CSE 211 Homework #2

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1.) a) $f(x_1) = f(x_2)$, assumed

$x_1 - 1 = x_2 - 1$

$\boxed{x_1 = x_2}$

For all integer numbers, there is only one image. Therefore, this function is one-to-one.

b) $f(x_1) = f(x_2)$, assumed

$x_1^3 = x_2^3$

$\boxed{x_1 = x_2}$

For all integers, there is only one image, Therefore, this function is one-to-one.

c) $f(x_1) = f(x_2)$, assumed

$x_1^2 + 1 = x_2^2 + 1$

$\boxed{x_1 = -x_2}$, $\boxed{x_1 = x_2}$

Two different integers have a same image. Therefore, this is not one-to-one.

d) $f(x_1) = f(x_2)$

$\lceil x_1/2 \rceil = \lceil x_2/2 \rceil$ lets $x_1 = 3$, $x_2 = 4$

$\lceil 1.5 \rceil = \lceil 2 \rceil$

$\boxed{2 = 2}$

Different integers in domain have same value in a image. Therefore, this function is not one-to-one.

2.) a) $y = n-1 \Rightarrow y+1 = n \Rightarrow f(n) = n+1$

$f(n) = n+1 \quad \forall n \in \mathbb{Z}, \forall f(n) \in \mathbb{Z}.$

Therefore, this is onto

b) $y = n^3 \Rightarrow n = y^3 \Rightarrow f(n) = \sqrt[3]{n}$

$f(n) = \sqrt[3]{n} \quad \forall n \in \mathbb{Z}, \forall f(n) \notin \mathbb{Z}$

Therefore, this is not onto.

c) $y = n^2+1 \Rightarrow y-1 = n^2 \Rightarrow n = \sqrt{y-1} \Rightarrow f(n) = \sqrt{n-1}$

$f(n) = \sqrt{n-1} \quad \forall n \in \mathbb{Z}, \forall f(n) \notin \mathbb{Z}.$

Therefore, this is not onto.

d) Let $f(n) = \lceil n/2 \rceil = a$, $a \in \mathbb{Z}$

if a is even, $n = 2a$. Then

$f(n) = \lceil \frac{2a}{2} \rceil = a$. $\forall a \mid a = 2k \in \mathbb{Z}$

$f(n) = a$.

if a is odd, $n = 2a+1$. Then

$f(n) = \lceil \frac{2a+1}{2} \rceil = a+1$. $\forall a \mid a = 2k+1 \in \mathbb{Z}$

$f(n) = a+1$

3.) a) $f(m,n) = 2m - n = k \quad k \in \mathbb{Z}$

$2m = n + k$ so $n+k$ is even number.

let $n = 0$, k is even. $2m = k \Rightarrow m = \frac{k}{2}$. $\forall k;\ k = 2t \in \mathbb{Z},\ m \in \mathbb{Z},\ n \in \mathbb{Z}$.

let $n = 1$, k is odd. $2m = k+1 \rightarrow m = \frac{k+1}{2}$ $\forall k;\ k = 2t+1 \in \mathbb{Z},\ m \in \mathbb{Z},\ n \in \mathbb{Z}$.

b.) $f(m,n) = m^2 - n^2 = k \quad k \in \mathbb{Z},\ m \in \mathbb{Z},\ n \in \mathbb{Z}$.

$(m-n).(m+n) = k$. lets $k = 1$.

$(m-n).(m+n) = 1 \Rightarrow (m-n) \stackrel{?}{=} 1,\ (m+n) \stackrel{?}{=} 1$

$\boxed{m = n}$ $\boxed{m = -n}$

for $\forall k \in \mathbb{Z}$, there doesn't exist $m \in \mathbb{Z}$, $n \in \mathbb{Z}$ such that

$f(m,n) = m^2 - n^2 = k$

c.) $f(m,n) = m + n + 1 = k \quad k \in \mathbb{Z}$

$m + n = k - 1$.

for $\forall k \in \mathbb{Z}$. there exist $m \in \mathbb{Z}$, $n \in \mathbb{Z}$ such that $f(m,n) = k$.

d.) $f(m,n) = |m| - |n| = k \quad k \in \mathbb{Z}$

if $k \geq 0$, let $n = 0$, $m = k$. $\forall k \in \mathbb{Z}^+$, $m \in \mathbb{Z}^+$, $n \in \mathbb{Z}$

if $k < 0$, let $n = -k$, $m = 0$. $\forall k \in \mathbb{Z}^-$, $m \in \mathbb{Z}$, $n \in \mathbb{Z}^+$

for $\forall k \in \mathbb{Z}$, there exist m and n such that

$f(m,n) = k$

e.) $f(m,n) = m^2 - 4 = k \quad k \in \mathbb{Z}$

$m^2 = k+4 \Rightarrow m = -\sqrt{k+4}\ ,\ m = \sqrt{k+4}$

m may not be any integer such that $f(m,n) = k$

This is not onto.

