# Regression Analysis and Quantitative Trading Strategies: Quantitative Trading Project

# **Butterfly Spread Strategy**

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#### **STRATEGY**

This project analyses the use of a butterfly spread strategy to capture returns based on volatility. Specifically, implied volatilities of call and put options at all possible strikes are modeled for predicting movement in the at-the-money implied volatility.

The strategy buys (sells) a call and put at-the-money with the same expiration date. A call and a put are also sold (bought) at strikes *B* units above and below the at-the-money strike respectively. These latter options are the "wings" of the butterfly. These wings offset the costs and overall exposure of the strategy because the opposite side of the at-the-money trade is taken.

Other volatility capturing spreads exist (Hull Chapter 9 and Natenberg Chapter 8), however the butterfly has some desirable traits. Specifically, the butterfly has no view on the direction of the underlying (such as strips, straps, and backspreads), and risk management is incorporated into the spread (with wing stops).

Strikes on options are not continuous, therefore "at-the-money" means to the nearest strike based on the current underlying price. Strikes are in increments of 5 for the example used in this project.

This strategy is re-evaluated daily; closing prices of the options and the underlying are used for calculations to decide on whether to trade at next open of the markets.

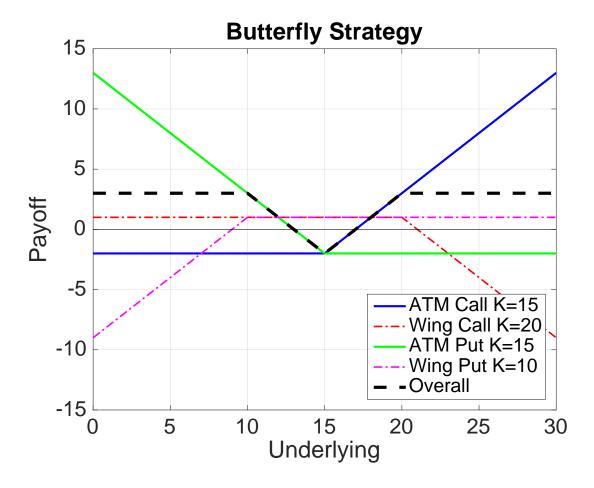
We are able to hedge the butterfly spread using delta hedging techniques, which will make a total of five trades. Further discussion is below on choosing to implement such a hedge.

Movements in implied volatilities are modeled using an exponentially weighted moving average (EWMA) approach. The number of days of history used in the model code is denoted as M. Differences in the EWMA prediction and option implied volatilities of today are used to decide on whether to buy or sell the butterfly tomorrow. Specifically, if the EWMA is greater than the implied volatility level today by l, then we presume implied volatilities of the options will rise. Therefore we *buy* the butterfly. If the EWMA is less than the implied volatility level today by k, then we presume implied volatilities of the options will fall. Therefore we *sell* the butterfly. The spread position is re-evaluated daily.

The rationale of this strategy is that prices (or volatilities) move along a path of least resistance (Lefevre). This strategy aims to identify this path. Burghardt and Lane provide insights into how volatility may be mispriced and how this can be exploited using volatility cones, however this report aims to test the more flexible EWMA approach.

The maximum cost of the strategy is *B* (discussed further later); capital is set at 3*B*.

Below is an illustration of the butterfly's payoff structure, given a long position in the spread. The underlying must move up or down by a certain amount for the payoff to be positive.



#### MODEL CONSTRUCTION

Here is a step-by-step walk-through of the code (written in Python) that implements the butterfly:

First the project is set up. To run this code locally, project\_path must be changed:

```
#~*~*~*~*~*~*****
# Michael Hyeong-Seok Beven
# University of Chicago
                                       #
# Financial Mathematics
# Quantitative Strategies and Regression #
# Final Project
#~*~*~*~*~*~*****
################
# python setup #
#################
import pandas as pd
import Quandl
import numpy as np
from numpy import sqrt, pi, log, exp, nan, isnan, cumsum, arange
from scipy.stats import norm
import datetime
import matplotlib.pyplot as plt
import keyring as kr # hidden password
key = kr.get_password('Quandl', 'mbeven')
import os
import warnings
warnings.filterwarnings('ignore')
pd.set_option('display.notebook_repr_html', False)
pd.set_option('display.max_columns', 10)
pd.set_option('display.max_rows', 20)
pd.set_option('display.width', 82)
pd.set_option('precision', 6)
project_path = '/Users/michaelbeven/Documents/06_School/2016 Spring/\
FINM_2016_SPRING/FINM 33150/Project'
images_directory = project_path+'/Pitchbook/Images/'
```

Parameters are then set for the project. The expiry on the chosen underlying and options (S&P 500 E-mini June 2016 futures and options) is June 17, 2016. M(integer), 1(non-negative real), k(non-negative real), and B(increments of 5) are all adjustable.

