# BinaryNet: Training Deep Neural Networks with Weights and Activations Constrained to +1 or -1

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### **Abstract**

We introduce BinaryNet, a method which trains DNNs with binary weights and activations when computing parameters' gradient. We show that it is possible to train a Multi Layer Perceptron (MLP) on MNIST and ConvNets on CIFAR-10 and SVHN with BinaryNet and achieve nearly state-of-the-art results. At run-time, BinaryNet drastically reduces memory usage and replaces most multiplications by 1-bit exclusive-not-or (XNOR) operations, which might have a big impact on both general-purpose and dedicated Deep Learning hardware. We wrote a binary matrix multiplication GPU kernel with which it is possible to run our MNIST MLP 7 times faster than with an unoptimized GPU kernel, without suffering any loss in classification accuracy. The code for BinaryNet is available.

### Introduction

Deep Neural Networks (DNNs) have substantially pushed Artificial Intelligence (AI) limits in a wide range of tasks, including but not limited to object recognition from images (Krizhevsky et al., 2012; Szegedy et al., 2014), speech recognition (Hinton et al., 2012; Sainath et al., 2013), statistical machine translation (Devlin et al., 2014; Sutskever et al., 2014; Bahdanau et al., 2015), Atari and Go games (Mnih et al., 2015; Silver et al., 2016), and even abstract art (Mordvintsev et al., 2015).

Today, DNNs are almost exclusively trained on one or many very fast and power-hungry Graphic Processing Units (GPUs) (Coates et al., 2013). As a result, it is often a challenge to run DNNs on target low-power devices, and

much research work is done to speed-up DNNs at run-time on both general-purpose (Vanhoucke et al., 2011; Gong et al., 2014; Romero et al., 2014; Han et al., 2015) and specialized computer hardware (Farabet et al., 2011a;b; Pham et al., 2012; Chen et al., 2014a;b; Esser et al., 2015).

We believe that the contributions of our article are the following:

- We introduce BinaryNet, a method which trains DNNs with binary weights and activations when computing the parameters' gradient (see Section 1).
- We show that it is possible to train a Multi Layer Perceptron (MLP) on MNIST and ConvNets on CIFAR-10 and SVHN with BinaryNet and achieve nearly state-of-the-art results (see Section 2).
- We show that, at run-time, BinaryNet drastically reduces memory usage and replaces most multiplications by 1-bit exclusive-not-or (XNOR) operations, which might have a big impact on both general-purpose and dedicated Deep Learning hardware. We wrote a binary matrix multiplication GPU kernel with which it is possible to run our MNIST MLP 7 times faster than with an unoptimized GPU kernel, without suffering any loss in classification accuracy (see Section 3).

## 1. BinaryNet

In this section, we detail our binarization function, how we use it to compute the parameters' gradient and how we backpropagate through it.

#### **Sign function**

BinaryNet constrains both the weights and the activations to either +1 or -1. Those two values are very advantageous from a hardware perspective, as we explain in Section 3. Our binarization function is simply the sign func-

tion:

$$x^{b} = \operatorname{Sign}(x) = \begin{cases} +1 & \text{if } x \ge 0, \\ -1 & \text{otherwise.} \end{cases}$$
 (1)

where  $x^b$  is the binarized variable (weight or activation) and x the real-valued variable. It is very straightforward to implement and works quite well in practice (see Section 2). Stochastic binarization could be used, as in (Courbariaux et al., 2015), and is more theoretically appealing but is also a more costly alternative because it requires the hardware to generate random bits when quantizing.

## Gradients computation and accumulation

A key point to understand about BinaryNet is that although we *compute* the parameters' gradient using binary weights and activations, we nonetheless *accumulate* the weights' real-valued gradient in real-valued variables, as per Algorithm 1. Real-valued weights are likely needed for Stochasic Gradient Descent (SGD) to work at all. SGD explores the space of parameters by making small and noisy steps and that noise is *averaged out* by the stochastic gradient contributions accumulated in each weight. Therefore, it is important to keep sufficient resolution for these accumulators, which at first sight suggests that high precision is absolutely required.

