Homework Assignment: The Hull White Model

Course: Fixed Income Derivatives

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Due Date: June 2, 2016

1 Hull-White Formulas

Derive the following results for the Hull-White model.

Volatility functions

$$\begin{aligned} b(t,T) &\triangleq \frac{1}{a}\left(1-e^{-a(T-t)}\right) \\ \Sigma_t^T &= -\sigma \cdot b(t,T) \end{aligned}$$

Short rate state

$$\begin{aligned} x_t &= \sigma \int_0^t e^{-a(t-s)} dW_s \\ r_t &= f_0^t + \frac{1}{2}\sigma^2 b(0,t)^2 + x_t \\ f_t^T &= f_0^T + \frac{1}{2}\sigma^2\left(b(0,T)^2 - b(t,T)^2\right) + \sigma\int_0^t e^{-a(T-s)} dW_s \\ &= f_0^T + \frac{1}{2}\sigma^2\left(b(0,T)^2 - b(t,T)^2\right) + e^{-a(T-t)}x_t \end{aligned}$$

Bond prices

$$\begin{aligned} A(t,T) &\triangleq \exp\left\{-\frac{1}{2}\sigma^2 b(t,T)\left(b(t,T)\frac{1-e^{-2at}}{2a} + b(0,t)^2\right)\right\} \\ P_t^T &= P_0^{t,T} A(t,T)\, e^{-b(t,T)x_t} \end{aligned}$$

Follow the same steps used in the notes for the Ho-Lee model.

2 Hull-White Variance Calculations

1. Show that the variance of the Hull-White short rate state x_t is given by
$$\mathrm{Var}\left[x_t\right] = \sigma^2 \frac{1 - e^{-2at}}{2a}.$$
2. Show that under Hull-White bond prices have a log-normal variance given by
$$\mathrm{Var}\left[\log P_t^T\right] = \sigma^2 b\left(t, T\right)^2 \frac{1 - e^{-2at}}{2a}.$$