Interest Rate Derivatives

Jeff Greco

jgreco@uchicago.edu

Copyright and Disclaimer

Interest Rate Derivatives

Interest rate derivatives are contingent claims on fixed income securities and/or interest rates.
I.e., they are financial instruments whose payments and valuation derive from interest rates.
Fundamental examples include:

- Bonds and notes
- Bills
- Treasury futures
- Deposits
- Forward rate agreements
- Eurodollar futures
- Interest rate swaps

Trading Agents

Who trades interest rate derivatives?

- Banks, broker dealers, and market makers
- Corporations and businesses
- Hedge funds, private equity, venture capitalists, and other investors and speculators
- Sovereign governments, agencies, and municipalities

Purpose for Interest Rate Derivatives

Why trade interest rate derivatives?

- Leverage and debt financing
- Cashflow, asset, and liability management
- Hedge interest rate exposure
- Speculation
- Utilize a relative funding advantage

Puzzle: Utilizing a Relative Funding Advantage

To be Continued ...

- Company A
  - Can borrow at a fixed rate of 7.00%.
  - Can borrow at a floating rate of LIBOR + 100bps.
  - Prefers floating.
- Company B
  - Can borrow at a fixed rate of 7.50%.
  - Can borrow at a floating rate of LIBOR + 125bps.
  - Prefers fixed.

How can they enter into an interest rate swap agreement to exploit a relative funding advantage?

What is the “Risk-Free Rate?”

There are several common misconceptions of the meaning of the term “risk-free rate.”

- ~~It means there is no possibility of loss due to credit default.~~ No possibility of default is often assumed but is neither necessary nor the meaning of risk-free rate.
- ~~There is a risk-free rate for each maturity date.~~ The risk-free rate always matures immediately.
- In fact, the *risk-free*, *short*, or *spot rate* is the interest rate of a hypothetical zero coupon bond with an immediate maturity date.
- “Risk-free” refers to there being no uncertainty in the value of this instantaneously maturing investment (it equals 1).
- By contrast, the values of longer maturity bonds fluctuate with time as the term curve shifts.
- There is a risk-free rate corresponding to each combination of currency and underlying credit quality.
- In practice, the overnight rate plays stand in for the risk-free rate since it is the shortest maturity term traded.

Risk-Free Rate Versus the Default-Free Rate

Many texts use the term “risk-free rate” to refer interchangeably to either an instantaneous/overnight rate or else a rate that is absent the possibility of credit default.

- We will use the term *risk-free rate* exclusively for continuously compounded instantaneous/overnight interest rates;
- and the terms *default risk free* or *riskless* to refer to rates in the absence of the possibility of loss due to credit or counterparty defaults.

London Inter-Bank Offer Rate (LIBOR)

The *London Inter-Bank Offer Rate (LIBOR)* represents the average Eurodollar deposit rate offered to major banks on the London market.

- Deposit terms[1] are O/N, 1w, 1m, 2m, 3m, 6m, and 1y.
- Currencies[2] are USD, EUR, GBP, CHF, and JPY.
- Published daily[3] by ICE Benchmark Administration (IBA), a subsidiary of NYSE Euronext.

---

[1] The 2w, 4m, 5m, 7m, 8m, 9m, 10m, and 11m tenors were discontinued after May 31, 2013.

[2] NZD was discontinued after February 28, 2013. DKK and SEK were discontinued after March 28, 2013. CAD and AUD were discontinued after May 31, 2013.

[3] IBA took over administration of LIBOR as of February 1, 2014 from the British Bankers' Association (BBA) for a token 1 GBP.

LIBOR Market Facts

- Benchmark index for interest rate derivatives globally and incorporated into standard documentation, e.g. those sponsored by the International Swaps and Derivatives Associations (ISDA).
- Around $350 trillion outstanding of LIBOR-based interest rate swaps.
- Over $10 trillion in loans indexed to LIBOR.
- Basis for settlement of contracts on many of the world's major futures and options exchanges.

LIBOR Calculation

- For each currency, a total of 8-16 contributor banks are selected annually by IBA.
- Contributors are asked to base their submissions on the question “at what rate could you borrow funds ... in a reasonable market size just prior to 11 am [London time].”
- The calculation agent (Thomson Reuters) calculates LIBOR as the trimmed arithmetic mean of the middle two quartiles.
- Submissions are not based on actual market transactions.
- LIBOR indicates the average rate at which leading banks (AA credit rating) can obtain unsecured funding for a given period and currency in the London market.

