Term Structure and Yield Curve Bootstrapping

Jeff Greco

jgreco@uchicago.edu

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Fixed Income Modeling Core Concept: Term Structure

- The term structure, or yield curve, is at the heart of fixed income derivatives modeling.
- Furthermore it is used for discounting and is therefore an important component in all other financial product areas.
- It can be expressed as the discount factors $P(t, T)$ and is an infinite dimensional stochastic process.
  - The infinite dimensional index is $T \geq t$.
  - The stochastic time variable is $t \geq 0$.
- Following the 2008 Global Financial Crisis (GFC), it has become a common requirement to price fixed income derivatives using two or more term structures simultaneously, e.g. both OIS and LIBOR.

Discount Factors

- Recall our definition of a *discount factor* $P(t, T)$. It is the present value at time t of 1 unit of currency payable at the future time T.
- I.e., it is the price of a zero coupon bond maturing at time T, with unit face amount, observed at time t.
- The mapping

  $$T \mapsto P(t, T), \qquad T \geq t$$

  is called the time t *discount curve*. It provides the prices for all possible zero coupon bonds of **a single credit quality** simultaneously.
- Typically, we will work with OIS and LIBOR discount curves.
- Time is measured in calendar days using an ACT/365 day count convention, unless specified otherwise.

Forward Discount Factors

- For fixed times $t \leq T \leq M$, define the *forward discount factor* as

$$P(t, T, M) = \frac{P(t, M)}{P(t, T)}.$$

- Let $F(t)$ represent the T-forward price of a bond maturing at time M.
- Consider the following portfolio:

Bond Maturity	**Face Amount**	**Settlement Time**	**Cashflow at t**	**Cashflow at T**	**Cashflow at M**
M	1	t (spot)	$-1 \times P(t, M)$	0	$+1$
T	$-P(t, T, M)$	t (spot)	$+P(t, T, M) \times P(t, T)$	$-P(t, T, M)$	0
M	-1	T (forward)	0	$+F(t)$	-1
		Totals	0	$F(t) - P(t, T, M)$	0

- This presents an arbitrage opportunity unless $F(t) = P(t, T, M)$. Therefore $P(t, T, M)$ must be the arbitrage-free forward bond price.

Forward Rates

- The *discrete forward rate* $f(t, T, M)$ is defined as

$$f(t, T, M) = -\frac{1}{M - T} \log P(t, T, M).$$

- The *instantaneous forward rate* $f(t, T)$ is defined as the limit

$$f(t, T) = \lim_{M \to T} f(t, T, M) = -\frac{\partial}{\partial T} \log P(t, T).$$

- Forward rates are continuously compounding.
- Discount factors can be recovered from forward rates

$$\begin{aligned} P(t, T, M) &= \exp\{-(M - T) \cdot f(t, T, M)\} \\ &= \exp\left\{-\int_T^M f(t, S)\, dS\right\}. \end{aligned}$$

Zero Rates

- The *zero rate* $R(t, T)$ is defined as

$$R(t, T) = f(t, t, T)$$

  and is the continuously compounding rate of return of the zero coupon bond.
- Discount factors can be expressed in terms of the zero rates

$$\begin{aligned} P(t, T) &= \exp\{-(T - t) \cdot R(t, T)\} \\ &= \exp\left\{-\int_t^T f(t, S)\, dS\right\}. \end{aligned}$$

The Spot Rate, Risk-Free Rate, or Short Rate

- The *spot, risk-free,* or *short rate* $r(t)$ is defined as

$$r(t) = f(t,t).$$

- It is the rate earned over an infinitesimal time period.
- It is the rate earned by a hypothetical *money market account*

