1. Why do we need "Quantum Gravity"?
(Introduction to Causal Set Theory)

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In the Proceedings of the Royal Society of London, in 1929, one of the giants of the twentieth-century scientific community, Paul Dirac narrated [1] at a moment: "The underlying physical laws necessary for the mathematical theory of a large part of physics and the whole of chemistry are thus completely known." However, the advancements in technology and innovative mathematical theories to study nature at its fundamentals made this statement quite vague, in essence, with the passage of time. People also stated the same when the discovery of the Higgs boson completed the mysterious puzzle of the standard model (SM) and made heuristic additions to our understanding of the universe, but then cosmology played its role to change the game and showed that the successful SM explains only around four percent of the universe while the nature and dynamics of the rest of the twenty-six percent dark matter and the seventy percent dark energy cannot be explained under its realm as shown in figure 1. These and some other compelling arguments again convinced the scientists to think of a more fundamental formalism to describe the underlying phenomena of nature, that is, a *theory of quantum gravity* – the merger of classical general relativity and quantum mechanics.

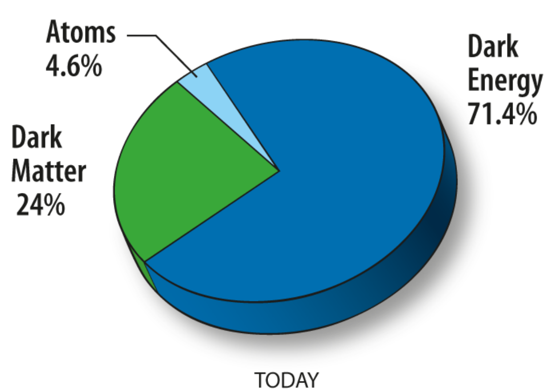

Figure 1: Pie Chart of the content of the Universe (*Credits:* NASA/WMAP Science Team)

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1 The Basic Idea

Since the formulation of Einstein's general relativity and quantum mechanics, both theories were (and are) successfully tested millions of times, which depicts how much important these theories are to modern day science. In the formalism and ideas, both are far apart. General relativity is classical in nature, while quantum mechanics upholds the idea of quantization and interference at the subatomic scale. Quantum (field) theory successfully accommodates all the interactions of nature in its framework but gravity. This framework is called the Standard Model of Particle Physics. However, general relativity is the only available tool to study all the manifestations of the gravitational field. Despite all the efforts, we are unable to wed general relativity to quantum theory. Anyhow some questions instantly pop out regarding their merger:

1. Why do we want to unify these two theories?
2. Is not it possible that gravity is the classical theory and has no quantum roots?
3. Is it really challenging to quantize gravity?

Skip the first two questions for a moment and come to the very last question. The answer is no. It is not difficult to quantize gravity. We know how to canonically quantize[1] fields. People have already done it for non-gravitational fields such as electromagnetic, weak, and strong nuclear. It is exquisite and simple machinery.

(i) Write down the classical Lagrangian density in terms of fields.

(ii) Calculate momentum and Hamiltonian densities as fields.

(iii) Treat these field densities as operators.

(iv) Impose commutation relations to make them quantum mechanical.

(v) Expand the fields in terms of ladder operators.

And that's it. The classical field has been quantized successfully.

The same trick can also be applied to the gravitational field. So, then? Would not this be the desired theory of quantum gravity? If it is straightforward, why physicists are struggling? To answer this, let us visit the first two questions in the next section.

1.1 Motivations for Quantum Gravity

Before proceeding, it would be essential to note that certain 'uniqueness theorems' say whatever theory of quantum gravity we develop must approximate to Einstein's GR (or Einstein-Cartan theory[2]) in some limit.

[1] Canonical quantization, introduced by Dirac [2], is one of the most direct and simplest ways to quantize classical theories.

[2] Einstein-Cartan theory is the natural generalization of GR. It is written in the language of tensors and spinors to accommodate fermions and quarks in gravity. (We might discuss it in detail at some later point.)