Beside that, adding noise to weights and activations when *computing* the parameters' gradient provides a form of regularization which can help to generalize better, as previously shown with variational weight noise (Graves, 2011), Dropout (Srivastava, 2013; Srivastava et al., 2014) and DropConnect (Wan et al., 2013). BinaryNet can be seen as a variant of Dropout, in which instead of randomly setting half of the activations to zero when computing the parameters' gradient, we binarize both the activations and the weights.

#### **Propagating Gradients Through Discretization**

The derivative of the sign function is 0 almost everywhere, making it apparently incompatible with backpropagation, since exact gradients of the cost with respect to the quantities before the discretization (pre-activations or weights) would be zero. Note that this remains true even if stochastic quantization is used. Bengio (2013) studied the question of estimating or propagating gradients through stochastic discrete neurons. They found in their experiments that the fastest training was obtained when using the "straight-through estimator", previously introduced in Hinton (2012)'s lectures.

We follow a similar approach but use the version of the straight-through estimator that takes into account the saturation effect and does use deterministic rather than stochastic sampling of the bit. Consider the sign function quantiAlgorithm 1 Training a DNN with BinaryNet. C is the cost function for minibatch,  $\lambda$  the learning rate decay factor and L the number of layers.  $\circ$  indicates element-wise multiplication. The function Sign() specifies how to binarize the activations and weights, Clip() how to clip the weights, BatchNorm() and BackBatchNorm() how to batch normalize and backpropagate through the normalization (See Algorithm 2), and Adam() how to update the parameters knowing their gradient (See Algorithm 3).

**Require:** a minibatch of inputs and targets  $(a_0, a^*)$ , previous weights W, previous BatchNorm parameters  $\theta$ , weights initialization coefficients from (Glorot & Bengio, 2010)  $\gamma$ , and previous learning rate  $\eta$ .

**Ensure:** updated weights  $W^{t+1}$ , updated BatchNorm parameters  $\theta^{t+1}$  and updated learning rate  $\eta^{t+1}$ .

```
{1. Computing the parameters' gradient:}
{1.1. Forward propagation:}
for k=1 to L do
    W_k^b \leftarrow \operatorname{Sign}(W_k) \\ s_k \leftarrow a_{k-1}^b W_k^b
     a_k \leftarrow \text{BatchNorm}(s_k, \theta_k)
     if k < L then
          a_k^b \leftarrow \operatorname{Sign}(a_k)
     end if
end for
 {1.2. Backward propagation:}
{Please note that the gradients are not binary.}
Compute g_{a_L} = \frac{\partial C}{\partial a_L} knowing a_L and a^*
for k = L to 1 do
     if k < L then
          g_{a_k} \leftarrow g_{a_k^b} \circ 1_{|a_k| \le 1}
     (g_{s_k}, g_{\theta_k}) \leftarrow \text{BackBatchNorm}(g_{a_k}, s_k, \theta_k)
     g_{a_{k-1}^b} \leftarrow g_{s_k} W_k^b
    g_{W_k^b} \leftarrow g_{s_k}^\top a_{k-1}^b
{2. Accumulating the parameters' gradient:}
\mathbf{for}\ k=1\ \mathrm{to}\ L\ \mathbf{do}
    \begin{array}{l} \boldsymbol{\theta}_k^{t+1} \leftarrow \operatorname{Adam}(\boldsymbol{\theta}_k, \boldsymbol{\eta}, g_{\boldsymbol{\theta}_k}) \\ \boldsymbol{W}_k^{t+1} \leftarrow \operatorname{Clip}(\operatorname{Adam}(\boldsymbol{W}_k, \gamma_k \boldsymbol{\eta}, g_{\boldsymbol{W}_k^b}), -1, 1) \end{array}
     \eta^{t+1} \leftarrow \lambda \eta
```

