Of course, the real world is rotating. And we should now look at how we have to modify the inertial reference frame equations to take this into account. So let's talk about the rotating reference frame of the earth.

So if we look down from space at the world with the North Pole at the center, of course the earth is rotating in the counterclockwise direction from this perspective. And looking at it from the side, we see the rotation axis given by this vertical axis here. The rotation having an angular velocity Omega, so we'll use uppercase Omega to denote the angular rotation rate of the planet.

And we'll define some coordinates here. We'll take a to be the mean radius of the earth. Of course, the earth isn't a precise sphere. The radius does vary between the equator and the pole. We'll come back to that in a minute. But for the moment, we'll talk about that as though it were constant.

And we'll define r here as the distance of any point on the surface from the rotation axis. Theta will be the latitude. And simple geometry tells us that r is equal to the radius, a times the cosine of theta.

Now the y direction, as it's been defined in the equations above, is the direction toward the North Pole parallel to the surface. We can also talk about an air velocity moving from west to east at a velocity, u. So let's rewrite the equation of motion for air moving in the north south direction in this rotating frame. And we start with what we had before, the acceleration is equal to minus alpha times pressure gradient. But we now have a term which is essentially the centrifugal force on that air. It's the net motion, u, from west to east times the sine of the latitude divided by the radius.

So u squared over r is the net centrifugal acceleration. And minus sine theta is its component in the y direction. So if we're right at the equator, of course, the centrifugal acceleration is outward, and that doesn't project at all on y. Whereas if we're at the pole, u is 0 at the pole, so we have another problem there. But sine theta takes into account the projection of this centrifugal acceleration in the direction that we're calling y.

So let's talk about this velocity, u. We've defined it as the total air velocity, including the fact that the planet is rotating. We can break that down into the complement that's owing to the fact that the Earth is rotating, that's Omega a cosine theta. It's just the component of rotation in the west to east direction. Plus what I'm calling u relative. It's the velocity of the air relative to the Earth. It's what we conventionally call the wind velocity.

And once again, r is equal to a cosine theta here. So we're going to substitute this total velocity expression, this breakdown into a component due to the Earth's rotation, plus the relative flow into the equation of motion in the y direction. And that gives us a new equation. That the acceleration dv by dt is equal to minus the inverse density or specific volume times the pressure gradient, as before.

But now we have these new terms. We have this term, which is proportional to Omega squared times the cosine of the latitude times the sine of the latitude times the radius of the earth. That doesn't depend on the relative flow at all. We have a second term that's twice the angular velocity of the earth's rotation times the sine of the latitude times the relative flow in the west to east direction. That's what we conventionally call the west east component of wind.

And then a final term proportional to the square of this relative flow. So we have these new terms to deal with. Now this first new term is kind of interesting. It tells us that there should be a constant acceleration toward the equator, regardless of whether the air is moving at all. This doesn't seem exactly right. It means that every object on the surface of the Earth should feel an acceleration toward the equator.

Why is that term there at all? Well this gets back to the fact that we've assumed from the beginning that the earth is a perfect sphere. Well it isn't, of course. It's an oblate sphere. And in practice, what we do is to take this term and combine it with another constant term, which is gravity. And when we do that, we discover that the sum of those two terms acts in a direction that's perpendicular to the actual oblate surface of the sphere. All right? So this term exists in this equation simply because we haven't been terribly careful and made a distinction between a sphere and an oblate sphere.

So what we do is we absorb that into gravity, and we slightly re-define y as parallel to the actual surface of the earth, and not parallel to its spherical equivalent. So we're going to absorb that into gravity. And we're going to write the result this way. So the acceleration in the north south direction is the pressure gradient times this term, which is proportional to the product of the angular velocity and the relative velocity of the wind. And a third term proportional to the square of the relative velocity.

Now if you simply compare the magnitudes of these two terms for atmospheric motions, one finds that this last term is quite small compared to the second term. And so for the purposes of our discussion here, we're going to ignore it. It's possible to take this into account. It doesn't change anything qualitatively. But it's small. So we'll ignore it here.

And then we get this simple equation with one additional term, which is called the Coriolis acceleration, named after a French scientist from the middle of the 19th century who first formulated it correctly. So we'll write the equation of motion in an earth-relative coordinate system as the acceleration is equal to minus alpha dp dy minus a quantity called f times u relative, where f is the Coriolis parameter, it's just twice the angular rotation rate of the earth times the sine of the latitude.

And so the Coriolis parameter is just a function of the latitude. And this acceleration is called the Coriolis acceleration. And it's an acceleration that, in general, tries to accelerate air to the right of its motion in the northern hemisphere, and to the left of its motion in the southern hemisphere.