Forward Rate Agreements (FRA)

- Cash settled forward Eurodollar deposit.
- Contract particulars
  - Currency, e.g. USD.
  - Notional amount, e.g. $1,000,000.
  - Forward settlement date.
  - Deposit term, e.g. 6m.
  - Index rate, e.g. LIBOR.
  - Forward strike rate.

Assume today is Thursday November 12, 2002 and you buy (borrow) a 3 x 9 FRA (settles in 3 months to 6m LIBOR) struck at 5.50% on $1,000,000 notional.
If 6m LIBOR is 5.74% in 3 months, how much compensation is due to you?

$$\$1,000,000 \times \frac{(0.0574 - 0.0550) \times \frac{180}{360}}{1 + 0.0574 \times \frac{180}{360}} = \$1166.52$$

Eurodollar Futures

- Futures contract on Eurodollar deposits.
- Traded on the Chicago Mercantile Exchange (CME).
- Cash settled to 3m LIBOR at expiration assuming \$1,000,000 notional per contract.
- Quoted as[4] 100-(rate).
- Marked-to-market daily to the closing futures price, i.e. \$25 per basis point[5] per contract.
- Listed out to 10 years, expiring every 3 months[6] in March, June, September, and December. Liquid out to 2-3 years.

[4] E.g. 93.50 corresponds to a rate of 6.50%.

[5] This is not a forward contract. Why?

[6] There are also four serial months listed. E.g., if it were June 2002, then July, August, October, and November 2002 contracts would also be listed.

Eurodollar Futures Example

Suppose you go long one Eurodollar futures contract at 93.50. At the end of the day it closes at 93.90, so you receive $\$25 \times 40 = \$1000$ in your margin account.
At expiration, 3m USD LIBOR is 6.00%. Therefore you will have received a total of

$$\$25 \times 100 \times ((100 - 6.00) - 93.50) = \$1250$$

throughout the time you held the position.
Suppose you had instead sold[7] (lend) a 3m FRA struck at 6.50%. Then you would have received a total of

$$\$1,000,000 \times \frac{(0.0650 - 0.0600) \times \frac{90}{360}}{1 + 0.0600 \times \frac{90}{360}} = \$1231.53$$

at settlement.

[7] Notice that the convention of buy/sell and long/short is reversed between Eurodollar futures and FRAs.

Eurodollar Futures Convexity Adjustment

Eurodollar futures and FRAs are very similar and are often thought of as interchangeable. There are two important distinguishing features to consider when comparing them:

- Eurodollar futures are marked-to-market daily and have a linear dependence on the underlying LIBOR rate

$$1,000,000 \times (1 - 0.25 \cdot L)$$

- FRAs have a single netted payment at the forward settlement date and have a non-linear dependence on LIBOR with (small) positive convexity

$$1,000,000 \times \frac{\tau\,(K - L)}{1 + \tau L}$$

As a consequence, there is a difference in the fair value of these securities, i.e. the futures price is not simply $100 \times (1 - K)$. This basis or difference is known as the *Eurodollar futures convexity adjustment*. Later in the course we will learn how to model and price this.

LIBOR Swaps

- Agreement between two counterparties to periodically exchange fixed versus floating LIBOR cashflows for a specified period of time.
- The cashflows are paid on a netted basis.
- The most typical USD interest rate swap has semi-annual fixed payments (30/360) versus quarterly floating rate payments (3m LIBOR Act/360).
- A payer (receiver) swap is one in which you agree to pay (receive) the fixed rate and receive (pay) the floating rate.
- Contract particulars
  - Notional amount, e.g. $1,000,000.
  - Currency, e.g. USD.
  - Swap term, e.g. 5y.
  - Fixed rate, e.g. 4.00%.

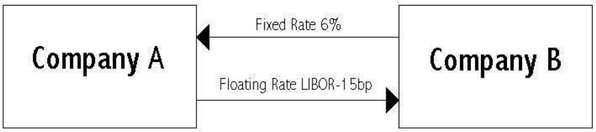
Puzzle: Utilizing a Relative Funding Advantage

LIBOR Swap Application

Primary market financing available to companies A and B.

	Company A	Company B
Fixed Rate Debt	7.00%	7.50%
Floating Rate Debt	LIBOR+100bp	LIBOR+125bp
Prefers to Issue	floating rate debt	fixed rate debt

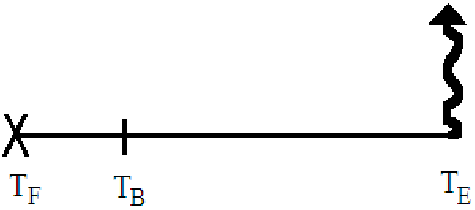
The companies A and B issue fixed and floating rate debt, respectively. Then they enter into a swap agreement.



