We're now in a position to develop a very simple radiative-convective model, which is a variation on the very simple radiative models that we discussed before in this class. So we're going to begin with a two layer model, which again is similar to the radiative models we've talked about before. In that we're going to consider two layers of gas, both of which are completely transparent to sunlight. But which are at the same time opaque to infrared radiation.

So the amount of sunlight coming into the system is represented by the Stefan Boltzmann constant sigma times the fourth power of the effective radiating temperature, T sub e. That's just our way of representing the solar flux. All of that solar flux is assumed to be absorbed by the surface.

Now unlike the radiative model, the surface gets rid of heat two ways. There is the usual way, radiating upward at sigma Ts to the fourth, where Ts is the surface temperature. But we're also going to include a convective flux, F sub s from the surface. Layer one radiates infrared radiation upward and downward at sigma times its temperature raised to the fourth power. It also receives a convective heat flux from the surface, F sub s. And it also transmits a convective flux upward toward layer two.

Layer two emits radiation upward and downward at sigma T2 to the fourth and receives convective heat flux from F sub c. So we have two more unknowns in this problem, which are the convective fluxes, but we're also going to put two more constraints, which we've learned from observations of the atmosphere and from the theory of convection. That is, we're going to assume that the radiative state is unstable to convection. And that the effect of convection here is to enforce a moist adiabatic lapse rate.

Now, for the sake of simplicity, we're going to assume that the temperature of the first layer, T sub 1, is equal to the temperature of the second layer plus a prescribed temperature difference, Delta T, which is meant to represent the difference of temperature along a moist adiabat. Likewise, we'll represent the surface temperature as the upper layer, which was twice Delta T.

Now, of course, the moist adiabatic lapse rate is not constant with height. But we're going to not deal with that complication here. And just stick to this very simple representation. So we have two more unknowns. But we have two more constraints. And we can solve this system.

So what do we have? Well at the top of the atmosphere, we have exactly the same balance as we had in the pure radiative model. That is, the only way for energy to get out of the system at the top of the atmosphere is by infrared radiation. So we have sigma T 2 to the fourth, must equal sigma T e to the fourth. Which means that, as before, the top layer temperature is equal to the effective black body temperature which is determined by sunlight and the albedo.

Once we've determined the temperature of the top layer by our moist adiabatic constraint, the temperatures are trivially determined for the other layers. So T1 is the effective black body temperature plus Delta T. And the surface temperature T sub s is the effective black body temperature plus 2 times Delta T. So that part's easy. All that remains is to determine the convective fluxes, which we get from energy balance of the other layers.

So, for example, energy balance at the surface demands that the surface convective flux plus the surface radiative flux equals the absorbed sunlight plus the infrared radiation coming down from layer one. So that's a surface energy balance. That right away tells us what the surface flux is, because we already know what the temperatures of the system are.

The balance in layer two is that the emission of infrared radiation by that layer, which is 2 sigma Te to the fourth, since T2 equals T e, must equal the absorption of infrared radiation coming from the first layer plus the convective flux coming from the first layer. Since we know T1-- T1 is just equal to Te plus Delta T-- this tells us what the convective flux is. So that is a very simple way of solving the system.

To write down the solution for the fluxes, I'm going to first define a new variable x, which is just Delta T divided by T sub e. The solution for the surface convective flux is just the solar flux, sigma Te to the fourth, times this function of x that you see here in brackets. Now what is that function?

Well, first of all, notice that when x equals zero, this will equal 1. We get 1 plus 1 minus 1. But if x becomes large, this term here will dominate, 2 x raised to the fourth power. And the flux will become negative. Which is unphysical -- convection has to produce a positive heat flux. So there'll be some particular value of x which makes the quantity in brackets 0. One can show that that value of x is associated with a Delta T, which is exactly what one would get in pure radiative equilibrium.

So for the solution to be viable, the radiative equilibrium temperature lapse rate, Delta T rad, you might call it, must be larger than the Delta T we specify along the moist adiabat. Likewise, the convective flux from layer one to layer two is given by this quantity that you see at the bottom here. And that also has a critical value of x associated with it. It can be positive or negative. But physically it has to be positive. And that will be true as long as the radiative equilibrium is unstable to convection.

Now as a first step toward a more complete calculation of radiative-convective equilibrium, we'll look back at, again, the paper by Manabe and Strickler, going all the way back to 1964, in which they used a model with many, many layers, which were not assumed to be opaque or perfectly transparent, but had realistic emissivities.

However, the convection in this particular model was represented fairly crudely. Very similar to the way we represented it in our two layer model. Which is to say that whenever the temperature gradient exceeds a particular critical value, which in this case is just 6 and 1/2 degrees Kelvin per kilometer, the model forced that lapse rate to go back to the moist adiabatic value.

Now as we saw earlier in this module, the moist convective equilibrium lapse rate is not a constant with height. It varies. It tends to be quite small near the surface and get larger as one goes up in altitude, so it's unrealistic in that respect. But the most unrealistic aspect of this calculation was that the water vapor content of the troposphere was simply specified. It completely ignores the fact that it's convection itself that lofts water vapor into the atmosphere, making it a very strongly two-way calculation.

This is simply a one-way calculation in which the water vapor profile together with all the other radiatively active trace gases, like carbon dioxide, methane and ozone, have been specified. Remember that this is the pure radiative equilibrium solution. This is the radiative dry convective equilibrium solution we've discussed before. And this is the radiative quasi-moist convective equilibrium solution, where we simply specify a uniform lapse rate that kind of stands in for a moist adiabat.

Notice in this case the surface temperature is even lower, 300 degrees Kelvin, corresponding to about 27 degrees centigrade. Not very unlike temperatures one encounters at the surface of the tropics. So the surface is cooler. But above a few kilometers, above this point here, the solution is warmer than the radiative equilibrium solution. When one gets above this point here, the solution is warmer than the pure radiative solution.

That means that the upper troposphere from this point up to the lower stratosphere here is cooling radiatively. And this is a characteristic of moist radiative-convective equilibrium. That the troposphere-- most of the troposphere-- is cooling radiatively. Notice that there are small differences in the stratosphere. And this results from the fact that whenever one changes the temperature of the troposphere, or, for that matter, any part of the column, radiative transfer is non-local. That will affect the radiative emissions to all other layers, potentially. And that can affect your temperature. So even though there's no convection in the stratosphere, one gets a slightly different temperature.