$$B_t = \exp\left\{\int_0^t r(s)\,ds\right\}.$$

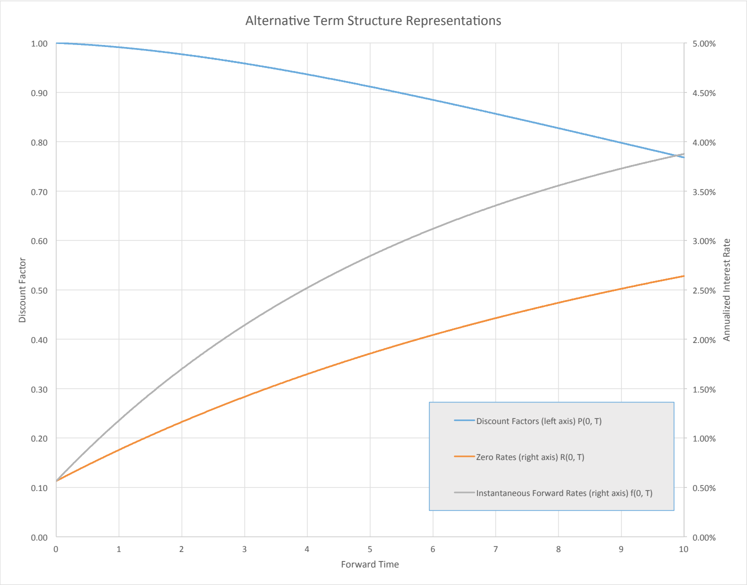
- The discount curve can <u>not</u> be recovered from the spot rate alone.

Term Structure

The *term structure* is interchangeably represented by any of the mappings

$$
\begin{aligned}
T &\longmapsto P(t, T) \\
(T, M) &\longmapsto P(t, T, M) \\
(T, M) &\longmapsto f(t, T, M) \\
T &\longmapsto f(t, T) \\
T &\longmapsto R(t, T)
\end{aligned}
$$

and may also be referred to as the *discount curve* or *yield curve*.



Alternative Term Structure Representations
Discount Factor
Annualized Interest Rate
Forward Time
1.00
0.90
0.80
0.70
0.60
0.50
0.40
0.30
0.20
0.10
0.00
5.00%
4.50%
4.00%
3.50%
3.00%
2.50%
2.00%
1.50%
1.00%
0.50%
0.00%
0
1
2
3
4
5
6
7
8
9
10
Discount Factors (left axis) P(0, T)
Zero Rates (right axis) R(0, T)
Instantaneous Forward Rates (right axis) f(0, T)

Term Structure Representations: Summary Table

Name	Symbol	Definition
discount factor	$P(t, T)$	given
forward discount factor	$P(t, T, M)$	$\dfrac{P(t, M)}{P(t, T)}$
discrete forward rate	$f(t, T, M)$	$-\dfrac{1}{M-T} \log P(t, T, M)$
instantaneous forward rate	$f(t, T)$	$\lim_{M \to T} f(t, T, M)$
zero rate	$R(t, T)$	$f(t, t, T)$
short rate	$r(t)$	$f(t, t)$

Yield Curve Construction: An Important Challenge

- Practitioners commonly take modeling of the term structure for granted, in some cases using any available implementation without considering the trade-offs.
- Just because yield curve construction is often ignored does not mean it is easy.
  - Infinite number of discount factors $P(0, t)$, $t \geq 0$ must be estimated from a small number of market quotes.
  - Underdetermined system.
  - Quoted securities can have overlapping time scales, contradictory prices, and complex relationships to the yield curve.
- The following slides present the yield curve construction technique known as *bootstrapping*.

Why Bootstrapping?

- There are many approaches to yield curve construction with advantages and disadvantages to each. Bootstrapping is one of the simpler methods employed, is widely used, and has a broad range of applications.

- Furthermore, it serves as a starting point for the majority of yield curve construction methods. It is an essential algorithm to master in order to understand the pros and cons of any method under consideration.

Outline of the Bootstrap Algorithm

The yield curve bootstrapping algorithm follows these steps.

1. Market quotes are chosen to construct the curve, e.g. published LIBOR indices, Eurodollar futures prices, and swap rates.
2. The chosen quotes are sorted in ascending order of their final tenors or maturity dates, e.g. starting with an overnight rate and ending with a 30 year swap rate. Each final tenor point/maturity date is called a *knot time*.
3. The front of the curve up to the first knot time is set to the initial quote, e.g. the effective Fed Funds rate. This establishes the risk-free rate.
4. The remainder of the curve is constructed recursively, extending the curve in each step to the next knot time in the sorted list of market quotes.

Choosing the Quotes

Care must be taken when choosing which quotes to use for constructing the yield curve. When possible, it is important to use liquid markets, minimize the use of maturity dates that are too close together, and avoid instruments with an overly complex or weak pricing relationship to the yield curve.