This result is an effective rate of LIBOR+85bp for Company A and 7.40% for Company B. They have now created synthetic liabilities in their preferred forms at superior funding levels.

LIBOR Swap Floating Payment Structure

- Notional amount, e.g. $\$100,000,000$.
- Index for fixing the floating rate, e.g. 3m LIBOR.



- Payment time T_E when cashflow is paid/received.
- Accrual period, comprised of a beginning time T_B and ending time T_E (with $[T_B, T_E]$ matching the LIBOR index tenor).
- Fixing time T_F when LIBOR index is observed to fix the floating rate (typically two business days prior to the accrual period beginning).

Overnight Index Swap (OIS) Reference Rate

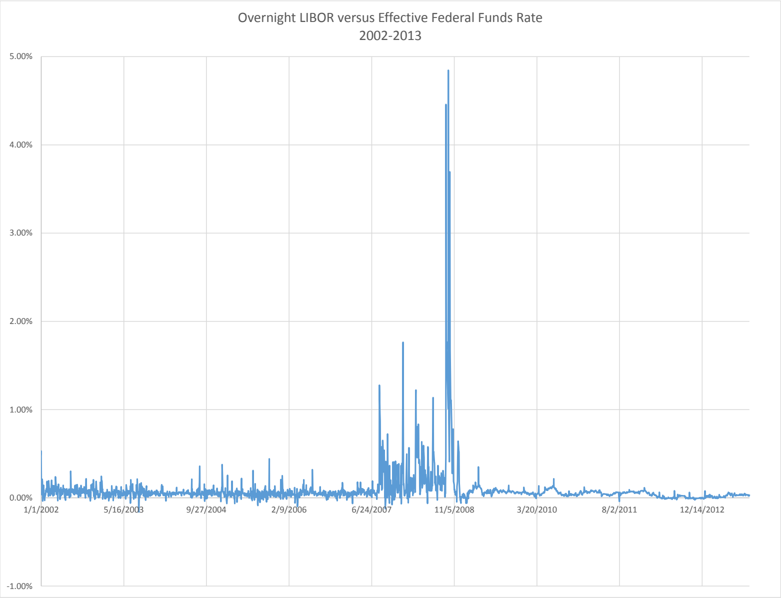
An emerging alternative index to LIBOR is the *Overnight Index Swap (OIS) Reference Rate*.

- Official overnight effective rate for capital reserve deposits. Determined daily by the local central bank as the weighted average of rates on brokered trades.[8]
- Available for most currencies and for many is the only liquid[9] interest rate index.
    - Examples: Federal Funds (USD), CORRA (CAD), EONIA (EUR), SONIA (GBP), CHOIS (CHF), TONA (JPY), AONIA (AUD), NZIONA (NZD), DKKOIS (DKK), SIOR (SEK), SONAR (SGD), HONIX (HKD).
- Like LIBOR, underlying market deposits are unsecured.
- Unlike LIBOR, based on actual market transactions.
- OIS discounting is the market standard for pricing collateralized trades and mandated by clearing houses.

[8] As of March 1, 2016, the USD OIS reference rate is a volume-weighted median of transactions. Prior to March 2016 it was a volume-weighted mean.

[9] LIBOR is still more liquid in its five remaining currencies.

Historical LIBOR-OIS Basis



LIBOR vs. OIS as the Benchmark Interest Rate Derivatives Index

- Interest rates need not be completely free of default risk, as we have previously discussed.
- However, interest rate derivatives are distinct from credit derivatives. The likelihood of default in the benchmark index should be kept to a minimum.
- AA rated banks possess a small but significant possibility of default.
- Traditionally the argument for LIBOR as an interest rate benchmark is derivatives are not long term investments in a single AA bank whose credit quality may deteriorate.
- Instead they embody rolling short term investments with a series of banks chosen each period to have a solid AA standing with no significant possibility of defaulting in the period.

LIBOR vs. OIS as the Benchmark

- Recent difficulties faced by LIBOR are not primarily due[10] to credit quality. They are primarily about published values:
  - not being based on actual market transactions
  - submissions being subjective
  - possibly manipulated during 2007-2009 credit crisis
- OIS has advantages and may supplant[11] LIBOR as the benchmark.
  - based on actual transactions
  - underlying investment period is shortened to one day
- OIS has already taken over as the funding rate for the collateral backing most derivative trades.