Next, the data is sourced. The future index, call and put prices were downloaded using Bloomberg. The risk-free rate (4 week bank discount) is taken from Quandl. Column names are cleaned in the code from their original form for ease of use. Option prices do not always exist for all strikes – these are interpolated for backtesting purposes. Note that a further investigation of this paper would be to compensate for this lack of liquidity:

```
#######
# data #
########
os.chdir(project_path) # set to local machine
calls = pd.read_csv('Data/Calls/ESM6C_All.csv',index_col='Date').iloc[::-1]
calls.index = pd.to_datetime(calls.index)
puts = pd.read_csv('Data/Puts/ESM6P_All.csv',index_col='Date').iloc[::-1]
puts.index = pd.to_datetime(puts.index)
future = pd.read_csv('Data/ESM6_IDX.csv',index_col='Date').Close.iloc[::-1]
future.index = pd.to_datetime(future.index)
future = future[(future.index >= calls.index[0]) & (future.index <= \</pre>
calls.index[-1])] # match date range
rfr = Quandl.get('USTREASURY/BILLRATES', returns='pandas', \
trim_start='2015-06-19',trim_end='2016-05-21').ix[:,0]/100
rfr = rfr.reindex(calls.index,method='bfill') # two missing rfr points-smoothed
def clean_colnames(df):
    11 11 11
    Removes 'Index' suffix at the end and replaces ' 'with '_'
    11 11 11
    for col in df.columns:
        df.rename(columns={col:col.replace(' Index','')},inplace=True)
    for col in df.columns:
        df.rename(columns={col:col.replace(' ','_')},inplace=True)
```

```
clean_colnames(calls)
clean_colnames(puts)

#smooth prices
for i in range(0,len(calls)):
    calls.ix[i] = calls.ix[i].interpolate()
    puts.ix[i] = puts.ix[i].interpolate()
```

Here is a list of options used (calls and puts):

```
>>> print(pd.DataFrame([calls.columns,puts.columns]).transpose())
             0
    ESM6C_1800 ESM6P_1800
0
1
    ESM6C_1805 ESM6P_1805
2
    ESM6C_1810 ESM6P_1810
3
    4
    ESM6C_1820 ESM6P_1820
5
    ESM6C_1825 ESM6P_1825
    ESM6C_1830 ESM6P_1830
6
7
    ESM6C_1835 ESM6P_1835
    ESM6C_1840 ESM6P_1840
8
9
    ESM6C_1845 ESM6P_1845
. .
99
    ESM6C_2295 ESM6P_2295
100
    ESM6C_2300 ESM6P_2300
101
    ESM6C_2305 ESM6P_2305
102
    ESM6C_2310 ESM6P_2310
103
    ESM6C_2315 ESM6P_2315
104
    ESM6C_2320 ESM6P_2320
105
   ESM6C_2325 ESM6P_2325
106 ESM6C_2330 ESM6P_2330
107
    ESM6C_2335 ESM6P_2335
    ESM6C_2340 ESM6P_2340
108
[109 rows x 2 columns]
```

We now begin the analysis. Below are two functions: an implied volatility calculator using the Newton-Raphsen Method, and a delta  $(N(d_1)$  for calls,  $N(d_1) - 1$  for puts) calculator for hedging purposes:

```
############
# analysis #
############
def impvol(w, price, S, K, tau, r):
    This function uses Newton's method to calculate implied vol.
    w: 1 for a call, -1 for a put
    price: price of the option
    S: price of the underlying
    K: strike
    tau: time to maturity in years (365 day convention)
    r: risk free rate (annualised 365 day convention)
    v = sqrt(2*pi/tau)*price/S
    for i in range(1, 10):
        d1 = (\log(S/K) + (r+0.5*pow(v,2))*tau)/(v*sqrt(tau))
        d2 = d1 - v*sqrt(tau)
        vega = S*norm.pdf(d1)*sqrt(tau)
        price0 = w*S*norm.cdf(w*d1) - w*K*exp(-r*tau)*norm.cdf(w*d2)
        v = v - (price0 - price)/vega
        if abs(price0 - price) < 1e-10 :
            break
    return v
def getdelta(cp, S, K, tau, r, v):
    This function calculates delta of a call or put
    cp: 0 for call, 1 for put
    S: price of the underlying
    K: strike
    tau: time to maturity in years (365 day convention)
    r: risk free rate (annualised 365 day convention)
    v: volatility
    d1 = (\log(S/K) + (r+0.5*pow(v,2))*tau)/(v*sqrt(tau))
    return norm.cdf(d1) - cp
```