- LIBOR quotes typically include
  - Published LIBOR indices
  - Eurodollar futures prices
  - LIBOR swap rates
- OIS quotes typically include
  - Average overnight reserves rate or effective Fed Funds rate
  - OIS swap rates
  - Fed Funds/LIBOR ("Feds") basis swap spreads

Other possibilities include FRA, Fed Fund futures, and even options prices when there is no liquid alternative.

Representing the Curve

The term structure can be represented interchangeably as either discount factors or interest rates, on either a spot or forward basis. In the interest of concreteness, we will use the following conventions with the bootstrap algorithm:

- Time $t \geq 0$ represents the number of years from spot, i.e. now.
- Time is measured using an ACT/365 accrual basis.
- The term structure is represented by continuously compounded instantaneous forward rates $t \mapsto f(0, t)$.

Parameterizing the Curve

We are solving an underdetermined system with an infinite number of solutions and no best choice. We will use interpolation to construct a “reasonable” yield curve consistent with observed market quotes. Any method may be used with bootstrapping, though for concreteness we will use:

- Piecewise constant forward rates. Specifically we will choose a left continuous step function.
- Sorted knot times $0 = t_0 < t_1 < t_2 < \cdots < t_n$ taken from the quotes’ final tenors.
- Flat extrapolation of the forward rates beyond t_n out to infinity.

Since $t \in (t_{i-1}, t_i] \Rightarrow f(0, t) = f(0, t_{i-1}, t_i)$, the problem is reduced to solving for a finite number of discrete forward rates

$$\{f(0, t_{i-1}, t_i)\}_{i=1}^{n}.$$

Bootstrapping Example: EUR OIS Curve

Steps 1&2: Choosing and Sorting the Market Quotes

December 27, 2013

Instrument	Accrual Begin	Accrual End	Accrual Basis[1]	Knot Time[2]	Market Quote
EONIA	12/27/13	12/30/13	0.00833	0.008	0.17100%
EUR OIS 3M	12/31/13	03/31/14	0.25000	0.258	0.16275%
EUR OIS 6M	12/31/13	06/30/14	0.50278	0.507	0.15140%
EUR OIS 1Y	12/31/13	12/31/14	1.01389	1.011	0.14820%
EUR OIS 2Y	12/31/13	12/31/15	2.02778	2.011	0.22480%
EUR OIS 3Y	12/31/13	12/30/16	3.04167	3.011	0.41857%
EUR OIS 5Y	12/31/13	12/31/18	5.07222	5.014	0.92097%
EUR OIS 10Y	12/31/13	12/29/23	10.13889	10.011	1.85693%

The knot times $0 = t_0 < t_1 < t_2 < \cdots < t_8$ are taken from the above table, with $t_0 = 0$ representing the spot date 12/27/13.

[1] The accrual basis for both EONIA and EUR OIS is ACT/360.

[2] Time is measured relative to 12/27/13 using an ACT/365 accrual basis.

Bootstrapping Example: EUR OIS Curve

Step 3: Setting the Front of the Curve

The goal is to determine the discrete forward rate $f(0,0,t_1)$ from the EONIA quote $R(0,0,t_1)$.

Denote the EONIA accrual basis (ACT/360) for the period between 0 (12/27/13) and t_1 (12/30/13) by $\tau_1 = \frac{3}{360}$.

Using the relationships

$$P(0,0,t_1) = e^{-f(0,0,t_1)t_1} \quad \textbf{and} \quad 1 + \tau_1 R(0,0,t_1) = P(0,0,t_1)^{-1},$$

with the later holding under matched discounting, we find

$$\begin{aligned} f(0,0,t_1) &= -\frac{1}{t_1} \log P(0,0,t_1) = \frac{1}{t_1} \log(1 + \tau_1 R(0,0,t_1)) \\ &= \frac{365}{3} \times \log\left(1 + \frac{3}{360} \times 0.17100\%\right) = 0.17337\%. \end{aligned}$$

Some Notes About OIS Conventions

- OIS are fixed/float swaps consisting of annual netted payments.
- Tenors of 1 year or less have only a single payment.
- Floating leg fixes to the daily compounded average overnight reserve deposit rate.
- Publication of the overnight rate occurs the following morning. Therefore OIS payments are delayed one business day[3] to accommodate the last observation.
  - In theory a convexity adjustment should be applied to OIS quotes.
  - In practice the effect is trivial ($< \frac{1}{10}$ basis point) and hence safely ignored.