---

[10] The difficulties are not *directly* about credit quality, though the motivations behind the alleged manipulations may be all about credit quality.

[11] The Financial Stability Board (FSB), representing G20 central banks, is considering alternatives to the LIBOR benchmark, including OIS, T-bill rate, and general collateral repo. The FSB may even choose different or multiple benchmarks per currency with some member jurisdictions considering doing so forcefully. Cf. `http://www.financialstabilityboard.org/wp-content/uploads/r_140722.pdf`.

Overnight Index Swaps (OIS)

- Exchange fixed versus floating OIS cashflows for a specified swap tenor.
- Shorter swap tenors of one year or less typically have a single cashflow at the end.
- Longer tenors have periodic cashflows paid on a netted basis.
- Each floating payment amount is determined as the compound rate of return of a daily investment at the OIS reference rate (geometric mean)

  $$\prod_{i=1}^{k} (1 + \tau_i R_i) - 1$$

  where k is the number of business days in the accrual period, R_i are the OIS reference rates, and τ_i are the accrual values.
- The payment occurs on a delayed $T + 2$ or $T + 1$ basis, depending on the currency.

Federal Funds/LIBOR Basis Swaps

- Periodic (typically quarterly) netted exchange of LIBOR versus Federal Funds (FF) floating rate payments for the specified swap tenor.
- Usually these swaps are referred to as simply "Feds."
- There is a two day rate cutoff[12] applied to the FF leg because of the one day lag in the release of the FF Effective rate, i.e. the final FF fixing is repeated and applied to both of the last two fixing days.
- A fixed rate spread S is negotiated and added to the FF leg with the payment based on an arithmetic[13] average, in contrast to the OIS compounding convention

$$\sum_{i=1}^{k} \tau_i \left(R_i + S \right)$$

where the variable definitions are the same as for the OIS swap floating payment.

[12] This is not necessary for ordinary OIS since payments are delayed.

[13] This implies the need for a convexity adjustment when pricing.

The Break-Even Rate

When trading a swap or an FRA, counterparties negotiate the rate.

- The strike rate is negotiated for an FRA.
- The fixed rate is negotiated for LIBOR swaps and OIS swaps.
- The spread is negotiated for Feds.

The *break-even rate (or spread)* is the theoretical level for which the instrument has a present value equal to zero.

- Trades usually occur at or near the break-even level.
- Deviation from break-even levels outside of bid-ask spread typically coincides with arbitrage opportunity.

Interest Rate Swap Valuation

Fixed Leg

The present value of a fixed leg at time t is

$$V_{\mathrm{FIX}}(t) = \sum_{i=1}^{n} c\tau_i P(t, t_i)$$

where n is the number of payment periods, c is the swap fixed rate, $t_1 < t_2 < \cdots < t_n$ are the fixed rate payment times, $\{\tau_i\}_{i=1}^{n}$ are the period accrual lengths in fractions[14] of a year, and a notional value of 1 is assumed.
The discounting

$$T \mapsto P(t, T)$$

is chosen to match the appropriate funding rate, e.g. OIS or LIBOR.

[14] These are not necessarily given by $\tau_i = t_i - t_{i-1}$, but depend on the day count convention used, e.g. 30/360.

Interest Rate Swap Valuation

Floating Leg

The present value of a floating leg at time t is

$$V_{\mathrm{FLOAT}}(t) = \sum_{i=1}^{n} R(t, t_{i-1}, t_i)\, \tau_i P(t, t_i)$$

where the variables are analogous to those used for a fixed leg and $R(t, t_{i-1}, t_i)$, $i = 1, 2, \ldots n$ are the forward floating rates for each float payment, i.e. either forward LIBOR or OIS depending on the floating payment indexing type.

The discounting $T \mapsto P(t, T)$ is again chosen to match the appropriate funding rate.

Interest Rate Swap Valuation

Matched and Mismatched Discounting and Indexing Types

The discounting (funding rate) will not necessarily match the float indexing type. For instance the forward rates $R(t, t_{i-1}, t_i)$ may fix to LIBOR and the discount factors $P(t, t_i)$ may be based on OIS.

- When the discounting matches the float indexing type then the forward floating rates simplify to[15]

  $$R\left(t, t_{j-1}, t_j\right) = \frac{P\left(t, t_{j-1}\right)/P\left(t, t_j\right) - 1}{\tau_j}, \qquad j = 1, 2, \ldots n \ .$$

  Notice that the value of the entire float leg simplifies to

  $$P\left(t, t_0\right) - P\left(t, t_n\right).$$

- When the discounting and indexing types differ, then no further simplification is available. The forward floating rates are determined by market supply and demand.

