In the computational solution of finite difference equations, there are certain advantages to be gained by defining different variables at different points in space. So let's consider a solution of a very simple two-dimensional system of equations on a simple Cartesian grid, in which we consider x, for example, to be longitude, and y to be latitude. And we're solving difference equations for temperature, meridional velocity, v, and zonal velocity, u.

Now, one particular grid on which these variables could be defined is given by the so-called "C grid," in which the temperatures are defined where these red dots appear. The east-west, or zonal motions, are defined on the blue squares. And the south to north, or v velocities, meridional velocities, are defined on these green diamonds. Doing it this way makes the solution of the difference equation somewhat more accurate, and it is common to stagger variables like this in solving difference equations.

This is an example of what a staggered grid might look like in three dimensions. So here we have a grid volume, and the variables are defined at the nodes within this volume.

So in this particular case, the x components of velocity are represented by these black squares, the y component of velocity by the black, upside-down triangles, and the z component of velocity by the dots-- whereas the densities, viscosities, and stress components tau xx, tau yy, tau zz, are defined on the open circles. And the other components of the stress are defined at the open squares, triangles and upside-down triangles. So one can show that staggering variables this way on finite difference grids presents certain advantages.

In the vertical, in both the atmosphere and the ocean, [in a climate model] we typically find fewer levels at which the variables are defined than we would find in a weather forecast model. These variables are not usually equally spaced in the vertical. There are typically more levels close to the ground-- and near the tropopause, where things tend to vary rapidly-- than in between.

A popular choice of vertical coordinate is to transform the equations so that the vertical coordinate, rather than being either altitude or pressure, is something called a sigma coordinate. Typically sigma is defined as the ratio of the pressure to the surface pressure for an atmospheric coordinate system.

So if you look at this, by definition sigma is equal to 1 at the surface. So this is a terrain-following coordinate. If we look at what those coordinate surfaces look like in physical space, we might see a picture like this.

So the lowest-- or zero sigma level-- is this curve at the bottom, which follows the topography. The next coordinate might look like this, and so forth. By the time we get high up in the atmosphere, that ratio doesn't have as much reflection of the surface topography as it does lower down.

This offers certain advantages. One doesn't have to worry about the coordinate surface ending where it intersects the topography. So the advantage is that it conforms to the natural terrain. So this is a way of representing mountains, both at the bottom of the atmosphere and potentially at the bottom of the ocean.

By definition, these coordinate surfaces never intersect the terrain, like a height coordinate system would. In some ways, it simplifies the mathematical equations of the model. But there are also limitations of this coordinate system.

It complicates certain computations, like the pressure gradient force in regions where there are slopes. And there can be problems such as land points extending artificially into the ocean due to smoothing that one needs to apply near mountainous terrain.

Now, of course in real climate models, these partial differential equations are either written as finite difference equations, or spectrally decomposed, but they're solved typically on a sphere. And here is an example of a classical spherical coordinate system that might be used for an atmospheric model. So each one of the little volumes that you see in this diagram like this is a grid volume of the model. The coordinates are latitude, longitude, and altitude.

But there are certain disadvantages of coordinate systems like this. For example, the grid volumes vanish at the poles, so the coordinate surfaces converge on the poles. You may have excessive spatial resolution there at the expense of not as much spatial resolution, for example, at the equator.

And so there are alternative grids. If we compare these classical spherical coordinates-- that you see, for example, in this demonstration on the left-- to other coordinate systems, we have, for example, a coordinate that's formed by conformally mapping cubes into spheres. So the grid surfaces of mapping like that is demonstrated in this diagram at the right.

And one can see that, while there are variations in the sizes of these grid volumes-- for example, they're relatively low here where the corners of the cube have been mapped onto the sphere, and they're relatively large in between these points.

There are also geodesic grids-- for example, this one developed at Colorado State University-- which have essentially geodesic tiles. They can be at various sizes giving different resolutions.

And there is even the idea of a grid based on the Fibonacci sequence. This is an example of such a grid. These grids tend to be very uniform and isotropic in space, but they have not yet worked their way into global climate models.