Let's first talk about how large scale atmospheric surface winds can drive ocean circulation. Here is a chart showing the direction and magnitude of the stress exerted by the winds on the ocean surface averaged over several years. The white arrows show the direction of the wind stress and the coloring shows the magnitude of the stress as indicated by the color scale at the bottom of this diagram.

This is a remarkable feat to have constructed a diagram like this in the first place. How do we actually measure wind stress at the surface of the ocean? There are not many measurements of wind out over the open oceans. This particular estimate of stress is derived from satellites from a remarkable instrument we talked about in one of the previous sections of the course which sends down beams of microwave radiation whose wavelength is comparable to that of capillary waves on the surface of the ocean, which are a direct measure of the wind stress at the surface. By measuring the amount of power returned as a function of the angle at which the measurement is made, it's possible to make an estimate of the amplitude and orientation of the capillary waves, whereby it is then possible to estimate the direction and magnitude of the stress.

So when we look at this diagram, we see that there is a westward stress in the northern part of the North Pacific and North Atlantic oceans. This corresponds to the mid-latitude Westerlies. We see an eastward stress in the tropical North Pacific, the tropical South Pacific, the tropical south Indian Ocean, and the tropical Atlantic. These are the trade wind regions. And finally, we see a belt in the southern ocean of very strong eastward directed stress corresponding to the Roaring Forties, the strong Westerlies that are persistent through most of the year over the southern ocean.

How does the wind cause the ocean to move on a rotating planet? We have a problem here because the fact that the ocean is on a rotating planet strongly inhibits motions toward the north or south. This is owing to the conservation of angular momentum in the ocean. So how does the ocean get around that constraint?

Well, let's look at the equations of motion governing a very slowly evolving ocean circulation. These are essentially the same as the equations we've already talked about for the atmosphere but with one additional important term. So here, for example, is an equation of motion for motion in the west to east direction. So u, in this case, is the west to east component of velocity in the ocean, d by dt is its time rate of change following a sample of fluid. This is the inverse of the density of seawater, alpha, which here we'll treat as a constant. Ocean water is nearly incompressible. And here is the zonal gradient of pressure in the ocean.

The second term, identical to the same term we see in the atmospheric equations, is the Coriolis term. It's twice the angular velocity of the Earth's rotation times the sine of the latitude times the south to north, or meridional, component of the ocean velocity v. The new term, which we didn't discuss in connection with the atmosphere, is the vertical gradient of the vertical flux of zonal momentum in the oceans. So tau sub x is a turbulent vertical flux of west to east momentum in the ocean.

Likewise, we have a similar equation governing motions in the north south direction v. So the acceleration of v, this term here, is given by minus the inverse density alpha, or specific volume, times the pressure gradient in the y direction minus the Coriolis term which is 2 omega sine theta times the zonal flow, u, and a similar stress term in the y direction. Coupled with those equations of motion is the mass continuity equation which, for the ocean, which is nearly incompressible-- and in that respect does differ appreciably from the atmosphere-- basically the mass continuity demands that the divergence of the three dimensional velocity vanish. That's what is shown by this equation.

Now, the total time derivative, the acceleration, can be broken up using the chain rule into a local derivative in time plus an advection, u d by dx plus v d by dy plus w d by dz, which can be written in shorthand as the partial derivative in time plus the velocity dotted with the gradient operator. So once again, the total time derivative denotes an actual acceleration in the reference frame of the parcel. The partial derivative in time is the timed tendency as measured by a fixed point in space.

Now, what we're going to do is to eliminate pressure between the first two equations and ignore local derivatives in time because we're looking at very slowly evolving circulations here. We're also going to ignore some small terms involving the vertical velocity w. Let's go back a minute and have a look at what we're going to do.

So we're going to eliminate pressure by taking the derivative of this first equation in y and subtracting that from a derivative of the second equation in x. By doing that, adding the two will eliminate the pressure terms. We're going to ignore time tendencies and certain terms that are small involving w, and we get this equation labeled here, equation one.

What does this say? It says v dot grad-- that's called the advection term of a quantity, which is the sum of the Coriolis parameter-- 2 omega sine theta-- plus zeta-- which is the vorticity, that's defined here. It's dv by the x minus du by dy. It's the vertical component of the curl of the velocity. It's a very important quantity in fluid dynamics. We'll refer to it as the vorticity.

So the horizontal advection of absolute vorticity, which is the sum of the [relative vorticity, zeta] and twice the local vertical projection of the Earth's angular velocity, is equal to the same term. That is, the absolute vorticity times dw by dz plus the vertical derivative of the curl of the stress.

Now, this term here is proportional to dw dz. That's sometimes called stretching. Mass continuity implies that where w is changing in z, fluid particles must be moving toward each other, converging or diverging. And by conservation of angular momentum, this will spin up or spin down the fluid. This vorticity can be thought of as a measure of the spin of the fluid. All right? The last term in equation one is the direct effect of the stress or the turbulent stress on the curl of the flow speed or the vorticity.