end for

**Algorithm 2** Batch Normalizing Transform (Ioffe & Szegedy, 2015), applied to activation x over a mini-batch.

**Require:** Values of x over a mini-batch:  $B = \{x_{1...m}\}$ ; Parameters to be learned:  $\gamma$ ,  $\beta$ 

Ensure: 
$$\{y_i = \text{BN}x_i\gamma, \beta\}$$
  
 $\mu_B \leftarrow \frac{1}{m} \sum_{i=1}^m x_i \text{ {mini-batch mean}}$   
 $\sigma_B^2 \leftarrow \frac{1}{m} \sum_{i=1}^m (x_i - \mu_B)^2 \text{ {mini-batch variance}}$   
 $\hat{x_i} \leftarrow \frac{x_i - \mu_B}{\sqrt{\sigma_B^2 + \epsilon}} \text{ {normalize}}$   
 $y_i \leftarrow \gamma \hat{x_i} + \beta \equiv \text{BN}x_i\gamma, \beta \text{ {scale and shift}}$ 

**Algorithm 3** ADAM learning rule (Kingma & Ba, 2014).  $g_t^2$  indicates the elementwise square  $g_t \circ g_t$ . Good default settings are  $\alpha = 0.001$ ,  $\beta_1 = 0.9$ ,  $\beta_2 = 0.999$  and  $\epsilon = 10^{-8}$ . All operations on vectors are element-wise. With  $\beta_1^t$  and  $\beta_2^t$  we denote  $\beta_1$  and  $\beta_2$  to the power t.

**Require:** Previous parameters  $\theta_{t-1}$  and their gradient  $g_t$ , and learning rate  $\alpha$ .

**Ensure:** Updated parameters  $\theta_t$ 

$$\begin{split} & \{ \text{Biased 1st and 2nd raw moment estimates:} \} \\ & m_t \leftarrow \beta_1 \cdot m_{t-1} + (1-\beta_1) \cdot g_t \\ & v_t \leftarrow \beta_2 \cdot v_{t-1} + (1-\beta_2) \cdot g_t^2 \\ & \{ \text{Bias-corrected 1st and 2nd raw moment estimates:} \} \\ & \hat{m} \leftarrow m_t/(1-\beta_1^t) \\ & \hat{v} \leftarrow v_t/(1-\beta_2^t) \\ & \{ \text{Updated parameters:} \} \\ & \theta_t \leftarrow \theta_{t-1} - \alpha \cdot \hat{m}/(\sqrt{\hat{v}} + \epsilon) \end{split}$$

zation

$$q = \operatorname{Sign}(r)$$

and assume that an estimator  $g_q$  of the gradient  $\frac{\partial C}{\partial q}$  has been obtained (with the straight-through estimator when needed). Then, our straight-through estimator of  $\frac{\partial C}{\partial r}$  is simply

$$g_r = g_q 1_{|r| < 1}. (2)$$

Note that this preserves the gradient's information and cancels the gradient when r is too large. The use of this straight-through estimator is illustrated in Algorithm 1. The derivative  $1_{|r| \le 1}$  can also be seen as propagating the gradient through *hard tanh*, which is the following piecewise linear activation function:

$$Htanh(x) = Clip(x, -1, 1) = max(-1, min(1, x))$$
 (3)

For hidden units, we use the sign function non-linearity to obtain binary activations, and for weights we combine two ingredients:

 Constrain each real-valued weight between -1 and 1, by projecting w<sup>r</sup> to -1 or 1 when the weight update brings w<sup>r</sup> outside of [-1,1], i.e., clipping the weights during training, as per Algorithm 1. The real-valued weights would otherwise grow very large without any impact on the binary weights.

• When using a weight  $w^r$ , quantize it using  $w^b = \operatorname{Sign}(w^r)$ .

This is consistent with the gradient canceling when  $|w^r| > 1$ , according to Eq. 2.

# A few helpful ingredients

A few elements of our experiments, although not absolutely necessary, significantly improve the accuracy of BinaryNets, as indicated in Algorithm 1:

- Batch Normalization (BN) (Ioffe & Szegedy, 2015), partly detailed in Algorithm 2, accelerates the training and also seems to reduces the overall impact of the weights scale. The normalization noise may also help to regularize the model.
- The ADAM learning rule (Kingma & Ba, 2014), detailed in Algorithm 3, also seems to reduce the impact of the weights scale.
- Lastly, scaling the weights' learning rates with the weights' initialization coefficients from (Glorot & Bengio, 2010) also seems to help, as suggested by Courbariaux et al. (2015).

