.421875 + AI = 105.634171$

The constant rate of return achieved by holding a bond until maturity is known as the yield to maturity. Current market price, par value, coupon payments, and time to maturity are taken into account.
The yield y for the **5 3/8s of 2/15/2031** satisfies the equation[1]

$$FP = N\left(1+\frac{y}{2}\right)^{-(n-\alpha)} + \frac{c}{2}\sum_{k=1}^{n}\left(1+\frac{y}{2}\right)^{-(k-\alpha)}$$

where $FP = 105.634171$ is the full price, $N = 100$ is the notional, $n = 57$ is the number of remaining coupons, $\alpha = \frac{83}{184}$ is the fraction of the next coupon already accrued, and $c = 5.375$ is the annual coupon amount. The solution $y = 5.0781\%$ can be found using a root finding technique such as Newton's method.

[1] This equation is the convention for US Treasury notes and bonds. Conventions vary somewhat by issuer but are qualitatively the same.

Modified duration is the relative sensitivity of a bond's price with respect to a change in the yield to maturity

$$D = -\frac{1}{FP}\frac{d}{dy}FP$$

$$= \frac{1}{FP}\left[N\frac{n-\alpha}{2}\left(1+\frac{y}{2}\right)^{-(n+1-\alpha)} + \frac{c}{2}\sum_{k=1}^{n}\frac{k-\alpha}{2}\left(1+\frac{y}{2}\right)^{-(k+1-\alpha)}\right]$$

Convexity is the second order relative sensitivity

$$\begin{aligned} C &= \frac{1}{FP}\frac{d^2}{d^2y}FP \\ &= \frac{1}{FP}\left[N\frac{(n-\alpha)(n+1-\alpha)}{4}\left(1+\frac{y}{2}\right)^{-(n+2-\alpha)}\right. \\ &\quad \left. +\frac{c}{2}\sum_{k=1}^{n}\frac{(k-\alpha)(k+1-\alpha)}{4}\left(1+\frac{y}{2}\right)^{-(k+2-\alpha)}\right] \end{aligned}$$

Duration and convexity are risk measures, as can be seen by the truncated Taylor series approximation

$$\frac{\Delta FP}{FP} \approx -D\Delta y + \frac{1}{2}C(\Delta y)^2$$

Deposits

A deposit is a security in which money is deposited for a known period of time at a fixed interest rate. It is similar to a zero coupon bond, except for two differences:

- A deposit is issued at par and pays back par plus interest at the end of the term. A zero coupon bond is issued at a discount and pays back only par at maturity
- Bonds trade in the secondary market, whereas deposits do not (deposits are not traded once established)

Example

Bank A makes a $3m$ Eurodollar deposit of $\$1,000,000$ with bank B at a rate of 5.50%, settling November 7, 2002. How much interest will be paid on February 7, 2003?

$$0.055 \times \frac{92}{360} \times \$1,000,000 = \$14,055.56$$

Treasury Bills (T-Bills)

- Issued weekly by the US Treasury via Dutch auction[2]
- Maturity dates of 4, 13, 26, and 52 weeks
- Exempt from state and local taxes
- Quoted using an annualized discount rate in simple interest terms (Act/360)
- \$2+ trillion outstanding, \$60 billion issued each week
- Very active, liquid secondary market.

[2] Bidders are filled in order from highest to lowest until the issuer is exhausted.

Treasury Notes and Bonds

- Fixed coupon bonds issued by the US Treasury
- Notes are issued with 2, 3, 5, and 10 year maturities
- Bonds are issued with 30 year maturities
- Exempt from state and local taxes
- Issued at (or very near) par value
- Pay semi-annual coupons determined by Dutch Auction
- Prices quoted in terms of par \$100 face value with $\frac{1}{32}$ of a dollar tick size using semi-annual compounding yield
- Identified by coupon and maturity, e.g. the 8 3/4s of May 2017
- \$10+ trillion outstanding
- Very active, liquid secondary market.

Fed(eral) Fund Deposits

- Same day overnight dollar deposits between US banks and/or the Federal Reserve
- Rate is pegged by the Federal Reserve
- Quoted as annualized simple interest (Act/360)
- Fed fund deposits are unsecured.

Eurodollar Deposits

- Dollar denominated deposits between major European banks
- Quoted as annualized simple interest (Act/360)
- Settlement date is two business days after trade date
- Eurodollar deposits are unsecured

Repurchase Agreements (Repo)

A repurchase agreement (repo) is a contract to sell a security to a counterparty with the promise to buy it back at a later date

- Most widely used securities are Treasury notes and bonds
- Amounts to a short term collateralized loan. Typically overnight but can be for as long as 3 months
- Often used to finance a (forward) position in the security
- The sale and repurchase prices are determined simultaneously on the trade date. Their difference equates with interest paid and is called the effective repo rate. By convention it is quoted in annualized simple interest terms using an Act/360 accrual basis
- The difference between the collateral's value and the sale price insulates the buyer from market risk and is referred to as the haircut
- The counterparties are referred to as the seller or buyer of the security at initiation
- The seller initiates a repo. The buyer initiates a reverse repo.

Repo Example

Example

Treasury bond **5 3/8s of 2/15/2031**
Trade Date: November 4, 2002
Initial Settlement Date: November 6, 2002
Repurchase Date: November 7, 2002
Bond Market Price: 104-13+
Haircut: 1%
Repo Rate: 1.25%

- Initial invoice price

$$(100\% - 1\%) \times \left(104 + \frac{13.5}{32} + 5\frac{.375}{2} \times \frac{83}{184}\right) = 104.577829$$

- Repurchase invoice price

$$104.577829 \times \left(100\% + 1.25\% \times \frac{1}{360}\right) = 104.581460$$