General relativity is a remarkably successful theory in its realm. Still, there are some rudimentary observational pieces of evidence that point out discrepancies in theoretical models and suggest a more fundamental viewpoint about gravity. Some of the primary motivations for quantum gravity are:

- **Unification:** Physicists are reductionists. The history of science shows that reductionism has been proven very successful. Standard Model is one of its prime examples where all the non-gravitational interactions can only be studied by only one gauge group, SU(3)×SU(2)×U(1). In the last century, we have seen the unification of electromagnetic and weak nuclear force (electroweak). Why would there be a reason gravity cannot be unified to others? It should be.
  Interestingly, there are also attempts to construct semiclassical frameworks in which all other forces are quantum while gravity remains essentially classical. However, all of these efforts have failed miserably. It indicates that classical and quantum concepts are likely incompatible, as inferred in the early days of quantum mechanics.

- **Universe:** All the attempts of gravity quantization face one severe problem – these are not UV complete. They break at high-energy scales. From the evolution history of universe[3], it is known that the present classical infrastructure of gravity tells us nothing about the initial conditions near the big bang or what happens at the sub–Planckian scale and final stages of black hole evolution. I want to discuss one example from cosmology and one from black holes in some detail.

  - **Cosmology:** There are cosmological solutions, by agreeing to the observations, which successfully predict an accelerated expansion of the universe[4]. From this result, it can be extrapolated that this expansion was started, at some zero cosmic time, from a highly dense point (or singularity) with no spatial dimensions. It is where classical GR fails to yield any feasible result(s). This inability to extract any information from this singularity and the subsequent Planck era signifies the need for the theory of quantized gravity.
  - **Black Holes:** Annoyingly mysterious black holes also qualify to be a testable region for quantum gravity. In the early seventies, Bekenstein interpreted the entropy of a black hole in terms of the area of event horizon [3] and Hawking conjectured that black holes emit thermal radiations having a black body spectrum [4]. It brings about an identicalness in the thermal behavior of black holes and the conventional laws of thermodynamics. Classical GR fails to explain it well because entropy is described under the notion of discrete states of a quantum system in traditional thermodynamics. It makes black hole entropy phenomenologically important in search of quantized gravity.

- **Time:** It is one of the most radical issues in the pursuit of QG. Quantum and generally covariant theories (as GR) have drastically different concepts of time. They are incompatible. In quantum theory, time is available as an external parameter. It is not described by an operator.

[3]https://mbilalazam.com/1-evolution-history-of-universe-a-story-from-zero-to-ten-seconds/

[4]We will discuss this problem in detail in the phenomenology of causal set theory.

It is kinematical. It is absolute and universal. Even in relativistic QFT, external Minkowski spacetime plays the role of absolute time. While in GR, time is a dynamical object. It is relative and non-absolute. It is not available as an external parameter. It needs to be constructed *naturally* by the theory itself. It is quite clear that the definition of time must need to be modified to develop a fully-fledged theory of quantum gravity. We will revisit the problem of time while discussing the role of background structures in quantum gravity.

- **Divergence:** Gravity is non-renormalizable. It breaks at high energy scales and yields infinities. It is still an open issue how to cure these divergences. There should be no divergences for any quantum version of gravity as there are none in quantum field theory.

2 Cut-Off Scales of Quantum Gravity

Length, mass, and time are fundamental, and the most basic physical quantities, and the speed of light (c), gravitational constant (G), and quantum of action ($\hbar$) are the fundamental constants of nature. Can we form these quantities from the constants of general relativity, c and G, or the constants of quantum theory, c and $\hbar$? No! There is no possible combination of these two sets of universal constants that can produce any of the fundamental quantities. But can we derive these quantities using all of these constants in any combination? Yes. We can. In honor of Max Planck, these units are called Planck units[5]. These units are:

$$\text{Planck length} = l_{\text{P}} = \sqrt{\frac{\hbar G}{c^3}} \approx 1.62 \times 10^{-33}\,\text{cm}, \tag{1}$$

$$\text{Planck time} = t_{\text{P}} = \frac{l_{\text{P}}}{c} = \sqrt{\frac{\hbar G}{c^5}} \approx 5.39 \times 10^{-44}\,\text{s}, \tag{2}$$

$$\text{Planck mass} = m_{\text{P}} = \frac{\hbar}{l_{\text{P}} c} = \sqrt{\frac{\hbar c}{G}} \approx 2.18 \times 10^{-5}\,\text{g}. \tag{3}$$

One may be bewildered by noticing that Planck mass is horrendously a larger quantity than Planck length and Planck time. But it is worth noting that m_{P} must be concentrated in a linear dimension l_{P}; otherwise, we will not be able to observe quantum effects of gravity. At this scale, classical properties become an emergent phenomenon of quantum theory. These scales provide the cut-offs to observe quantum gravity. We can easily realize that these scales are nearly impossible to detect and observe with the present technology. However, these scales do not provide stringent limits on gravity. These are just the most natural possible cut-offs. The unified theory of forces may also contain other parameters; for instance, in string theory, fundamental length scale is the string length, l_{S}, rather than Planck length, l_{P}. Nevertheless, it is an educated guess to set l_{S} equals to l_{P}.

