CSE 211 Homework #1

Muhammed Bilal Türk 210104004047

1.) a.

p	q	r	$q \land r$	$p \lor (q \land r)$	$\neg(p \lor (q \land r))$
1	1	1	1	1	0
1	1	0	0	1	0
1	0	1	0	1	0
1	0	0	0	1	0
0	1	1	1	1	0
0	1	0	0	0	1
0	0	1	0	0	1
0	0	0	0	0	1

p	q	r	$\neg p$	$\neg q$	$\neg r$	$\neg q \lor \neg r$	$(\neg p) \land (\neg q \lor \neg r)$
1	1	1	0	0	0	0	0
1	1	0	0	0	1	1	0
1	0	1	0	1	0	1	0
1	0	0	0	1	1	1	0
0	1	1	1	0	0	0	0
0	1	0	1	0	1	1	1
0	0	1	1	1	0	1	1
0	0	0	1	1	1	1	1

$$\neg(p \lor (q \land r)) \equiv (\neg p) \land (\neg q \lor \neg r)$$

Both tables have same results. Therefore, this expression is proved.

b.

p	q	r	$q \lor r$	$p \land (q \lor r)$	$\neg(p \land (q \lor r))$
1	1	1	1	1	0
1	1	0	1	1	0
1	0	1	1	1	0
1	0	0	0	0	1
0	1	1	1	0	1
0	1	0	1	0	1
0	0	1	1	0	1
0	0	0	0	0	1

p	q	r	$\neg p$	$\neg q$	$\neg r$	$\neg q \lor \neg r$	$(\neg p) \lor (\neg q \lor \neg r)$
1	1	1	0	0	0	0	0
1	1	0	0	0	1	1	1
1	0	1	0	1	0	1	1
1	0	0	0	1	1	1	1
0	1	1	1	0	0	0	1
0	1	0	1	0	1	1	1
0	0	1	1	1	0	1	1
0	0	0	1	1	1	1	1

$$\neg(p \land (q \lor r)) \not\equiv (\neg p) \lor (\neg q \lor \neg r)$$

The results of tables are not same. Therefore, this expression is disproved.

CamScanner ile tarandı

2.) To prove a 'AND' statement is true, both side of the statement must be true. Therefore, both $(p \vee q \vee r)$ and $(\neg p \vee \neg q \vee \neg r)$ statements must be true. For left side of the 'AND' statement, at least one true is required to prove 'OR' statement is true. It is given that at least one of p, q and r is true so that left side of the 'AND' statement must be true. For right side of the 'AND' statement, at least one true is required. For the inverse of a condition is true, one false condition is required. It is given that at least one of p, q and r is false. Therefore, $(p \vee q \vee r) \wedge (\neg p \vee \neg q \vee \neg r)$ statement must be true when at least one of p, q and r is true and one of p, q and r is false.

When these three variables have the same truth value, one of the side of 'AND' statement must be false. Therefore, $(p \vee q \vee r) \wedge (\neg p \vee \neg q \vee \neg r)$ statement can't be true when p, q and r is same truth value.

3.) Let $g(x) = c$ and $h(x) = k$. When $c = 0$ and $k = 0$,

$f(x)$ becomes; $f(x) = 0^2 + 0^2$. When this is simplified $f(x)$ becomes, $f(x) = 0$ which is not positive polynomial function. Therefore, it is disproved that $f(x)$ is always positive if and only if there exist $g(x)$ and $h(x)$ such that $f(x) = g(x)^2 + h(x)^2$.

4.) a.) $\bigcup_{n=1}^{\infty} A_n = (0,1) \Rightarrow A_1 \cup A_2 \cup A_3 \cup A_4 \cdots \cup A_k \quad , k = \infty$

$(0,1) \cup (0,1/2) \cup (0,1/3) \cup (0,1/4) \cup \ldots\ldots \cup (0,1/k)$

$\lim_{k \to \infty} \frac{1}{k} = 0$, so $1 > 1/2 > 1/3 > \ldots\ldots > \lim_{k \to \infty} \frac{1}{k}$

1 includes every number smaller than itself. Therefore, $\bigcup_{n=1}^{\infty} A_n = (0,1)$ is proved.

b.) $\bigcap_{n=1}^{\infty} A_n = \phi \Rightarrow A_1 \cap A_2 \cap A_3 \cap \ldots\ldots \cap A_k \quad , k = \infty$

$(0,1) \cap (0,1/2) \cap (0,1/3) \cap \ldots\ldots \cap (0,1/k)$

$\lim_{k \to \infty} \frac{1}{k} = 0$ so $1 > 1/2 > 1/3 > \ldots\ldots > 1/k$

k is infinite number so that every number has a number smaller than itself. Therefore, $\bigcap_{n=1}^{\infty} A_n = \phi$ is proved.