[3] The payment delay is two business days in some jurisdictions, e.g. USD.

Bootstrapping Example: EUR OIS Curve

Step 4: Extend the Curve to the 3M, 6M, and 1Y Points

Set $t' = \frac{4}{365}$ to correspond to the EUR OIS accrual begin date 12/31/13 and denote the accrual bases in order by $\tau_2, \tau_3, \ldots, \tau_8$ as provided in the table. The market quotes have a one day gap between t_1 and t'. Fortunately the step function parametrization ensures $f(0, t_1, t_2) = f(0, t_1, t') = f(0, t', t_2)$.

The 3M, 6M, and 1Y quotes provide

$$
\begin{aligned}
f(0, t_1, t_2) &= -\frac{1}{t_2 - t'} \log P(0, t', t_2) = \frac{1}{t_2 - t'} \log\left(1 + \tau_2 R(0, t', t_2)\right) \\
&= \frac{365}{90} \times \log\left(1 + \frac{90}{360} \times 0.16275\%\right) = 0.16498\% \\
f(0, t_2, t_3) &= \frac{1}{t_3 - t_2} \log \frac{1 + \tau_3 R(0, t', t_3)}{1 + \tau_2 R(0, t', t_2)} = 0.14204\% \\
f(0, t_3, t_4) &= \frac{1}{t_4 - t_3} \log \frac{1 + \tau_4 R(0, t', t_4)}{1 + \tau_3 R(0, t', t_3)} = 0.14690\%.
\end{aligned}
$$

Bootstrapping Example: EUR OIS Curve

Step 4: Extend the Curve Recursively to Swaps Beyond 1Y

Assume we have bootstrapped the curve out to t_{i-1} and wish to extend it to t_i using the next quoted mufti-payment OIS breakeven rate[4,5]

$$c = \frac{P(0, t') - P(0, s_m)}{\sum_{j=1}^{m} \widehat{\tau}_j P(0, s_j)}$$

with $\{s_j\}_{j=1}^{m}$ denoting the ordered swap fixed payment times and $\{\widehat{\tau}_j\}_{j=1}^{m}$ the corresponding accrual bases. A little algebraic rearrangement shows we need to solve

$$c = \frac{P(0, t', t_{i-1})^{-1} - e^{-f(0,t_{i-1},t_i)(t_i - t_{i-1})}}{\sum_{j=1}^{\overline{m}} \widehat{\tau}_j P(0, s_j, t_{i-1})^{-1} + \sum_{j=\overline{m}+1}^{m} \widehat{\tau}_j e^{-f(0,t_{i-1},t_i)(s_j - t_{i-1})}}$$

for $f(0, t_{i-1}, t_i)$ where $\overline{m}$ is chosen such that $s_{\overline{m}} \leq t_{i-1}$ and $s_{\overline{m}+1} > t_{i-1}$.

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[4] This formula holds under matched discounting.

[5] Alternate variables s_j and $\widehat{\tau}_j$ are used to avoid collision with those previously defined. Note that $s_m = t_i$.

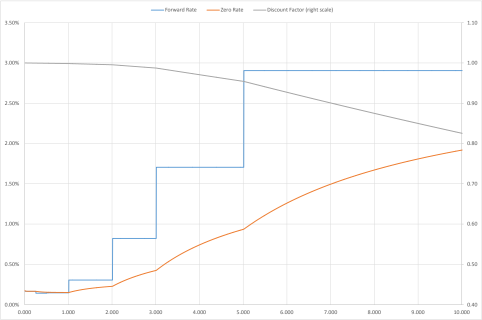
Bootstrapping Example: EUR OIS Curve

Final Results

It is not generally possible to analytically solve[6] for $f\,(0, t_{i-1}, t_i)$ in the breakeven rate equation. Therefore a numeric root finding technique is needed, e.g. Newton-Raphson.

EUR OIS 12/27/13

Knot Time	Forward Rate
0.008	0.17337%
0.258	0.16498%
0.507	0.14204%
1.011	0.14690%
2.011	0.30536%
3.011	0.82153%
5.014	1.70509%
10.011	2.90506%



[6]This is due to the second summation in the denominator. It can be solved for analytically in cases where it contains only one term, e.g. the 2Y and 3Y OIS in our example.