So if we now ignore non-linear terms, that is, if we look at this equation and ignore places where there are products of velocity-- for example, the product of vorticity in dw by dz or the v dot grad of the vorticity-- we can do that if the motions are sufficiently weak. We get an approximation to this equation, which can be written this way. v beta is equal to f dw dz plus the vertical gradient of the curl of the stress.

Now, the symbols we've introduced here for shorthand are beta, which is just 2 omega cosine theta. It's the derivative of the 2 omega sine theta or Coriolis term in latitude. We'll just refer to that as beta. And f, which is 2 omega sine theta, we'll use for the Coriolis parameter. OK? So this introduces some shorthand.

Now, the next thing we're going to do to make an interesting point is to take this equation we've just developed-- equation two-- and integrate it vertically from the bottom of the ocean to the top of the ocean. And we're going to ignore any turbulent stress that might occur at the bottom of the ocean for the purposes of this. Now, when we integrate this term vertically, we're going to get f times the difference between w at the top of the ocean and w at the bottom. But w at the top and w at the bottom vanish because we can't have water flowing up through the top of the ocean or down through the bottom of the ocean.

So when we do that integral, we get beta times the vertical integral of v-- so that's the total north south motion of a vertical column of the ocean-- is equal, basically, to the curl of the surface stress. Which, of course, is exerted on the ocean by the wind. So we have a remarkable relationship that the net vertically integrated north south motion of the water on slow time scales is proportional to the curl of the wind stress of the surface divided by beta. This remarkable relation was developed in the 1940s by the physical oceanographer Harold Sverdrup and is known as the Sverdrup Relation.

What does that imply? Well, if we go back to our yearly climatology of winds stress-- that's the white arrows with the magnitude given by the coloring-- well, in this region here, for example, the curl of the wind stress is negative. If we look at the wind stress, it's varying rapidly from north to south here. If we look at derivatives, we see that the curl of the wind stress is negative. And that implies, by the equation we developed in the previous slide, that there is a vertically integrated southward transport of ocean water.

Likewise, in the far North Atlantic, north of the strong Westerlies, there's a positive curl of the wind stress. And that would imply that water in the net should be moving northward there. If we go to the Atlantic we see a similar pattern, across the sub tropics there's a strong negative curl of the wind stress implying water moving southward in the subtropical Atlantic, whereas in the far North Atlantic the curl of the wind stress is positive and we'd expect that to drive a northward ocean current in the far North Atlantic.

Now, the southern hemisphere is more complicated. There is a strong positive curl of the wind stress. And that should drive ocean water northward from the equator-ward side of the southern ocean. But on the other hand, if we take the mass continuity equation and integrate it vertically, this says that wherever there is a non-zero vertical integral of the meridional velocity, with some derivative in the north or south direction, there has to exist an east west velocity. All right?

So how are we going to actually use this equation? We sort of know what the right hand side of the equation is from the Sverdrup relationship. How will we use this equation to compute the zonal part of the vertically integrated ocean velocity? Well, we can simply integrate this equation in x, whereupon we get the integral over the depth of the ocean of the zonal velocity, is equal to minus the integral of d by dy of the vertical integral of the meridional velocity in x plus an integration constant which can be a function of y.

How do we determine that integration constant? Well, for one thing, we have to insist that there be no flow through a north south boundary. That's a boundary condition and we can help to use that. But this problem is not solvable, strictly speaking, if we talk about a closed ocean basin with north south walls on the east and west sides. Because we only have one integration constant to work with.

So we can't satisfy u equals 0 at both the eastern and western boundaries. And Henry Stommel showed that if we have to make that choice, we have to enforce that u equals 0 specifically on the eastern boundary, and invoke a different set of equations in a narrow boundary layer on the western side where all the return flow is concentrated. So he actually showed that these equations imply that there have to be strong concentrated, generally pole-ward currents, along the western margins of closed ocean basins. And this basically is the explanation for currents like the Gulf Stream.

So let's have a look at that in the context, again, of a map of the time mean flow of currents at the ocean surface. This is not, strictly speaking, vertically integrated ocean currents. This comes from an atlas that was produced in the 1940s but will serve our purpose here. We see that we have, in the Atlantic for example, as predicted by the Sverdrup relationship, basically a southward flow through most of the Atlantic. To satisfy mass continuity, we have to have eastward motion up here, westward motion down there, no flow through the boundary. And as predicted by Henry Stommel, we have a narrow current-- return current, the Gulf Stream-- on the western flank which don't obey the Sverdrup equations because of their intensity and the importance of bottom friction and lateral friction.

These boundary currents specifically occur on the west side of ocean basins. That asymmetry, the fact that it's west and not east, is because of the particular sine of beta. Beta is a positive number.

We see the same kind of thing in the North Pacific ocean with this anticyclonic gyre here. Generally southward motion in the interior of the extra tropics, southward motion along the eastern periphery, and very strongly concentrated boundary current, the Kuroshio current here on the west side. One can see similar things in the southern hemisphere. So this remarkably simple set of equations, with supplements, predicted that there should be, in mid-latitudes, these big anticyclonic ocean gyres with very strong concentrated pole-ward flow on their western flanks.