Forward LIBOR

FRA = forward contract on a LIBOR rate

FRAs have zero value at contract-time $t=0$.

Forward LIBOR is the rate that makes the value of the corresponding FRA be zero today.

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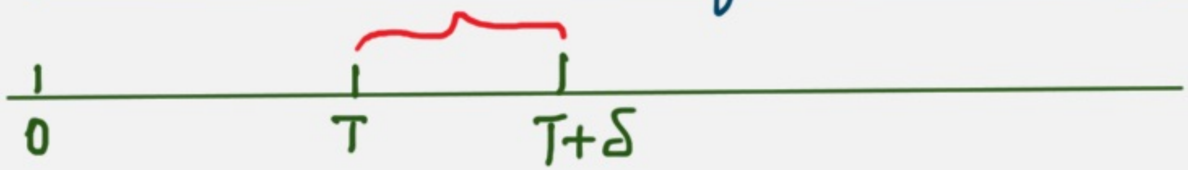
FRA written on the spot LIBOR $L(T, T+\delta)$



If the payment occurs at $T+\delta$: FRA in-arrears

$T+\delta$: Payoff $= N\delta\left[L(T,T+\delta) - L(0,T,T+\delta)\right]$

$$0 = \frac{V_0(FRA)}{P(0,T+\delta)} = \mathbb{E}^{T+\delta}\left[\frac{N\delta\left[L(T,T+\delta)-L(0,T,T+\delta)\right]}{\underbrace{P(T+\delta,T+\delta)}_{=1}}\right]$$

$$N\delta\, \mathbb{E}^{T+\delta}\left[L(T,T+\delta)-L(0,T,T+\delta)\right] = 0$$

$$L(0,T,T+\delta) = \mathbb{E}^{T+\delta}\left[L(T,T+\delta)\right]$$

$$L(t,T,T+\delta) = \mathbb{E}^{T+\delta}\left[L(T,T+\delta) \mid \mathcal{F}_t\right]$$

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Payment occurs at T:

$$\text{Payoff} = N\delta\left[\frac{L(T,T+\delta) - L(0,T,T+\delta)}{1+\delta L(T,T+\delta)}\right]$$

$$0 = \frac{V_0(FRA)}{P(0,T)} = N\delta\,\mathbb{E}^T\left[\frac{L(T,T+\delta) - L(0,T,T+\delta)}{1+\delta L(T,T+\delta)}\cdot\frac{1}{\underbrace{P(T,T)}_{=1}}\right]$$

$$\mathbb{E}^T\left[\frac{L(T,T+\delta) - L(0,T,T+\delta)}{1+\delta L(T,T+\delta)}\right] = 0$$

$$\mathbb{E}^T\left[\frac{L(T,T+\delta)}{1+\delta L(T,T+\delta)}\right] = L(0,T,T+\delta)\,\mathbb{E}^T\left[\frac{1}{1+\delta L(T,T+\delta)}\right]$$

$$L(0,T,T+\delta) = \mathbb{E}^T\left[\frac{L(T,T+\delta)}{1+\delta L(T,T+\delta)}\right] \Big/ \mathbb{E}^T\left[\frac{1}{1+\delta L(T,T+\delta)}\right] \neq \mathbb{E}^T\left[L(T,T+\delta)\right]$$

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