## 2. Benchmark results

Method	Test error rate
Binary expectation backpropagation (Cheng et al., 2015)	2.12%
Bitwise Neural Networks (Kim & Smaragdis, 2016)	1.33%
BinaryConnect (Courbariaux et al., 2015)	$1.18 \pm 0.04\%$
BinaryNet (this work)	0.96%
Deep L2-SVM (Tang, 2013)	0.87%

Table 1. Test error rates of MLPs trained on the permutation invariant MNIST (without knowledge that inputs are images and without unsupervised learning) depending on the method. BinaryNet achieves near state-of-the-art results with only a single bit per weights and activations. This result suggests that augmenting the number of hidden units can compensate for the discretization noise.

Method	Test error rate
BinaryNet (this work)	11.40%
BinaryConnect (Courbariaux et al., 2015)	8.27%
Gated pooling (Lee et al., 2015)	7.62%

Table 2. Test error rates of ConvNets trained on the CIFAR-10 (without data-augmentation) depending on the method.

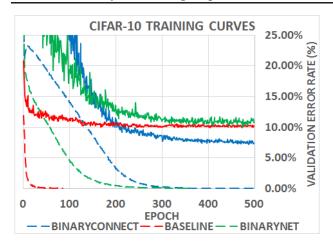


Figure 1. Training curves of a ConvNet on CIFAR-10 depending on the method. The dotted lines represent the training costs (square hinge losses) and the continuous lines the corresponding validation error rates. Intriguingly, BinaryNet is faster to train and yields worse results than BinaryConnect (Courbariaux et al., 2015), suggesting that it is slightly overfitting and might benefit from additional noise (e.g. Dropout).

Method	Test error rate
BinaryNet (this work)	2.80%
BinaryConnect (Courbariaux et al., 2015)	2.15%
Gated pooling (Lee et al., 2015)	1.69%

Table 3. Test error rates of ConvNets trained on the SVHN depending on the method.

We obtained near state-of-the-art results with BinaryNet on MNIST, CIFAR-10 and SVHN benchmarks. The code for reproducing these results is available <sup>1</sup>.

#### MLP on MNIST

MNIST is a benchmark image classification dataset (Le-Cun et al., 1998). It consists in a training set of 60K and a test set of  $10K\ 28 \times 28$  gray-scale images representing digits ranging from 0 to 9. In order for this benchmark to remain a challenge, we did not use any convolution, data-augmentation, preprocessing or unsupervised learning.

The MLP we train on MNIST consists in 3 hidden layers of 4096 binary units (see Section 1) and a L2-SVM output layer; L2-SVM has been shown to perform better than Softmax on several classification benchmarks (Tang, 2013; Lee et al., 2014). We regularize the model with Dropout (Srivastava, 2013; Srivastava et al., 2014).

The square hinge loss is minimized with the ADAM adap-

CIFAR-10 ConvNet architecture  Input: $32 \times 32$ - RGB image $3 \times 3$ - 128 convolution layer  BatchNorm and Binarization layers $3 \times 3$ - 128 convolution and $2 \times 2$ max-pooling layers  BatchNorm and Binarization layers
$3\times3$ - 128 convolution layer BatchNorm and Binarization layers $3\times3$ - 128 convolution and $2\times2$ max-pooling layers
ž –
$3\times3$ - 256 convolution layer BatchNorm and Binarization layers $3\times3$ - 256 convolution and $2\times2$ max-pooling layers BatchNorm and Binarization layers
$3\times3$ - 512 convolution layer BatchNorm and Binarization layers $3\times3$ - 512 convolution and $2\times2$ max-pooling layers BatchNorm and Binarization layers
1024 fully connected layer BatchNorm and Binarization layers 1024 fully connected layer BatchNorm and Binarization layers
10 fully connected layer BatchNorm layer (no binarization) Cost: Mean square hinge loss