Sample USD OIS Market Quotes

December 31, 2013

Instrument	Market Quote	Accrual Begin	Accrual End
Effective FF Rate	0.08000%	12/31/13	01/02/14
USD OIS 3M	0.09200%	01/03/14	04/03/14
USD OIS 6M	0.10200%	01/03/14	07/03/14
USD OIS 1Y	0.13000%	01/03/14	01/05/15
USD OIS 2Y	0.28000%	01/03/14	01/04/16
FF/LIBOR Basis Swap 3Y	0.23640%	01/03/14	01/03/17
USD LIBOR Swap 3Y	0.87000%		
FF/LIBOR Basis Swap 5Y	0.27060%	01/03/14	01/03/19
USD LIBOR Swap 5Y	1.78000%		
FF/LIBOR Basis Swap 10Y	0.32191%	01/03/14	01/03/24
USD LIBOR Swap 10Y	3.09000%		

USD OIS Curve Construction: A Special Case

- USD OIS beyond 2 years are not liquid in today's market.
- Fortunately FF/LIBOR basis swaps, called "Feds," offer the world's most liquid longer dated OIS market.
- Unfortunately the Feds, due to a longer history, have different conventions than the rest of the OIS marketplace.
  - Float/float basis swaps between FF and LIBOR legs. Each spread must be combined with a USD LIBOR swap rate to supply a (synthetic) pure OIS rate to the bootstrap algorithm.
  - Quarterly FF leg pays an arithmetic average of the daily effective FF rate. This requires a convexity adjustment[7].
  - Last FF fixing of each payment uses the day old rate. This handles the morning after rate publication differently than OIS. Its effect can be included in the convexity adjustment at no extra burden, though its contribution is negligible.

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[7] The current 10Y FF/LIBOR convexity adjustment is roughly 1/2 basis point. In periods of higher volatility the 30Y adjustment can exceed 5 basis points. Many market participants choose to ignore or are unaware of this adjustment. This could prove to be a costly mistake.

Extending the Curve with Feds Quotes

Extending the USD OIS curve with FF/LIBOR basis swap quotes involves solving the breakeven rate formula

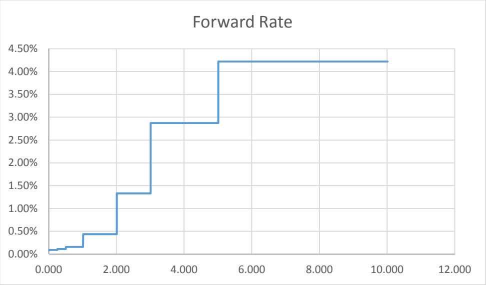
$$
\begin{aligned}
& c_{\mathrm{LIBOR}} - c_{\mathrm{FF/LIBOR}} + \gamma \\
& \quad = \frac{\sum_{j=1}^{J} P\left(0, s_{j,K_j}\right) \sum_{k=1}^{K_j} \left(P\left(0, s_{j,k-1}, s_{j,k}\right)^{-1} - 1\right)}{\sum_{j=1}^{m} \widehat{\tau}_j P\left(0, s_j\right)}
\end{aligned}
$$

where $c_{\mathrm{FF/LIBOR}}$ denotes the Feds quote, c_{LIBOR} denotes the LIBOR swap quote, and γ is the calculated Feds convexity adjustment.

Sample USD OIS Bootstrapped Curve

USD OIS 12/31/13

Knot Time	Forward Rate
0.005	0.08111%
0.255	0.09327%
0.504	0.11363%
1.014	0.15953%
2.011	0.43771%
3.011	1.33107%
5.011	2.87127%
10.014	4.22001%



Bootstrapping LIBOR Projection Curves

- Full collateraliztion and mandatory swap central clearing has changed the meaning of LIBOR swap quotes.
- LIBOR swaps now require mismatched OIS discounting.
- Swap quotes are no longer (theoretically) suitable for implying a LIBOR credit quality discount curve.
- Quotes **only** capable of representing projection curves for forward LIBOR in secured trades with collateral earning the OIS reference rate.
- OIS discount curve must be constructed first and supplied as input when bootstrapping LIBOR projection curves.

Bootstrapping LIBOR Projection Curves

LIBOR Basis Swaps

Another swap type we have not yet mentioned is the LIBOR basis swap.