[5] For the sake of completeness - Planck units are defined exclusively in terms of four universal constants. Fourth one is the Boltzmann constant, k_{B}. Using the four constants, one can also define Planck units of temperature, density, the fine structure constant of gravity and charge.

3 Approaches Quantum Gravity

Since 1930s, there is no shortage of proposed theories for quantization of gravitational fields[6]. No idea is complete; no theory is entirely consistent with quantum fields. No theory provides a way to put it to experimental tests; every approach suffers from conceptual problems. Every theory meets observational discrepancies. Issues are there, but even today, the charm of QG is not faded away. Nothing is more fascinating than the problems of gravity. It is challenging to review all the approaches of QG in just one section; however, I will try to briefly discuss its main research directions in QG. The subsequent sections may contain some *fancy* technical terms which I will, intentionally, try to oversimplify, and it may cause the text to lose its essence (so the advanced readers are advised to see [7, 8] and references therein, upon which this section is based).

3.1 Three-and-Half Directions

Research in QG can be mapped onto three main and some *neglected* lines:

- covariant,
- canonical,
- sum-over-histories,
- others.

This list does not qualify to be a precise way to describe research directions since these names are somewhat misleading and sometimes are used interchangeably. Still, they possess specific scientific rationality, developmental logic, and methodological unity. Often, one approach becomes a hot topic, and others get neglected, and this is repeatedly happening over the years. However, all of these approaches share some common features, and many things cannot fit into any of these directions.

- **Covariant Approach:** Started in the early thirties by Rosenfeld, Fierz, and Pauli, it is one of the most dominant research directions in the pursuit of QG. It is an attempt to construct a quantum field theory of gravity. In covariant approaches, fluctuations of the metric element are studied over a four-dimensional Lorentzian flat spacetime or some other appropriate background metric. Rules and equations were established in the sixties by Feynman, Faddeev, and others, while the approach was proved to be non-renormalizable by 't Hooft and others in the seventies. The search for renormalizability gave birth to string theory in the late eighties, whose sun is not set to date.

- **Canonical Approach:** Initiated by Dirac and pioneered by Wheeler, DeWitt, and Ashtekar, it is the Hamiltonian (or canonical) formulation of GR. In this approach, phase space variables are promoted to quantum operators on usual Hilbert space, and these operators correspond to the full or some functions of the metric. The canonical approach started gaining attention when the framework of loop quantum gravity (LQG) was established. LQG is also one of the most significant competitors of string theory.

[6] A list of candidate theories can be found on the following Wikipedia page: https://en.wikipedia.org/wiki/Quantum_gravity#Candidate_theories. However, this is not an exhaustive list.

- **Sum-Over-Histories Approach:** Feynman pioneered path integral or SOH formulation, and it proved itself as one of the most successful formalisms of quantum theory. In quantum world, there are infinite ways (paths) to go from some initial point to final point, and the probabilities are given by adding contributions from all the possible trajectories (or histories). Similarly, in gravity, this sum is taken over all the possible geometries of spacetime. Some discrete causal approaches to QG are based on different versions of SOH, for instance, spinfoam formalism or causal set approach.

- **Other Approaches:** These include effective field theories (such as noncommutative geometry), Penrose's twistor theory, Regge calculus, causal dynamical triangulation, causal sets, and many more. Some of the alternate approaches have suffered from the serious and fundamental crisis. Some of these are too mathematical to describe physical reality. Some of these ideas also provide an astonishing insight into the phenomenology of quantum gravity and bridge the gap between theoretical and observational discrepancies.

It is also important to note that only string theory claims to be the unified version of all fundamental forces or, loosely speaking, the theory of everything, while all other approaches provide different ways to merge quantum theory with general relativity. None of these approaches have been delved into fully-fledged quantum versions of gravity. All ideas are still in development. Some of the proposed versions of gravitational theories have an established kinematical side, but they lack a dynamical framework. For avid history readers, the evolution of the research in quantum gravity is summed up in figure 2, taken from [8].

