

CSE211 Discrete Mathematics

Homework I

(Due November 12, 2023 23:59)

1. Use a truth table to prove or disprove the following statements:

(a) $\neg(p \vee (q \wedge r)) = (\neg p) \wedge (\neg q \vee \neg r)$

(b) $\neg(p \wedge (q \vee r)) = (\neg p) \vee (\neg q \vee \neg r)$

2. Explain, without using a truth table, why $(p \vee q \vee r) \wedge (\neg p \vee \neg q \vee \neg r)$ is true when at least one of p, q, and r is true and at least one is false, but is false when all three variables have the same truth value.
3. Let $f(x), g(x), h(x)$ be polynomials with real coefficients. Prove or disprove the following: $f(x)$ is always positive if and only if there exists $g(x)$ and $h(x)$ such that $f(x) = g(x)^2 + h(x)^2$.
4. Consider the following collection of sets indexed by \mathbb{N} :

$$A_1 = (0, 1), \quad A_2 = \left(0, \frac{1}{2}\right), \dots, A_n = \left(0, \frac{1}{n}\right), \dots$$

Prove:

(a) $\bigcup_{n=1}^{\infty} A_n = (0, 1) .$

(b) $\bigcap_{n=1}^{\infty} A_n = \emptyset .$

Please upload a scan of your handwritten answers on the assignments of the course Teams page.

Tülay Ayyıldız