Implied volatilities are then calculated *for all* option prices. These are the bedrock of the strategy since they form the foundation of modeling and trade decisions:

```
#implied vol tables
```

```
impvol_calls = calls*nan # empty table, same shape
for i in range (0,impvol_calls.shape[0]):
    for j in range (0,impvol_calls.shape[1]):
        impvol_calls.ix[i,j] = impvol(1,calls.ix[i,j],future.ix[i],\
        float(calls.columns[j][-4:]),\
        (expiry-calls.index[i].to_datetime()).days/365,rfr[i])

impvol_puts = puts*nan
for i in range (0,impvol_puts.shape[0]):
    for j in range (0,impvol_puts.shape[1]):
        impvol_puts.ix[i,j] = impvol(-1,puts.ix[i,j],future.ix[i],\
        float(puts.columns[j][-4:]),\
        (expiry-puts.index[i].to_datetime()).days/365,rfr[i])
```

The Pandas function for EWMA is used for volatility predictions. Predictions for implied volatility are calculated for *each strike separately*, for both calls and puts:

```
#calculate EWMA
ewma_calls = calls*nan #empty table
for column in impvol_calls:
    ewma_calls[column] = pd.ewma(impvol_calls[column],M)

ewma_puts = puts*nan
for column in impvol_puts:
    ewma_puts[column] = pd.ewma(impvol_puts[column],M)
```

At-the-money options are based on the current price of the underlying. Since strikes are in increments of 5, we must round the E-mini future:

```
#round underlying to strike increments
df = pd.concat((future,future.apply(lambda x: 5*round(float(x)/5))),axis=1)
df.columns = ['future','future_rounded']
```

Around 85% of the change in option prices is attributable to the volatility *level* (Sinclair). Based on this, the volatility level around the at-the-money strike is calculated as the average of the at-the-money implied volatility, as well as two neighboring strikes above and below. This is also an average of both calls and puts. These averages are calculated for the actual implied volatility and EWMA predicted implied volatility. The difference between these averages later decides trades based on 1 and k:

```
#compare means of EWMA to means of actual impvol each day
df['impvol_avg'] = nan #empty vector
df['ewma_avg'] = nan
#vol levels using 2 strikes on either side (call and put)
for i in range(0,df.shape[0]):
    df.impvol_avg[i] = (impvol_calls['ESM6C_'+str(df.future_rounded.ix[i])][i]+\
            impvol_puts['ESM6P_'+str(df.future_rounded.ix[i])][i] +\
            impvol_calls['ESM6C_'+str(df.future_rounded.ix[i]-10)][i] +\
            impvol_puts['ESM6P_'+str(df.future_rounded.ix[i]-10)][i] +\
            impvol_calls['ESM6C_'+str(df.future_rounded.ix[i]-5)][i] +\
            impvol_puts['ESM6P_'+str(df.future_rounded.ix[i]-5)][i] +\
            impvol_calls['ESM6C_'+str(df.future_rounded.ix[i]+5)][i] +\
            impvol_puts['ESM6P_'+str(df.future_rounded.ix[i]+5)][i] +\
            impvol_calls['ESM6C_'+str(df.future_rounded.ix[i]+10)][i] +\
            impvol_puts['ESM6P_'+str(df.future_rounded.ix[i]+10)][i])/10
    df.ewma_avg[i] = (ewma_calls['ESM6C_'+str(df.future_rounded.ix[i])][i] +\
            ewma_puts['ESM6P_'+str(df.future_rounded.ix[i])][i] +\
            ewma_calls['ESM6C_'+str(df.future_rounded.ix[i]-10)][i] +\
            ewma_puts['ESM6P_'+str(df.future_rounded.ix[i]-10)][i] +\
            ewma_calls['ESM6C_'+str(df.future_rounded.ix[i]-5)][i] +\
            ewma_puts['ESM6P_'+str(df.future_rounded.ix[i]-5)][i] +\
            ewma_calls['ESM6C_'+str(df.future_rounded.ix[i]+5)][i] +\
            ewma_puts['ESM6P_'+str(df.future_rounded.ix[i]+5)][i] +\
            ewma_calls['ESM6C_'+str(df.future_rounded.ix[i]+10)][i] +\
            ewma_puts['ESM6P_'+str(df.future_rounded.ix[i]+10)][i])/10
```