- Exchanges floating rate payments of one LIBOR tenor for payments of another LIBOR tenor, e.g. 6-month vs. 3-month.
- Credit risk increases with a Eurodollar deposit's tenor.
- To compensate for this, a fixed spread is added to the leg with shorter tenor payments.
- Existence of LIBOR basis swaps market is proof an entire term structure is needed for **each** published LIBOR tenor.

Bootstrapping LIBOR Projection Curves

Different Term Structure Representation Needed

- Following example continues to use variables P and f to represent OIS discount factors and OIS forward rates.
- Marketplace no longer directly trades fixed income derivatives of LIBOR credit quality.
- Mismatched discounting prevents implementation of replicating strategies to determine LIBOR discount factors, forward rates, and zero rates.
- Only available information is forward LIBOR $L(t, T, M)$ used for projecting swap floating rate cash flows.
- Typically several forward LIBOR curves are constructed, one for each $\mathrm{tenor} \in \{\mathrm{O/N}, 1\mathrm{w}, 1\mathrm{m}, 2\mathrm{m}, 3\mathrm{m}, 6\mathrm{m}, 1\mathrm{y}\}$

  $$T \mapsto L(0, T, M(T; \mathrm{tenor}))$$

  where $M(T; \mathrm{tenor})$ represents the LIBOR accrual end date corresponding to begin date T and fixed tenor.
- We assume $T \mapsto L(0, T, M(T; \mathrm{tenor}))$ is a left continuous step function.

Bootstrap Example: 3m USD LIBOR Projection Curve

The Quotes

December 31, 2013

Instrument	Accrual Begin	Accrual End	Knot Time	Market Quote
USD LIBOR 3M	01/03/14	04/03/14	0.008	0.24610%
EDH4	03/19/14	06/19/14	0.214	99.7250
EDM4	06/18/14	09/18/14	0.463	99.6850
EDU4	09/17/14	12/17/14	0.712	99.6350
EDZ4	12/17/14	03/17/15	0.962	99.5700
EDH5	03/18/15	06/18/15	1.211	99.4700
EDM5	06/17/15	09/17/15	1.460	99.3250
EDU5	09/16/15	12/16/15	1.710	99.1300
EDZ5	12/16/15	03/16/16	1.959	98.8750
USD LIBOR Swap 3Y	01/03/14	01/03/17	2.759	0.87000%
USD LIBOR Swap 5Y	01/03/14	01/03/19	4.759	1.78000%
USD LIBOR Swap 10Y	01/03/14	01/03/24	9.762	3.09000%

Bootstrap Example: 3m USD LIBOR Projection Curve

Initial LIBOR Index

The initial 3-month LIBOR quote $L(0, t_1, M(t_1; 3\mathrm{m}))$ is extended to all earlier times

$$L(0, t, M(t; 3\mathrm{m})) = L(0, t_1, M(t_1; 3\mathrm{m})) = 0.24610\%, \quad t \leq t'.$$

Bootstrap Example: 3m USD LIBOR Projection Curve

Continuing with Eurodollar Futures

For each futures we extend the curve from t_{i-1} to t_i

- Eurodollar futures prices settle to 100 $- (\mathrm{rate})$, where the rate equals 3-month LIBOR at contract expiration.
- Prior to expiration, pricing corresponds closely with an FRA. However, there are subtle differences due to the linear payoff and the payment timing (mark-to-market).
- For each futures we extend the curve from t_{i-1} to t_i with

  $$L(0, t_i, M(t_i; 3\mathrm{m})) = \frac{100 - \mathfrak{F}}{100} - \gamma$$

  where $\mathfrak{F}$ is the futures price and γ is the futures convexity adjustment.
- The convexity adjustment is larger for longer dated futures and increases with interest rate volatility. It can reach values as high as 25 basis points. For futures expiring within 2 years, the current amount is $< 1/2$ basis point.

Bootstrap Example: 3m USD LIBOR Projection Curve

LIBOR Swaps

- Extending the curve to LIBOR swaps requires solving the breakeven rate formula under mismatched discounting

$$c = \frac{\sum_{j=1}^{m'} L\left(0, s'_{j-1}, s'_j\right) \hat{\tau}'_j P\left(0, s'_j\right)}{\sum_{j=1}^{m} \hat{\tau}_j P\left(0, s_j\right)}$$

  where the discount factors $P(0, t)$ are taken from the OIS curve constructed previously.