4.) a) $f(m,n) = m+n = k \quad k \in \mathbb{Z}$

Let $m = k$, $n = 0$. $f(m,n) = k + 0 = k$

$\forall k \in \mathbb{Z}$, there exist $m \in \mathbb{Z}$, $n \in \mathbb{Z}$ such that $f(m,n) = k$.

b) $f(m,n) = m^2 + n^2 = k \quad k \in \mathbb{Z}$

Let $k = 6$. There doesn't exist any $m \in \mathbb{Z}$, $n \in \mathbb{Z}$ such that $f(m,n) = k$

c) ~~Answer is in the following page.~~

d) $f(m,n) = |n| = k \quad k \in \mathbb{Z}$

if $k \geq 0$, $m = 0$, $n = k$. Then $f(m,n) = |k| = k$

if $k < 0$, $m = 0$, $n = -k$. Then $f(m,n) = |-k| = k$

for $\forall k \in \mathbb{Z}$, There exist $m \in \mathbb{Z}$, $n \in \mathbb{Z}$ such that $f(m,n) = k$.

e) $f(m,n) = m - n = k \quad k \in \mathbb{Z}$

if $m = k$, $n = 0$. Then $f(m,n) = k - 0 = k$.

for $\forall k \in \mathbb{Z}$, there exist $m \in \mathbb{Z}$, $n \in \mathbb{Z}$ such that $f(m,n) = k$.

c) $f(m,n) = m = k \quad k \in \mathbb{Z}$

Let $m = k$, $n = 0$. Then $f(m,n) = k = k$.

For $\forall k \in \mathbb{Z}$, there exist $m \in \mathbb{Z}, n \in \mathbb{Z}$ such that $f(m,n) = k$.

6.) $n = 1 \qquad f_2 . f_0 - f_1^2 = (-1)^n \qquad f_0 = 0,\ f_1 = 1,\ f_2 = 1$

$$0 - 1^2 = (-1)^1$$

$$-1 = -1$$

assume $n = k \quad f_{k+1} . f_{k-1} - f_k^2 = (-1)^k$, prove $n = k+1$

$$f_{k+2}\, f_k - f_{k+1}^2 = (-1)^{k+1}$$

$$(f_k + f_{k+1}) . f_k - f^2_{k+1} = (-1)^{k+1}$$

$$f_k^2 + f_1 \; f_{1+1} - f^2_{k+1} = (-1)^{k+1}$$

$* \; f_k^2 = f_{k+1} . f_{k-1} - (-1)^k \Rightarrow f_k^2 = f_{k+1} . f_{k-1} + (-1)^{k+1}$

$$f_{k+1} . f_{k-1} + (-1)^{k+1} + f_k . f_{k+1} - f^2_{k+1} = (-1)^{k+1}$$

$$f_{k+1} (f_{k-1} + f_k - f_{k+1}) + (-1)^{k+1} = (-1)^{k+1}$$

$$f_{k+1} (f_{k+1} - f_{k+1}) + (-1)^{k+1} = (-1)^{k+1}$$

$$f_{k+1} (0) + (-1)^{k+1} = (-1)^{k+1}$$

$$(-1)^{k+1} = (-1)^{k+1} \checkmark$$

6.) $g: A \to B$, $f: B \to C$

a) $x, y \in A$, if $x = y$ then $g(x) = g(y)$ $g(x), g(y) \in B$

$g(x), g(y) \in B$, if $g(x) = g(y)$ then $f(g(x)) = f(g(y))$ $f(g(x)), f(g(y))$

$x = y$ so that $f(g(x)) = f(g(y))$. Therefore, $f \circ g$ is one-to-one

b) $a \in A$, $b \in B$, for $\forall b \in B$ $g(a) = b$

$b \in B$, $c \in C$, for $\forall c \in C$ $f(b) = c \Rightarrow f(g(a)) = c$

$\forall b \in B$ $g(a) = b$, then $\forall c \in C$ $f(g(a)) = c$, Therefore, $f \circ g$ is onto.

$\sum_{n=1}^{n=3} x$ $x(1) + x_2 + x_3$

$1 + 2 + 3 =$

$\sum_{n=1}^{n=2} x + \sum_{n=2}^{n=3} x$

7.)

a) Basis: $n=0$ $\sum_{k=0}^{n=0} ar^k \stackrel{?}{=} \frac{ar^{n+1} - a}{r-1}$

$$ar^0 = \frac{ar - a}{r-1} \Rightarrow a = \frac{a(r-1)}{r-1} \Rightarrow a = a \checkmark$$

Inductive: assume $n = t$ $\sum_{k=0}^{n=t} a.r^k = \frac{ar^{t+1} - a}{r-1}$

$n = t+1$

$$\sum_{k=0}^{n=t+1} a.r^k \stackrel{?}{=} \frac{ar^{t+2} - a}{r-1} \Rightarrow \sum_{k=0}^{n=t} ar^k + ar^{t+1} = \frac{ar^{t+2} - a}{r-1}$$

$$\frac{ar^{t+1} - a}{r-1} + \frac{ar^{t+1}}{(r-1)} = \frac{ar^{t+2} - a}{r-1} \Rightarrow \frac{ar^{t+1} - a}{r-1} + \frac{ar^{t+2} - ar^{t+1}}{r-1} = \frac{ar^{t+2} - a}{r-1}$$

$$\frac{ar^{t+2} - a}{r-1} = \frac{ar^{t+2} - a}{r-1} \checkmark$$

b.) $\sum_{k=32}^{80} 2r^k \Rightarrow \sum_{k=0}^{80} 2r^k - \sum_{k=0}^{31} 2r^k \Rightarrow \frac{2r^{81}-2}{r-1} - \frac{2r^{32}-2}{r-1}$

$\sum_{k=0}^{n} ar^k \,(r\neq 0)$ is $\frac{ar^{n+1}-a}{r-1}, r\neq 1$

$\Rightarrow \frac{2(r^{81}-r^{32})}{r-1}$

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