Duration.

Price-Yield Relationship.

$Cf_i = 0.5cN \quad Cf_n = 0.5cN + N \quad Df_i(y) = e^{-yt_i} \quad B_0(y) = \sum_{i=1}^{n} Cf_i \, Df_i(y)$

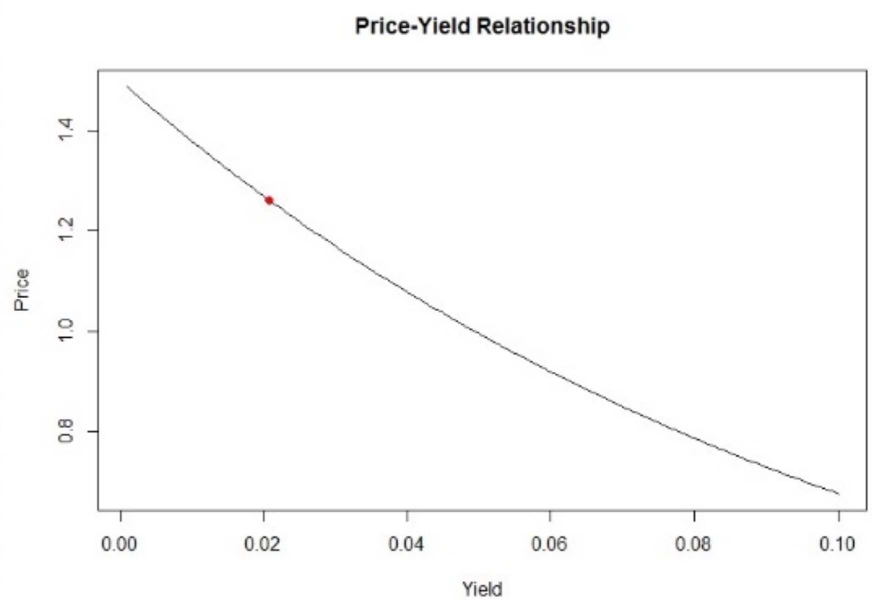
0.5: time between coupons

$D = \frac{1}{B_0} \sum_{i=1}^{n} t_i \, Cf_i Df_i = \sum_{i=1}^{n} t_i \left[\frac{Cf_i Df_i}{B_0} \right] = \sum_{i=1}^{n} t_i w_i$

$\sum_{i=1}^{n} w_i = 1 \Rightarrow D = E[T]$, T - Cash flow waiting time

POISSON.... `=EXP(-$B$3*(A8-$A$6))`

	A	B	C	D	E
1	Notional	1			
2	Coupon	0.05			
3	Yield	0.0208			
4	Price	1.2612891			
5	Time	Cash flows	Discount Factors	Present Values	
6	0	0	1	0	
7	0.5	0.025	0.989653893	0.024741347	
8	1	0.025	=EXP(-B3*(A8-$	0.024485371	
9	1.5	0.025	0.969281697	0.024232042	
10	2	0.025	0.959253405	0.023981335	
11	2.5	0.025	0.949328867	0.023733222	
12	3	0.025	0.939507009	0.023487675	
13	3.5	0.025	0.929786769	0.023244669	
14	4	0.025	0.920167095	0.023004177	
15	4.5	0.025	0.910646948	0.022766174	
16	5	0.025	0.901225297	0.022530632	
17	5.5	0.025	0.891901124	0.022297528	
18	6	0.025	0.88267342	0.022066835	
19	6.5	0.025	0.873541186	0.02183853	
20	7	0.025	0.864503435	0.021612586	
21	7.5	0.025	0.85555919	0.02138898	
22	8	0.025	0.846707483	0.021167687	
23	8.5	0.025	0.837947357	0.020948684	
24	9	0.025	0.829277864	0.020731947	
25	9.5	0.025	0.820698067	0.020517452	
26	10	1.025	0.812207037	0.832512213	
27					



Duration as sensitivity to change in yield.

$$\Delta B \approx \frac{dB}{dy}\Delta y \quad \frac{dB}{dy} = \frac{d}{dy}\sum_{i=1}^{n} Cf_i e^{-yt_i} = \sum_{i=1}^{n} Cf_i \frac{d}{dy} e^{-yt_i} = -\underbrace{\sum_{i=1}^{n} Cf_i t_i e^{-yt_i}}_{D}$$

Then $\Delta B \approx -\Delta y D$ or $\underbrace{\frac{\Delta B}{B}}_{\text{\% change in price}} = -D \underbrace{\Delta y}_{\text{abs. change in yield}}$

Duration can be used for understanding risk of a bond portfolio to changing yields and hedging it.

Modified duration.

Simple compounding is more commonly used than continuous.

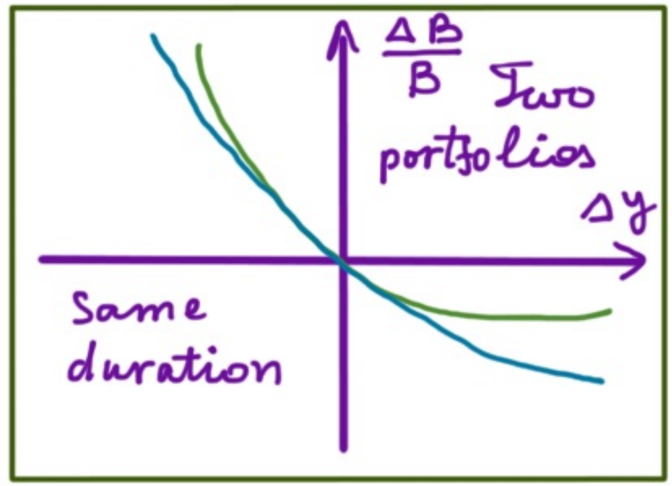
$\Delta B = -\frac{BD\Delta y}{1+\frac{y}{m}} = -BD^*\Delta y$, $D^* = \frac{D}{1+\frac{y}{m}}$, m-compounding frequency.

Convexity.

For larger changes Δy duration may become not accurate and needs a correction.

Second term in Taylor expansion can provide it.

$$C = \frac{1}{B}\frac{d^2B}{dy^2} = \frac{1}{B}\sum_{i=1}^{n} Cf_i\, t_i^2 e^{-y t_i}$$



Taylor expansion

$$\Delta B = \frac{dB}{dy}\Delta y + \frac{1}{2}\frac{d^2B}{dy^2}\Delta y^2$$

$$\frac{\Delta B}{B} = -D\,\Delta y + \frac{1}{2} C\,(\Delta y)^2$$

Convexity is larger when cash flows are evenly distributed over longer time.