- This can be re-written to solve for $L(0, t_i, M(t_i; 3\mathrm{m}))$ directly

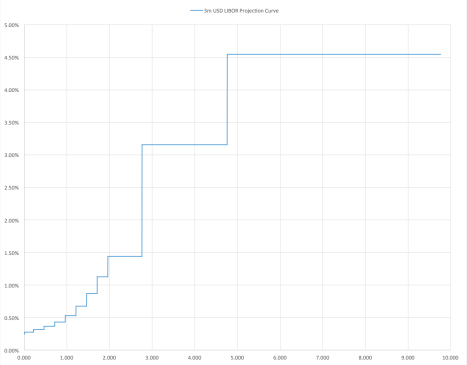
$$L(0, t_i, M(t_i; 3\mathrm{m})) = \frac{c \sum_{j=1}^{m} \hat{\tau}_j P\left(0, s_j\right) - \sum_{j=1}^{\overline{m}} L\left(0, s'_{j-1}, M\left(s'_{j-1}; 3\mathrm{m}\right)\right) \hat{\tau}'_j P\left(0, s'_j\right)}{\sum_{j=\overline{m}+1}^{m'} \hat{\tau}'_j P\left(0, s'_j\right)}$$

Bootstrap Example: 3m USD LIBOR Projection Curve

Final Results

3m USD LIBOR 12/31/13

Knot Time	Forward LIBOR
0.008	0.24610%
0.214	0.27500%
0.463	0.31500%
0.712	0.36500%
0.962	0.43000%
1.211	0.53000%
1.460	0.67500%
1.710	0.87000%
1.959	1.12500%
2.759	1.43905%
4.759	3.15647%
9.762	4.54488%



Constructing LIBOR Curves for Multiple Tenors

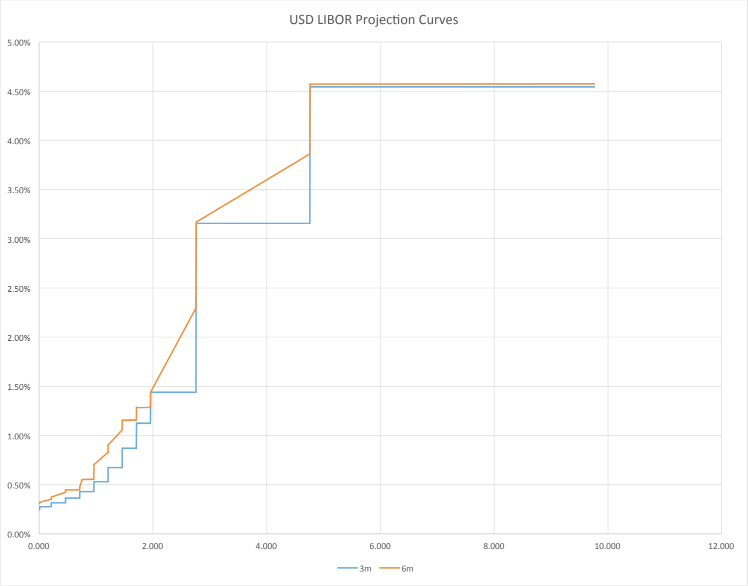
- Each LIBOR currency has a default payment frequency for vanilla fixed/float LIBOR swaps (3m for USD and 6m for EUR, GBP, CHF, and JPY).

- Construction of projection curves for alternate tenors requires that the default LIBOR tenor curve is constructed first.

- Alternate tenor curves are constructed relative to the default tenor curve using LIBOR basis swap quotes.

- In practice care must be taken to handle quoted instrument tenor granularity differences among constructed curves.

Sample 6m USD LIBOR Projection Curve



Further Remarks on Yield Curve Construction

- These slides merely scratch the surface for yield curve construction methods.
- Actual implementations typically include many additional important elements.
  - Turn of year effects.
  - Alternative interpolation (and extrapolation) techniques.
  - Weighting and other schemes to handle noisy or illiquid pricing.
- Often a different parameterization is more appropriate, e.g. spread levels.
- Care should be taken for related curves to handle incompatible interpolation methods and knot time granularity differences, e.g. for the OIS curve together with multiple LIBOR curves for a single currency.
- Most implementations go beyond bootstrapping to build curves that have a higher level of regularity or smoothness. Yield curve smoothing is an ongoing area of financial industry research and development.