Table 4. Architecture of our CIFAR-10 ConvNet. We only use "same" convolutions, as in VGG (Simonyan & Zisserman, 2015).

tive learning rate method (Kingma & Ba, 2014). We use an exponentially decaying global learning rate and also scale the weights learning rates with the weights initialization coefficients from (Glorot & Bengio, 2010), as per Algorithm 1. We use Batch Normalization with a minibatch of size 100 to speed up the training. As typically done, we use the last 10K samples of the training set as a validation set for early stopping and model selection. We report the test error rate associated with the best validation error rate after 1000 epochs (we do not retrain on the validation set). The results are in Table 1.

#### **ConvNet on CIFAR-10**

CIFAR-10 is a benchmark image classification dataset. It consists in a training set of 50K and a test set of 10K 32 × 32 color images representing airplanes, automobiles, birds, cats, deers, dogs, frogs, horses, ships and trucks. We do not use any preprocessing or data-augmentation (which can really be a game changer for this dataset (Graham, 2014)).

The architecture of our ConvNet is detailed in Table 4. It is the same architecture as Courbariaux et al. (2015)'s except for the activations binarization. Courbariaux et al. (2015)'s architecture is itself greatly inspired from VGG (Simonyan & Zisserman, 2015).

The square hinge loss is minimized with ADAM. We use an exponentially decaying learning rate, like for MNIST. We scale the weights learning rates with the weights initialization coefficients from (Glorot & Bengio, 2010). We use Batch Normalization with a minibatch of size 50 to speed up the training. We use the last 5000 samples of the train-

¹https://github.com/MatthieuCourbariaux/
BinaryNet/tree/master/Train-time

ing set as a validation set. We report the test error rate associated with the best validation error rate after 500 training epochs (we do not retrain on the validation set). The results are in Table 2 and Figure 1.

#### ConvNet on SVHN

SVHN is a benchmark image classification dataset. It consists in a training set of 604K examples and a test set of 26K  $32 \times 32$  color images representing digits ranging from 0 to 9. We follow the same procedure that we used for CIFAR-10, with a few notable exceptions: we use half the number of units in the convolution layers and we train for 200 epochs instead of 500 (because SVHN is a much bigger dataset than CIFAR-10). The results are shown in Table 3.

## 3. Much faster at run-time

{3. Output layer:}

**Algorithm 4** Running a MLP trained with BinaryNet. L is the number of layers.

**Require:** a vector of 8-bit inputs  $a_0$ , the binary weights  $W^b$  and the BatchNorm parameters  $\theta$ .

```
 \begin{split} &\textbf{Ensure:} \text{ the MLP output } a_L. \\ & \{1. \text{ First layer:} \} \\ & a_1 \leftarrow 0 \\ & \textbf{ for } n = 1 \text{ to } 8 \textbf{ do} \\ & a_1 \leftarrow a_1 + 2^{n-1} \times \text{XnorDotProduct}(\mathbf{a}_0^n, \mathbf{W}_1^b) \\ & \textbf{ end for} \\ & a_1^b \leftarrow \text{Sign}(\text{BatchNorm}(a_1, \theta_1)) \\ & \{2. \text{ Remaining hidden layers:} \} \\ & \textbf{ for } k = 2 \text{ to } L - 1 \textbf{ do} \\ & a_k \leftarrow \text{XnorDotProduct}(a_{k-1}^b, W_k^b) \\ & a_k^b \leftarrow \text{Sign}(\text{BatchNorm}(a_k, \theta_k)) \\ & \textbf{ end for} \end{split}
```

 $a_L \leftarrow \text{XnorDotProduct}(a_{L-1}^b, W_L^b)$ 

 $a_L \leftarrow \text{BatchNorm}(a_L, \theta_L)$ 

BinaryNet, by drastically reducing memory usage and also replacing most 32-bit multiplications by 1-bit exclusive-not-or (XNOR) operations, is much faster to run than 32-bit float neural networks, both on general-purpose and dedicated hardware. We wrote a binary matrix multiplication GPU kernel with which it is possible to run our MNIST MLP 7 times faster than with an unoptimized GPU kernel, without suffering any loss in classification accuracy. The code for reproducing these results is available <sup>2</sup>.

