Type 1 and Type 2 Error and Power

TRUTH TABLE

Decision		Truth: H_0	Truth: H_A
		H_0	H_A
Decision	$P \le \alpha$ Reject H_0 and accept H_A. There *is statistically significant* evidence that [H_A] (P-value).	Type 1 error α = *Type 1 error rate* $= P\{\text{Reject } H_0\}$ given H_0 true.	Correct decision $(1-\beta)$ = *power* $= P\{\text{Reject } H_0\}$ given H_A true.
Decision	$P > \alpha$ Not reject H_0 and Not accept H_A. There *is not statistically significant* evidence that [H_A] (P-value).	Correct decision	Type 2 error β = *Type 2 error rate* $= P\{\text{Not reject } H_0\}$ given H_A true.

α = ***significance level*** = ***Type 1 error rate***
= probability of ***rejecting*** a ***true*** null hypothesis.

β = operating characteristic = ***Type 2 error rate***
= probability of ***not rejecting*** a ***false*** null hypothesis.

$(1-\beta)$ = ***power*** = sensitivity = probability of ***rejecting*** a ***false*** null hypothesis.

δ = ***effect*** = difference between true and hypothetical value of the parameter of interest, e.g., = $(\theta - \theta_0)$, for H_0: $\theta = \theta_0$. Note that H_0 can be stated in terms of the effect as H_0: $\delta = 0$. The symbol θ denotes a generic parameter of interest.

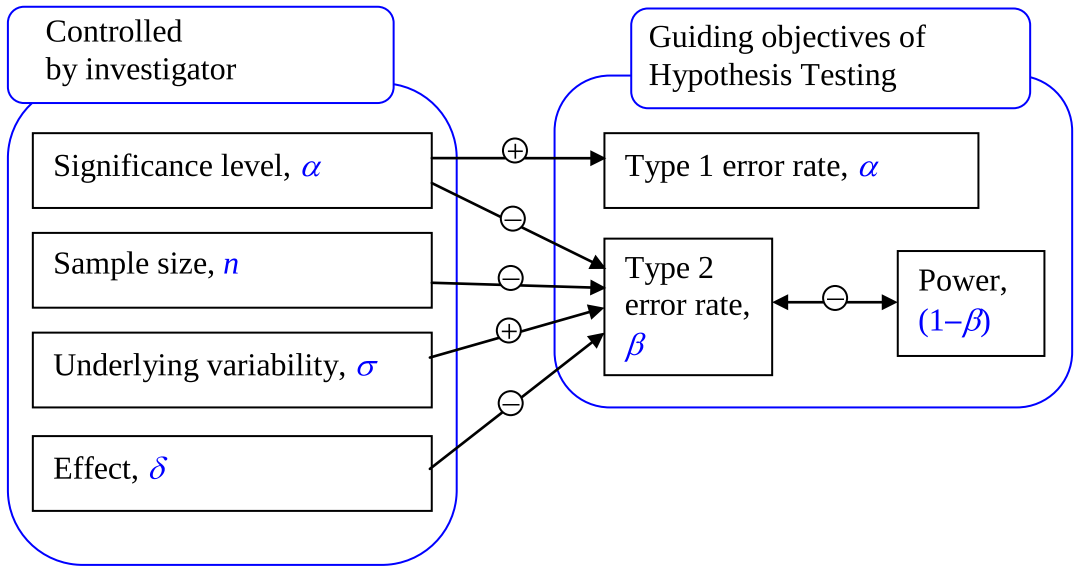
We can achieve a

1. Small Type 1 error rate, α by choosing a small significance level, α.
2. Small Type 2 error rate, β, and large power $(1-\beta)$ by way of a
    a. large significance level, α
    b. large sample size, n
    c. small underlying standard deviation, σ
    d. large effect, δ

Statistical Significance (P-value < α) versus Practical Importance (δ large)

A result can be *statistically significance* (P-value < α) but *not of practical importance* (δ small) if the test is extremely powerful and sensitive [$(1-\beta)$ large]. This is neither a Type 1 error nor a Type 2 error.

A result can be *not statistically significant* (P-value > α) but *of practical importance* (δ large) if the test lacks power and lacks sensitivity [$(1-\beta)$ small]. This is a Type 2 error.



$a \xrightarrow{\oplus} b$ denotes a *positive relationship*, i.e., if *a* increases, then *b* increases, and if *a* decreases, then *b* decreases.

$a \xrightarrow{\ominus} b$ denotes a *negative relationship*, i.e., if *a* increases, then *b* decreases, and if *a* decreases, then *b* increases.