Lastly, we have briefly reviewed the idea of quantum gravity, its need, present problems, and some of the possible directions. Readers may visit the reference (and references therein) for a detailed outlook of the topic. This series is dedicated to *causal set theory*, initiated by Rafael Sorkin in the early eighties, and from the next post, we will try to discuss it from the very basics. We will discuss the need of discrete causal approaches and causal metric hypothesis, which will lead us to one of the most important theoretical breakthroughs of the last century: *metric recovery theorems*.

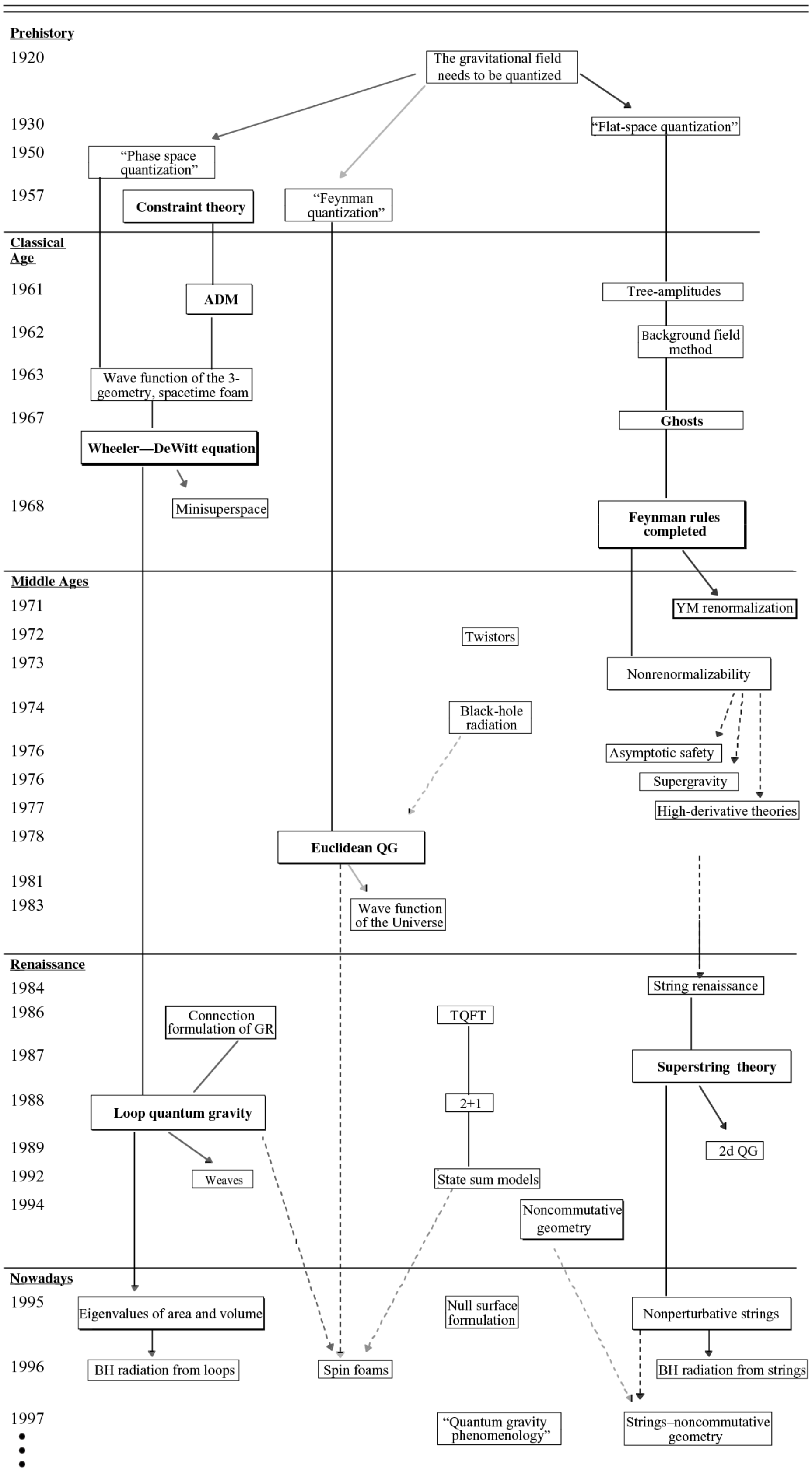

Figure 2: The search for a quantum theory of the gravitational field

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