The strategy is now implemented below. Buy and sell signals (df.signal) are set based on 1 and k. The strike at entry (df.entry\_strike) is 0 if there is no trade signal. A loop is then run that tracks the at-the-money and wing options (prices and deltas) depending on df.entry\_strike. The previous day's option prices are also tracked, because when rebalancing we must exit the previous day's position. The profit of the strategy is then based on the rebalancing of the butterfly:

```
#build butterfly spread with delta hedge
df['signal'] = nan

df.signal[df.ewma_avg - df.impvol_avg > 1] = 1 #buy butterfly

df.signal[df.ewma_avg - df.impvol_avg < -k] = -1 #sell butterfly

df['entry_strike'] = df.future_rounded*(1-isnan(df.signal))

df.signal[isnan(df.signal)] = 0

# track entry strike; find call and put prices accordingly

df['call_ATM'] = nan

df['put_ATM'] = nan

df['call_wing'] = nan</pre>
```

```
df['put_wing'] = nan
df['call_ATM_delta'] = nan
df['put_ATM_delta'] = nan
df['call_wing_delta'] = nan
df['put_wing_delta'] = nan
for i in range(1,len(df)):
    if df.signal[i] == 0:
        df.call_ATM[i] = 0
        df.put_ATM[i] = 0
        df.call_wing[i] = 0
        df.put\_wing[i] = 0
        df.call_ATM_delta[i] = 0
        df.put_ATM_delta[i] = 0
        df.call_wing_delta[i] = 0
        df.put_wing_delta[i] = 0
    else:
        df.call_ATM[i] = calls.ix[i]['ESM6C_'+str(df.entry_strike[i])]
        df.call_ATM_delta[i] = getdelta(0, df.future[i], df.entry_strike[i],\
        (expiry-df.index[i].to_datetime()).days/365,rfr[i], \
        impvol_calls.ix[i]['ESM6C_'+str(df.entry_strike[i])])
        df.put_ATM[i] = puts.ix[i]['ESM6P_'+str(df.entry_strike[i])]
        df.put_ATM_delta[i] = getdelta(1, df.future[i], df.entry_strike[i],\
        (expiry-df.index[i].to_datetime()).days/365,rfr[i], \
        impvol_puts.ix[i]['ESM6P_'+str(df.entry_strike[i])])
        df.call_wing[i] = calls.ix[i]['ESM6C_'+str(df.entry_strike[i]+B)]
        df.call_wing_delta[i] = getdelta(0, df.future[i], df.entry_strike[i]+B,\
        (expiry-df.index[i].to_datetime()).days/365,rfr[i],\
        impvol_calls.ix[i]['ESM6C_'+str(df.entry_strike[i]+10)])
        df.put_wing[i] = puts.ix[i]['ESM6P_'+str(df.entry_strike[i]-B)]
        df.put_wing_delta[i] = getdelta(1, df.future[i], df.entry_strike[i]+B,\
        (expiry-df.index[i].to_datetime()).days/365,rfr[i],\
        impvol_puts.ix[i]['ESM6P_'+str(df.entry_strike[i]-10)])
df['delta_hedge'] = df.call_ATM_delta + df.put_ATM_delta - (df.call_wing_delta\
+ df.put_wing_delta)
# track previous day's position value
df['call_ATM_old'] = nan
df['put_ATM_old'] = nan
df['call_wing_old'] = nan
df['put_wing_old'] = nan
for i in range(1,len(df)):
    if df.signal[i-1] == 0:
        df.call_ATM_old[i] = 0
        df.put_ATM_old[i] = 0
        df.call_wing_old[i] = 0
```

```
df.put_wing_old[i] = 0
else:
    df.call_ATM_old[i] = calls.ix[i]['ESM6C_'+str(df.entry_strike[i-1])]
    df.put_ATM_old[i] = puts.ix[i]['ESM6P_'+str(df.entry_strike[i-1])]
    df.call_wing_old[i] = calls.ix[i]['ESM6C_'+str(df.entry_strike[i-1]+B)]
    df.put_wing_old[i] = puts.ix[i]['ESM6P_'+str(df.entry_strike[i-1]+B)]

df = df[2:]

df['butterfly'] = df.signal*((df.call_wing + df.put_wing) - \
    (df.call_ATM + df.put_ATM))

df['butterfly_old'] = df.shift(1).signal*((df.call_wing_old + \
    df.put_wing_old) - (df.call_ATM_old + df.put_ATM_old))

df['profit'] = df.shift(1).butterfly - df.butterfly_old

df.profit[0] = df.signal[0]*((df.call_wing[0] + df.put_wing[0]) - \
    (df.call_ATM[0] + df.put_ATM[0]))

df['cum_profit'] = cumsum(df.profit)
```

