FINM 33601: Homework 6 – Black's Model

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1.

We know that a cap is a strip of caplets – call options on successive LIBOR rates – corresponding to the floating cashflows of a swap. Similarly, we know that a floor is a strip of floorlets – put options on successive LIBOR rates – corresponding to the floating cashflows of a swap. We can therefore attempt to analytically derive put-call-parity for caplets and floorlets in general, which in turn shows the put-call-parity relationship between caps and floors.

From the notes, a caplet written on the LIBOR rate $L(T, T+\tau)$ has payoff:

$$\text{Payoff}_{\text{caplet}}(T+\tau) = N\tau[L(T, T+\tau) - K]_+$$

where occurring at time $T + t$, the end of the period, where N is the notional of the swap, t is the day basis corresponding to the time interval $[T, T+t]$, K is the cap strike rate. Likewise, for the floorlet:

$$\text{Payoff}_{\text{floorlet}}(T+\tau) = N\tau[K - L(T, T+\tau)]_+$$

Now we create a portfolio by buying the caplet and shorting a floorlet at the same rate, giving the following overall payoff:

$$\begin{aligned}
\text{Payoff}_{\text{overall}}(T+\tau) &= \text{Payoff}_{\text{caplet}}(T+\tau) - \text{Payoff}_{\text{floorlet}}(T+\tau) \\
&= N\tau[K - L(T, T+\tau)] \\
&= \text{Payoff}_{\text{swap}}
\end{aligned}$$

Indeed, this portfolio confirms put-call-parity for a caplet and floorlet since the payoff is the same as a swap, which naturally extends to put-call-parity for caps and floors.

2.

The payoff of a payer's swaption is:

$$\text{Payoff}_{\text{payer's swaption}} = \left(P(t_0, t_0) - P(t_0, t_n) - c \sum_{k=1}^{n} \tau_k P(t_0, t_k)\right)_+$$

The payoff of a receiver's swaption is:

$$\text{Payoff}_{\text{receiver's swaption}} = \left(c \sum_{k=1}^{n} \tau_k P(t_0, t_k) - P(t_0, t_0) + P(t_0, t_n)\right)_+$$

We apply the same methodology as in 1., where we create a portfolio that is long the payer's swaption and short the receiver's swaption:

$$\begin{aligned}\text{Payoff}_{\text{overall}}(T + \tau) &= \text{Payoff}_{\text{payer's swaption}} - \text{Payoff}_{\text{receiver's swaption}} \\ &= \left(P(t_0, t_0) - P(t_0, t_n) - c \sum_{k=1}^{n} \tau_k P(t_0, t_k)\right)\end{aligned}$$

where this is the forward-starting swap, as given on slide 70 of the *Black's Model* notes. This is the put-call-parity relationship for European swaptions.