Because the earth is spherical, particularly for the atmosphere, there's an advantage to representing the continuous variability of variables in the horizontal as orthogonal spectral functions. This is an alternative to using finite differences.

So let's go back, just to get the general idea of this, to our very simple partial differential equation that states that the time derivative of a variable, u, is equal to minus some constant c times its derivative in x.

Now Fourier's theory says that we can represent any reasonably smooth distribution of a variable in space as a sum of orthogonal functions, such as sines and cosines. So for example, we can represent the variable u as a sum over some potentially large number n of different modes in the x direction. And these modes are represented by the sine and cosine functions.

So here is an amplitude function which varies with n. This is the sine of n pi x over L, where L you might think of as the size of the box in which this equation is going to be solved. And likewise, we have cosine functions here.

So generally speaking, these orthogonal functions are designed to automatically satisfy the boundary conditions. Now, if we simply substitute that form for the variables u into the original PDE, that reduces it to basically a set of coupled ordinary differential equations for the amplitudes a and b here, which can be solved somewhat more easily.

So we're representing the continuous distribution of the variable u in x by a finite number of cosines and sines given by n. Now, when we do this in a climate model, we're not solving the equations in a box, but on a sphere. So the orthogonal functions that we use, rather than being sines and cosines, are spherical harmonics-- which are the solution of simple wave equations on the sphere.

Because the boundary conditions in the vertical are complicated, and we can't represent the variation of different variables as periodic in the z direction, we don't generally use spectral methods to describe the vertical variability of the various variables in either the ocean or the atmosphere. And instead we resort to finite differences. But of course, as we discussed before, these may be in a variety of different kinds of vertical coordinate systems, such as just plain altitude, or pressure, or the so-called sigma coordinate.

When we go to solve these equations, we have to satisfy various different constraints to ensure that the equations are well behaved. One of the most fundamental of these constraints is called the Courant-Friedrichs-Lewy condition or CFL condition. It states that c times the time step, divided by the distance between node points in the horizontal, must be smaller than 1, where c is the phase speed of the fastest wave that the equations can represent in the system.

So delta t is a time step. Delta x is the internode spacing. And this means that we have to choose a time step that's small enough to satisfy this condition. If we do not do that, for most kinds of differencing schemes, the equations will become unstable. That is, the difference equations will become unstable, and we really won't be able to solve them.

There are many other interesting aspects of solving partial differential equations numerically, which we will not have time to go into in this course. But it is safe to call this a subdiscipline of computational fluid dynamics. There's a lot to it, and a lot that goes into the formulation and solution of equations in climate models.

How big are climate models? What are the computational demands of them? Well, typically a climate model will solve for the variation in time of between 10 and 15 variables. There are some models that solve fewer variables than this, and others that go outside the range of 10 to 15, but generally speaking, that's what we're looking at.

A typical climate model today may solve the equations on 20 levels in the vertical, but that varies a great deal as well. Grid points might be spaced on the order of 100 kilometers or so apart. And that yields, if we collect all the variables at all the grid points on a sphere, somewhere between 1 and 5 million variables that we're solving for.

Typically, a time step in the model-- that is, the time step we use to advance the equations to tell us the variables at discrete intervals in time-- is about 20 minutes. And that yields somewhere between 70 and 350 million variables per simulated day.

So this is quite a computational demand. One thing to notice about this list is if one increases by a factor of 2 the resolution in all three spatial directions, that entails a factor of 8 increase in variables. But the situation is actually worse than this, because increasing the resolution according to the CFL condition usually requires us to reduce the time step. So the actual increase might be well in excess of a factor of 8, just by doubling the resolution in all three spatial dimensions.