Finally, we view performance metrics:

```
#performance
df.cum_profit[-1]/K #total profit
df.cum_profit[-1]/K*(365/(df.index[-1]-df.index[0].to_datetime()).days)#annual
(df.profit/K).std() * sqrt(365) # annualised standard dev of returns
(df.profit.mean() / df.profit.std()) # sharpe
df.profit.mean() / df.profit[df.profit < 0].std() # sortino
i = np.argmax(np.maximum.accumulate(df.cum_profit) - df.cum_profit)#drawdown end
j = np.argmax(df.cum_profit[:i]) #drawdown start
max_drawdown = (df.cum_profit[i] - df.cum_profit[j])/K</pre>
```

Here is a description summary of each major data frame/series created:

- calls prices of calls on each day for strikes 1800 to 2335 in increments of 5
- puts prices of puts on each day for strikes 1800 to 2335 in increments of 5
- impvol\_calls implied volatilities for all of calls
- impvol\_puts implied volatilities for all of puts
- ewma\_calls EWMA predictions for all of impvol\_calls
- ewma\_puts EWMA predictions for all of impvol\_puts
- df
  - future future price

- future\_rounded future rounded to nearest 5
- impvol\_avg average implied volatility (as described above)
- ewma\_avg average ewma volatility (as described above)
- signal buy/sell signal
- entry\_strike strike at entry of trade
- call\_ATM at-the-money call price at entry of trade
- put\_ATM at-the-money put price at entry of trade
- call\_wing wing call price at entry of trade
- put\_wing wing put price at entry of trade
- call\_ATM\_delta at-the-money call delta at entry of trade
- put\_ATM\_delta at-the-money put delta at entry of trade
- call\_wing\_delta wing call delta at entry of trade
- put\_wing\_delta wing put delta at entry of trade
- delta\_hedge sum of deltas
- call\_ATM\_old previous day's current at-the-money call price
- put\_ATM\_old previous day's current at-the-money put price
- call\_wing\_old previous day's current wing call price
- put\_wing\_old previous day's current wing put price
- butterfly total value for the spread
- butterfly\_old total updated value for the spread from previous day
- profit difference in today's spread value and yesterday's spread value
- cum\_profit cumulative profit

#### **INVESTMENT UNIVERSE**

The butterfly spread strategy is adaptable to many underlying assets and respective options, such as: commodities, equities, indexes, foreign exchange, fixed income and so on. The strategy requires that options in particular are liquid. This is essential for properly building the butterfly, with two options at-the-money and two at the wings. These options in the wings act as stops; they stop potential profits in a long butterfly and stop potential losses in a short butterfly. These wings ultimately limit the investment capital required for the strategy, expanding the investment universe to more expensive products.

This strategy can also employ a delta hedge, however one must be careful in using this hedge. The hedge removes risk captured by directional exposure in the options versus the underlying, however this risk is very small given the symmetry of the butterfly. The dominant risk in this strategy is thus based on volatility (which we clearly do not want to hedge).

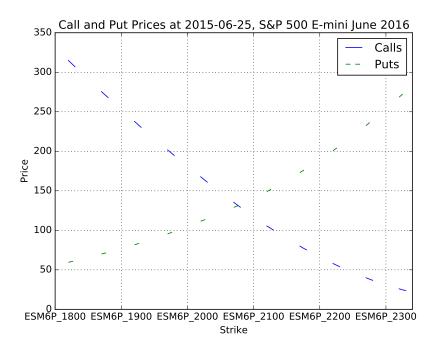
The strategy also easily adapts to shorter investment horizons, such as intraday.

