# TOTIENTS

THE AUTHOR

## 1. Totient Sums

x Suppose we want to find the sum of totients $\varphi(i)$ for $1 < i \leq n$
Andy in Stack exchange:

Let
F(N) = cardinality$\{a, b : 0 < a < b \leq N\}$
R(N) = cardinality$\{a, b : 0 < a < b \leq N, gcd(a, b) = 1\}$

$$R(N) = F(N) - \sum_{m=2}^{N} R(\lfloor \frac{N}{m} \rfloor)$$

```
def R2(N,X2={}):
    if N==1:
        return 0
    try:
        return X2[N]
    except KeyError:
        fsum = F(N)
        m=2
        while 1:
            x = N//m
            nxt = N//x
            if(nxt >= N):
                result=fsum - (N-m+1)*R2(N//m,X2)
                X2[N]=result
                return result
            fsum -= (nxt-m+1) * R2(N//m,X2)
            m = nxt+1

def F(N):
    return N*(N-1)//2

#returns sum of totients of x<=n
```

```
#wrapper for R2
#sum of totient(x) for x<=n
def totientSum(n):
    return R2(n)+1
```

$$\sum_{k=2}^{n} k^j R_j\left(\left\lfloor \frac{n}{k} \right\rfloor\right)$$

$$= \sum_{k=2}^{\lfloor n/q \rfloor} k^j R_j\left(\left\lfloor \frac{n}{k} \right\rfloor\right) + \sum_{m=1}^{q-1} \sum_{k=\lfloor \frac{n}{m+1} \rfloor+1}^{\lfloor \frac{n}{m} \rfloor} k^j R_j(m)$$

$$= \sum_{k=2}^{\lfloor n/q \rfloor} (S_j(k) - S_j(k-1)) R_j\left(\left\lfloor \frac{n}{k} \right\rfloor\right) + \sum_{m=1}^{q-1} \left( S_j\left(\left\lfloor \frac{n}{m} \right\rfloor\right) - S_j\left(\left\lfloor \frac{n}{m+1} \right\rfloor\right)\right) R_j(m)$$

Daniel Fischer writes:

$$R(N) = F(N) - F\left(\left\lfloor \frac{N}{2} \right\rfloor\right) - \sum_{k=1}^{\lfloor \frac{N-1}{2} \rfloor} R\left(\left\lfloor \frac{N}{2k+1} \right\rfloor\right)$$