Quantum Computing and Cryptography

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Outline

Outline

Introduction

- Cryptography=TODO
- TODO: secret

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- TODO: secret

$\rightarrow$ Science of secret

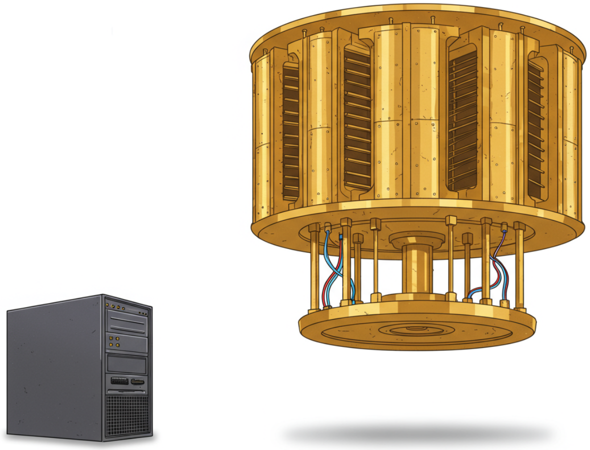
Outline

RSA

Outline

1. Intro
2. RSA
3. Quantum computing
   - Introduction to the quantum world
   - Quantum algorithms
4. Post-Quantum cryptography
   - Intro to PQ cryptography
   - Lattice cryptography
     - What is a lattice ?
   - Limits of PQ cryptography
5. Conclusion

Quantum computing



Outline

Qubits

Classical bit

$b \in \{0, 1\}$

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- $|-\rangle \rightarrow 0$ (50%), 1 (50%)

NOT gate

X gate

- $X\left|0\right\rangle \rightarrow \left|1\right\rangle$
- $X\left|1\right\rangle \rightarrow \left|0\right\rangle$

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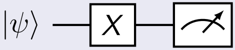
Circuit representation



Hadamard gate

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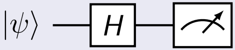

Outline

Bernstein-Vazirani Problem

Problem Definition

Given an oracle for a function f:

$$f : \{0, 1\}^n \rightarrow \{0, 1\}$$

$$f(\mathrm{x}) = \mathrm{x} \cdot \mathrm{s}$$

where s is a secret bit string. Find s with the fewest oracle calls. ($\cdot$ is the bitwise dot product, XOR sum).

Classical Algorithm - Example

Example (n=2)

To find s $= s_0 s_1$:

Requires 2 queries.

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To find s $= s_0 s_1$:

- Query $f(10)$. Result: s_0.

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Example (n=2)

To find s $= s_0 s_1$:

- Query $f(10)$. Result: s_0.
- Query $f(01)$. Result: s_1.

Requires 2 queries.

Classical Algorithm - General Case

Complexity: $\mathcal{O}(n)$

We need to isolate each bit of s by querying with inputs that have a single '1'. This requires n queries for an n-bit string.

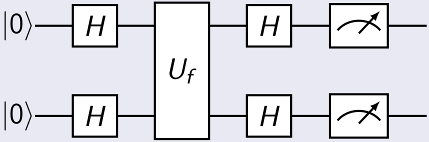
Quantum Algorithm - Overview

Complexity: $\mathcal{O}(1)$

The quantum algorithm can find s with just **one** query. It uses superposition to query all possible inputs simultaneously.

Quantum Algorithm - Circuit

Circuit Diagram



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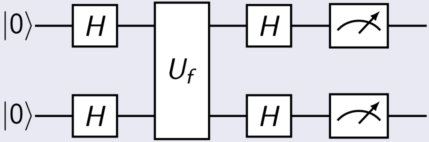

Explanation

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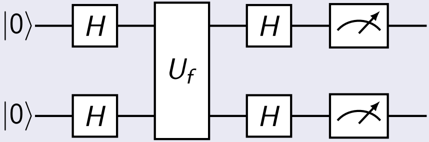

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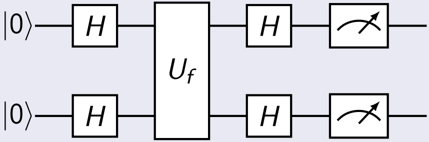

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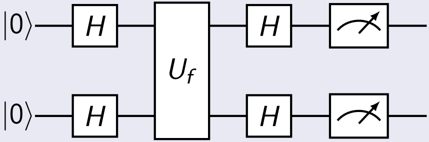

Explanation

- $H^{\otimes n}$: Hadamard gates on all n input qubits (creates superposition).
- U_f : The quantum oracle.
- Final Hadamards and measurement reveal s.

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Complexity Gain

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Requirements

- Requires a large number of high-quality (low-error) qubits (roughly $2n$ for an n-bit number).
- We currently don't have quantum computers large and stable enough to break practical RSA encryption.

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What is PQ cryptography

- Based on (other) mathematical problems
- Considered unsolvable by a quantum computer

What it is not :

- Cryptography **using** quantum technologies

The problems

- Codes
- Hash functions
- Multivariates polynomials systems
- Isogenies
- Lattices

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Why lattices ?

- Well spread
- Good results

Encryption/Key encapsulation	
Crystals-Kyber	Lattices
Signatures	
Crystals-Dilithium	Lattices
Falcon	Lattices
Sphincs+	Hash

Outline

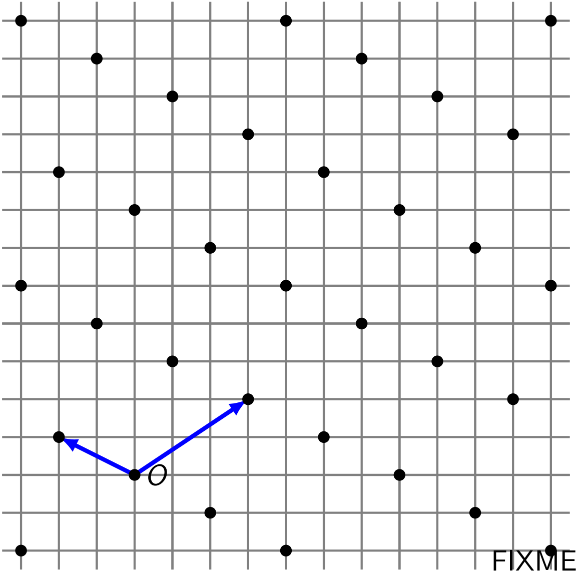
Some definitions

- A discret subgroup of RR^n

Like vector spaces, we have :

- Vectors and matrices
- Linear combination

Example



FIXME : Basis:

Learning with error problem

TODO

(Fully) homomorphic encryption

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- Used to manipulate private data (e.g. Medical data, data science)

Outline

Sizes of the keys and data

TODO	

Not necessarly robust to classical computer

- Example : Supersingular isogenies Diffie-Hellman key exchange

Outline

Conclusion