Let's now talk about a concept called geostrophic balance. In geostrophic balance, we assume that there's no acceleration at all. And in that case, the horizontal pressure gradient, alpha dp by dy, has to be balanced by this Coriolis term. That is, there has to be enough West-East motion to balance the pressure gradient.

This is a fundamental balance that we see in both the ocean and the atmosphere for motions that evolve on time scales much larger than one day. And that includes most of the large-scale overturning circulations and the eddies in both fluids. This is the fundamental reason why on weather maps, for example, in the northern hemisphere, we find air circulating clockwise around high pressure systems and counterclockwise around low pressure systems. This is a direct reflection of geostrophic balance.

Again, f is the Coriolis parameter. And if we were to go through the equations of motion and do things carefully, we'd find a very similar expression for the balance of East-West pressure gradients, which don't exist in our idealized world, but do exist in the real world, which simply says that alpha dp by dx is plus the Coriolis parameter times the relative velocity.

From this equation of geostrophic balance, we can derive something called the thermal wind equation. And what we are going to do is to combine expressions for the geostrophic balance in the y-direction, so the pressure gradient in y that we have thanks to the fact that temperature varies in the North-South direction, is equal to minus f times the West-East wind component-- that's geostrophic balance. And then our old friend hydrostatic balance, which can be written alpha dp by dz is minus g.

Now by cross-differentiating, we can eliminate pressure. When we do that, we get something called the thermal wind equation, which says that the Coriolis parameter times the vertical derivative of the West-East wind is equal to minus the acceleration of gravity times the gradient in y of the log of the temperature. And this gradient has to be taken on a constant pressure surface.

This is called the thermal wind equation. And what it implies is that, if the temperature decreases poleward in either hemisphere, the zonal wind, that is, the West-East component of the wind, should increase with altitude. And since we're taking the wind to be zero at the surface, this implies that where there exist zones in which the temperature decreases toward the pole, the zonal wind must increase with altitude. And that means that we can construct a completely consistent nonlinear solution to all of the equations that govern the thermodynamics and the dynamics of the atmosphere.

So we start with our radiative-convective equilibrium state. Here, again, is the equator. Here's the North Pole-- a mirror image for the South Pole. The radiative-convective equilibrium, naturally, has higher temperatures toward the equator, lower temperatures toward the pole. This results in pressures that decrease at altitude toward the pole. And the accelerations that would normally result from these North-South pressure gradients are balanced by Coriolis forces, which require westerly winds increasing with height.

So in this diagram, I've denoted the winds by these yellow circles, whose diameters are increasing with altitude. So at altitude, we have strong west winds. And as we go down, these become weaker. And the thermal wind equation predicts that these winds will be strongest where the temperature gradient is strongest, but weighted toward the equatorward side of that, where the Coriolis parameter tends to be smaller. Remember the Coriolis parameter appears in this thermal wind balance.

So there's a prediction that in this nice, nonlinear equilibrium solution, we have westerly winds increasing with height, which, as you perhaps know, is a characteristic of the observed atmosphere in middle latitudes. Now, because these winds blow from West to East, they don't transport energy in the North-South direction. And so this is a perfectly general nonlinear equilibrium solution of all the equations we know about for our idealized planet, which isn't very different from the Earth. It lacks seasons, and it lacks continents. But we're going to start from that point, and we're going to ask, well if this is such a beautiful solution, how come we don't observe it?

Well, we could say, well, of course, the real world has seasons, and it has continents. But we can and have created computer models of our idealized planet, and we still don't observe solutions like this, so what gives? Let's just review.

There is an exact solution. Every individual column of the atmosphere and the surface beneath it are in radiative-convective equilibrium. Surface pressure doesn't vary across the planet. And pressure above the surface decreases poleward, more rapidly at higher altitudes. The pressure gradient is balanced geostrophically with a west wind. So we predict that in this idealized solution, the west winds increase with altitude in middle latitudes where there's a strong temperature gradient.

There are essentially two problems with this solution. The first is that there might not be enough angular momentum in the whole system available for the West-East wind component necessary to balance the temperature gradient through thermal wind. The second is that the equilibrium, although it might be a perfectly good equilibrium solution, may be unstable. That is, the slightest perturbation to it might result in its evolution to a completely different regime. We're going to look at both of these possible problems.