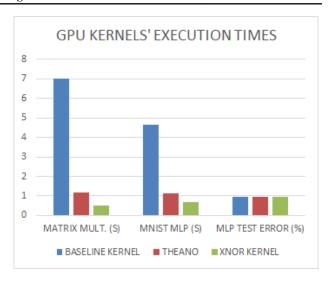


Figure 2. The first three columns represent the time it takes to perform a  $8192 \times 8192 \times 8192$  (binary) matrix multiplication on a GTX750 Nvidia GPU, depending on which kernel is used. We can see that our XNOR kernel is significantly faster than our baseline and Theano's (Bergstra et al., 2010; Bastien et al., 2012) kernels. The next three columns represent the time it takes to run the MLP from Section 2 on the full MNIST test set. As MNIST's images are not binary, the first layer's multiplications are always performed by the baseline kernel. The last three columns show that the MLP accuracy does not depend on which kernel is used.

## First layer

In a BinaryNet, both the weights and activations are binary. As the output of one layer is the input of the next, all the layers inputs are binary, with the exception of the first layer. However, we do not believe this to be a major issue. Firstly, in computer vision, the input representation typically has much fewer channels (e.g. Red, Green and Blue) than internal representations (e.g. 512). As a result, the first layer of a ConvNet is often the smallest convolution layer, both in terms of parameters and computations (Szegedy et al., 2014).

Secondly, it is relatively easy to handle continuous-valued inputs as fixed point numbers, with m bit of precision. For

<sup>2</sup>https://github.com/MatthieuCourbariaux/ BinaryNet/tree/master/Run-time

example, in the common case of 8-bit fixed point inputs:

$$s = x \cdot w^b \tag{4}$$

$$s = \sum_{i=1}^{1024} x_i w_i^b \tag{5}$$

$$s = \sum_{i=1}^{1024} \left( \sum_{n=1}^{8} 2^{n-1} x_i^n w_i^b \right) \tag{6}$$

$$s = \sum_{n=1}^{8} \left( 2^{n-1} \sum_{i=1}^{1024} \left( x_i^n w_i^b \right) \right) \tag{7}$$

$$s = \sum_{n=1}^{8} 2^{n-1} (x^n \cdot w^b)$$
 (8)

Where x is a vector of 1024 8-bit inputs,  $x_1^8$  the most significant bit of the first input,  $w^b$  a vector of 1024 1-bit weights and s the resulting weighted sum. This trick is used in Algorithm 4.

#### **XNOR-accumulate**

Applying a DNN mainly consists in convolutions and matrix multiplications. The key arithmetic operation of deep learning is thus the multiply-accumulate operation. Artificial neurons are basically multiply-accumulators computing weighted sums of their inputs. With BinaryNet, both the activations and the weights are constrained to either -1 or +1. As a result, most of the 32-bit floating point multiplications are replaced by 1-bit XNOR operations. This could have a huge impact on deep learning dedicated hardware. For instance, a 32-bit floating point multiplier costs about 200 FPGA slices (Govindu et al., 2004; Beauchamp et al., 2006), whereas a 1-bit XNOR gate only costs a single slice.

#### 7 times faster on GPU at run-time

It is possible to speed-up GPU implementations of DNNs trained with BinaryNet, by using a method some people call SIMD (single instruction, multiple data) within a register (SWAR). The basic idea of SWAR is to *concatenate* groups of 32 binary variables into 32-bit registers, and thus obtain a 32 times speed-up on bitwise operations (e.g. XNOR). Using SWAR, it is possible to evaluate 32 connections with only 4 instructions:

$$a_1 + = \text{popcount}(\text{not}(\text{xor}(a_0^{32b}, w_1^{32b})))$$
 (9)

Where  $a_1$  is the resulting weighted sum, and  $a_0^{32b}$  and  $w_1^{32b}$  the concatenated inputs and weights. Those 4 instructions take 7 clock cycles on recent Nvidia GPUs (and if they were to become a fused instruction, it would only take a single clock cycle). As a result, we get a theoretical Nvidia GPU speed-up of  $32/7 \approx 4.6$ . In practice, this speed-up is

quite easy to obtain as the memory bandwidth to computation ratio is also increased by 7 times.