[15] See the appendix for proof using arbitrage arguments.

Interest Rate Swap Valuation

Combining the Legs (Fixed/Float Swap with Matched Discounting)

- Fixed/float swap value with matched discounting

$$V_{\mathrm{FIX}}(t) - V_{\mathrm{FLOAT}}(t) = \sum_{i=1}^{n} c\tau_i P(t, t_i) - (P(t, t_0) - P(t, t_n))$$

- Setting the above to zero and solving for c yields the break-even swap rate

$$c = \frac{P(t, t_0) - P(t, t_n)}{\sum_{i=1}^{n} \tau_i P(t, t_i)}$$

Interest Rate Swap Valuation

Combining the Legs (LIBOR Swap with OIS Discounting)

- LIBOR receiver swap value under OIS discounting[16]

$$V_{\mathrm{FIX}}(t) - V_{\mathrm{LIBOR}}(t) = \sum_{i=1}^{n} c\tau_i P(t, t_i) - \sum_{j=1}^{m} L\left(t, t'_{j-1}, t'_j\right) \tau'_j P\left(t, t'_i\right)$$

- Break-even swap rate

$$c = \frac{\sum_{j=1}^{m} L\left(t, t'_{j-1}, t'_j\right) \tau'_j P\left(t, t'_i\right)}{\sum_{i=1}^{n} \tau_i P(t, t_i)}$$

[16] The fixed and floating leg payment frequencies, number of payments, and/or accrual bases may not be the same. Thus the indexing varies between the two legs, and primes have been added to differentiate the floating leg payment times and accrual bases from those of the fixed leg. The variable L has been substituted in place of R to emphasize the forward floating rates are LIBOR based.

Interest Rate Swap Valuation

Further Notes

- The swap valuation formulas presented apply directly when the valuation and trade dates match, i.e. spot market swaps.
- The formulas must be modified slightly when applied to seasoned swaps, i.e. for any valuation date following the trade date.
- In particular, at any point in time, up to the first two immediately following cash flows of each floating payment leg may have been already “fixed.”
- In this case the appropriate forward rate(s) $R(t, t_{i-1}, t_i)$ should be replaced by the known payment amounts.

Interest Rate Swap Valuation (continued)

Further Notes

- Further, OIS cash flows do not “fix” all at once. A portion fixes daily during its accrual period. A single payment amount will ultimately be the daily compounded OIS reference rate

$$\prod_{i=1}^{k} \left(1 + \tau_i R\left(t_{i-1}, t_{i-1}, t_i\right)\right) - 1$$

- The terms in the product need to be partitioned into the known “fixed” rates and unknown rates remaining in the accrual period. I.e. choose the index k^- such that $t_{k^-} \leq t < t_{k^-+1}$, then the OIS payment can be represented as

$$\prod_{i=1}^{k^-} \left(1 + \tau_i R\left(t_{i-1}, t_{i-1}, t_i\right)\right) \prod_{i=k^-+1}^{k} \left(1 + \tau_i R\left(t_{i-1}, t_{i-1}, t_i\right)\right) - 1$$

Appendix

Present Value of a Swap Floating Payment with Matched Discounting (Funding Rate)

Following is a proof of the arbitrage-free price of a swap floating payment under matched discounting. Without loss of generality, the floating payment indexing and discounting/collateral funding rate are assumed to be LIBOR derived.

Consider the following strategy for the time line $t \leq T_F \leq T_B \leq T_E$:

- At time t, trade the following portfolio:
  - Buy 1 notional of the T_B-maturity bond.
  - Sell 1 notional of the T_E-maturity bond.
  - Agree to pay a single period LIBOR swap floating cash flow, fixing at time T_F and covering the period $[T_B, T_E]$.
- At time T_F agree to make a Eurodollar deposit for the amount 1 and covering the period $[T_B, T_E]$.

Appendix (continued)

Present Value of a Swap Floating Payment with Matched Discounting (Funding Rate)

This strategy has zero net cash flow for all times except at time t for the net amount of

$$P(t, T_E) - P(t, T_B) + \tau L(t, T_B, T_E) P(t, T_E).$$

Therefore, in the absence of arbitrage, this must sum to zero, and the fair present value of the forward LIBOR rate is as expected

$$L(t, T_B, T_E) = \frac{P(t, T_B) / P(t, T_E) - 1}{\tau}.$$

This verifies of the definition of forward LIBOR presented in the swap discussion above.