

# MSFM, Fixed Income Derivatives (33601)

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## Homework Assignment on Statistical Model

This assignment helps understanding construction of the statistical model of interest rates and its properties

This assignment is individual

### 1. Prepare the data (5%)

Fund the project data are in the file `StatisticalModelData2014.csv`.

The format of the data is shown below:

##	USGG3M	USGG6M	USGG2YR	USGG3YR	USGG5YR	USGG10YR	USGG30YR
## 1/5/1981	13.52	13.09	12.289	12.28	12.294	12.152	11.672
## 1/6/1981	13.58	13.16	12.429	12.31	12.214	12.112	11.672
## 1/7/1981	14.50	13.90	12.929	12.78	12.614	12.382	11.892
## 1/8/1981	14.76	14.00	13.099	12.95	12.684	12.352	11.912
## 1/9/1981	15.20	14.30	13.539	13.28	12.884	12.572	12.132
## 1/12/1981	15.22	14.23	13.179	12.94	12.714	12.452	12.082

### 2. Estimate the 3-factor model using PCA (15%)

2.1. Define factor and factor loadings

2.2. Calculate relative importance of factors (see slide 17)

2.3. Plot and interpret the shapes of factor loadings (see slide 18)

### 3. Calculate historical volatilities and correlation coefficients of factors (15%)

**3.1. Use the whole period of history to calculate**

$Var[\Delta f_i(t)]$ ,  $Cor[\Delta f_i(t), \Delta f_j(t)]$ ,  $i = 1, 2, 3$ , where  $\Delta f_i(t)$  is a one-day increment of the factor (see slides 20,27)

**3.2. Calculate the same variables using a rolling window approximately 1 month****4. Find historical estimates of volatilities of the first 3 factors corresponding to the last month of the observed period (10%)****5. Calculate time series of each of 7 rates predicted by the model (5%)****6. Fit parametric forms from slide 32 to each of the first 3 vectors of factor loadings (10%)**

Include parameters  $a$ ,  $b$  in the report.

**Hint.** You can use the following parameter values as initial guesses.

Loading.1

```
##          a          b
## 1  0.320  0.070
## 2  0.006  0.285
## 3 36.550 -0.292
```

Loading.2

```
##          a          b
## 1 0.650 -1.130
## 2 0.004  0.539
```

Loading.3

```
##          a          b
## 1 4.200e-01 -1.920
## 2 5.000e-08  0.620
## 3 5.000e-01 -0.410
## 4 2.876e+00  3.035
```

**Hint.** Restrict optimization to searching  $a > 0$ .

**7. Calculate time series of instantaneous forward rates with maturity 5 years and discount bonds with maturity 4.5 years for the whole period of observation. Calculate the histograms of one-day increments (20%)**

**8. Calculate correlations between the short rate and instantaneous forward rates (20%)**

8.1. See slides 34-36.

8.2. Calculate

$$Cor(\Delta F(t, 0), \Delta F(t, \tau))$$

as function of  $\tau$  for increasing  $\tau > 0$ .

8.3. Repeat the calculations using one-, two- and three-factor models.

8.4. Plot the graph of correlations.