In order to validate those theoretical results, we wrote two GPU kernels:

- The first kernel (baseline) is a quite unoptimized matrix multiplication kernel.
- The second kernel (XNOR) is nearly identical to the baseline kernel, except that it uses the SWAR method, as in Equation 9.

The two GPU kernels return exactly the same output when their inputs are constrained to -1 or +1 (but not otherwise). The XNOR kernel is about 14 times faster than the baseline kernel and 2.5 times faster than Theano's (Bergstra et al., 2010; Bastien et al., 2012), as shown in Figure 2. Last but not least, the MLP from Section 2 runs 7 times faster with the XNOR kernel than with the baseline kernel, without suffering any loss in classification accuracy (see Figure 2).

## 4. Related work

Courbariaux et al. (2015); Lin et al. (2015) train DNNs with *binary weights* when computing the parameters' gradient. In some of their experiments, they also exponentially quantize the activations during some parts (but not all) of the computations. By contrast, we train DNNs with *binary weights and activations*, which can be much more efficient in terms of hardware (see Section 3). Moreover, their method (BinaryConnect) is slower to train than ours (see Figure 1), yields worse results on MNIST (see Table 1) but better results on CIFAR-10 and SVHN (see Tables 2 and 3).

Soudry et al. (2014); Cheng et al. (2015) do not train their DNN with Backpropagation (BP) but with a variant called Expectation Backpropagation (EBP). EBP is based on Expectation Propagation (EP) (Minka, 2001), which is a variational Bayes method used to do inference in probabilistic graphical models. Let us compare their method to ours:

- Like ours, their method binarizes both the activations and the weights when computing the parameters' gradient.
- Their method optimizes the weights posterior distribution (which is not binary). In this regard, our method is quite similar as we accumulate the weights' gradient in real-valued variables.
- In order to obtain a good performance, their method requires to average over the output of a few binary neural networks sampled from the weights posterior distribution (which might be costly).

 Their method (binary expectation backpropagation) yields a good classification accuracy for fully connected networks on MNIST (see Table 1), but not (yet) for ConvNets.

Hwang & Sung (2014); Kim et al. (2014) *retrain* neural networks with *ternary* weights and *3-bit* activations, i.e.:

- They train a neural network with high-precision,
- After training, they ternarize the weights to three possible values -H, 0 and +H and adjust H to minimize the output error,
- And eventually, they retrain with ternary weights and 3-bit activations when computing the parameters' gradient.

By comparison, we *train all the way* with *binary* weights and activations, i.e., our training procedure could be hardware accelerated as it only needs very few multiplications, as in (Lin et al., 2015), and our binary DNNs are likely more efficient at run-time.

Similarly, Kim & Smaragdis (2016) *retrain* deep neural networks with *binary* weights and activations. Their method (bitwise neural networks) yields a good classification accuracy for fully connected networks on MNIST (see Table 1), but not (yet) for ConvNets.

## Conclusion

We have introduced BinaryNet, a method which trains DNNs with binary weights and activations when computing the parameters' gradient (see Section 1). We have shown that it is possible to train an MLP on MNIST and ConvNets on CIFAR-10 and SVHN with BinaryNet and achieve nearly state-of-the-art results (see Section 2). Moreover, at run-time, BinaryNet drastically reduces memory usage and replaces most multiplications by 1-bit exclusive-not-or (XNOR) operations, which could have a big impact on both general-purpose and dedicated deep learning hardware. We wrote a binary matrix multiplication GPU kernel with which it is possible to run our MNIST MLP 7 times faster than with an unoptimized GPU kernel, without suffering any loss in classification accuracy (see Section 3). Future works should explore how to extend the speed-up to train-time (e.g. by binarizing some gradients), and also extend benchmark results to other models (e.g. RNN) and datasets (e.g. ImageNet).

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