#### **COMPETITIVE EDGE**

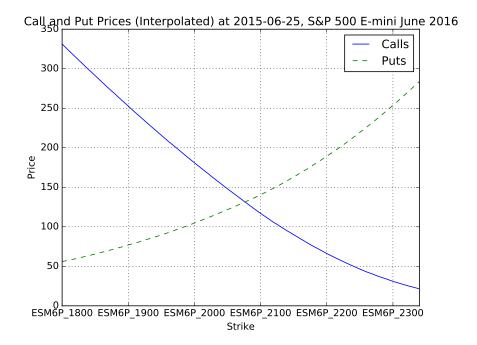
The EWMA approach for measuring what implied volatility *is* versus what it *ought to be* is flexible and easily adjusted (changing historical days). This strategy is also mechanically different from many positions in investment portfolios (e.g. value or growth stocks based portfolios) since the trade driver is option implied volatilities. Therefore, this strategy may add alpha to portfolios. A diversified portfolio of butterfly spreads could also be created. As previously mentioned, the butterfly also has a low cost structure. Another great feature of this spread is based in its symmetry; a view on market direction is not required.

#### **EMPIRICAL EXPLORATION**

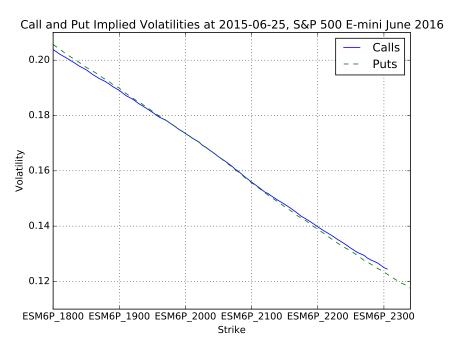
Here, the S&P 500 E-mini June 2016 contracts are assessed in detail. Option contracts can only exist at certain strikes when far from maturity:



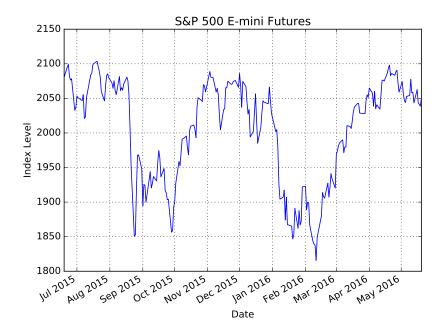
This poses a limitation to the strategy, which would have to be investigated further. In order to properly backtest the strategy, missing prices can be interpolated:



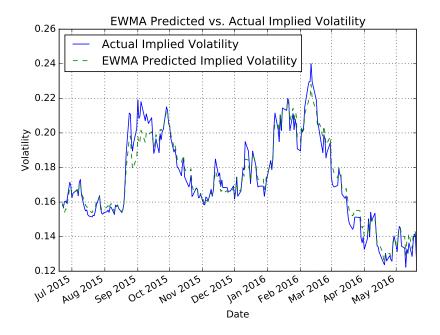
Theoretically, implied volatilities of calls and puts should be equal. In practice these are slightly different:



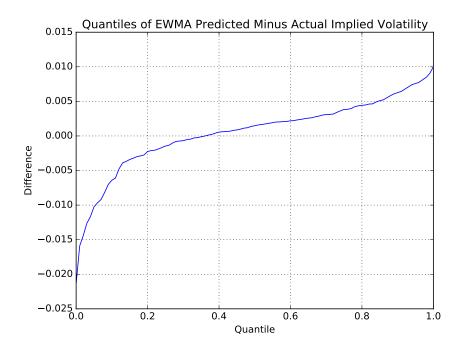
We therefore use an average of both when calculating the implied volatility level. Next is the time series of the underlying. The average and standard deviation over the life of the June 2016 options are 2006 and 73.75 respectively:



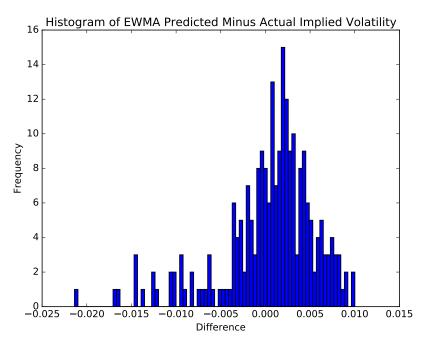
There are clear deviations between the EWMA predicted implied volatility and actual volatility in the chart below. These are indications (and the basis for our trade) to exploit mispriced options. Using a shorter history for EWMA increases the agility of capturing differences, however there is a tradeoff in the stability of EWMA; more noise is introduced with a shorter EWMA history:



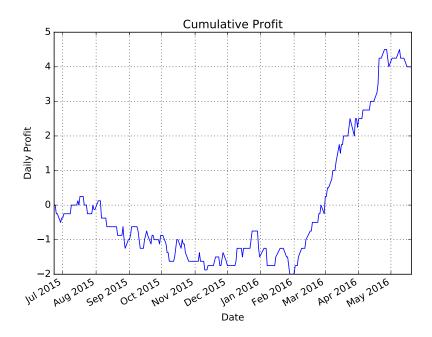
Choosing thresholds to buy/sell the butterfly is quantiles based:



EWMA predictions are higher on average compared to realized implied volatility. The difference is also negatively skewed:



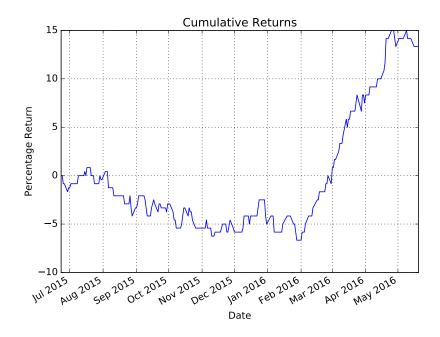
Below is a chart of cumulative profits. Butterfly wingspan: 20, EWMA history: 10 days, buy/sell thresholds: \$0.3 standard deviations from the mean. 185 trades are made:



Overall, the analysis suggests that the strategy performs best when closer to maturity (within 6 months). Based on arbitrage arguments, the cost of the butterfly can never be greater than B. One can see this by looking at the payoff diagram on page 3. If the spread costs more than B, then the payoff can never be positive. On the other hand, the cost can clearly not be above zero. Therefore, the capital level set to execute this strategy is 3B.

#### IMPLEMENTATION AND INVESTMENT PROCESS

We assume that the strategy is implemented with with capital at 3B. The chart below calculates returns using this numeraire:



This strategy can be applied across several asset classes. Candidate assets should be selected based on high levels of liquidity for each option strike, as well as the underlying. The strategy is reassessed separately for different expiration dates.

#### **BACKTESTING**

- Backtesting is done for June 2015 to May 2016
- Fees and costs are to be incorporated
- Both long and short butterfly positions are taken (114 long trades and 71 short trades)
- Trade frequency is most easily changed by tweaking *l* and *k*
- Cumulative return: 13.3%
- Annualized cumulative return: 14.7%
- Annualized standard deviation of returns: 11.7%
- Sharpe ratio: 0.09
- Sortino ratio: 0.18
- Maximum drawdown: 7.5%

#### **RISK MANAGEMENT**

Positions are reviewed daily to monitor profits and losses. A stop-loss can be set, based on some fraction of K to exit and reassess the strategy for a particular butterfly. The wings of the butterfly inherently act as stops for daily positions, such that losses on any given day are limited to B. Creating a diversified portfolio of multiple butterflies for different assets and maturities would dampen idiosyncratic risk, and may allow for economies of scale in setting the overall capital requirement. Such a diversified portfolio should be based on diversifying through different origins of volatility. This strategy is mostly exposed to gamma and vega, i.e. volatility or lack thereof. If realized gamma is relatively high and we are short the butterfly, then the strategy will lose money. If realized gamma is relatively low and we are long the butterfly, then the strategy will lose money. Note that the strategy "bleeds" capital before making a significant comeback in profits. Further analysis is desirable into the extent of this effect, as well as its triggers. Risk measures can then be set accordingly based on this.

## **ABOUT THE MANAGER**

MICHAEL BEVEN graduated as a National Merit Scholar from the Australian National University in 2012 with a Bachelor of Actuarial Studies and a Bachelor of Finance (Majors in Quantitative and Corporate Finance). He is currently completing a Master of Science in Financial Mathematics at the University of Chicago. Prior to starting at the University of Chicago, Michael was an Actuarial Analyst in Sydney at Quantium, a data analytics focused actuarial consultancy. He has also interned at Macquarie Group and Westpac Institutional Bank in quantitative risk analytics. Michael is deeply interested in electronic trading and sees Chicago as a great place to progress his career.

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