MATH 211/502 : Discrete Mathematics **(Due Date: 23.11.2023, 23:59)**

Homework #2

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Question 1.
Determine whether each of these functions from $\mathbb{Z}$ to $\mathbb{Z}$ is one-to-one.
a) $f(n) = n - 1$
b) $f(n) = n^3$
c) $f(n) = n^2 + 1$
d) $f(n) = \lceil n/2 \rceil$

Question 2.
Which functions in Question 1 are onto?

Question 3.
Determine whether $f : \mathbb{Z} \times \mathbb{Z} \to \mathbb{Z}$ is onto if **a)** $f(m, n) = 2m - n$
b) $f(m, n) = m^2 - n^2$
c) $f(m, n) = m + n + 1$
d) $f(m, n) = |m| - |n|$
e) $f(m, n) = m^2 - 4$

Question 4.
Determine whether $f : \mathbb{Z} \times \mathbb{Z} \to \mathbb{Z}$ is onto if **a)** $f(m, n) = m + n$
b) $f(m, n) = m^2 + n^2$
c) $f(m, n) = m$
d) $f(m, n) = |n|$
e) $f(m, n) = m - n$

Question 5.
Let f_n be the n^{th} fibonacci number. Show that $f_{n+1}f_{n-1} - f_n^2 = (-1)^n$ when n is a positive integer.

Question 6.
Suppose that g is a function from A to B and f is a function from B to C.
a) Show that if both f and g are one-to-one functions, then $f \circ g$ is also one-to-one.
b) Show that if both f and g are onto functions, then $f \circ g$ is also onto.

Question 7.
a) Prove that the close form of the sum $\sum_{k=0}^{n} ar^k (r \neq 0)$ is

$$\frac{ar^{n+1} - a}{r - 1}, r \neq 1.$$

b) Calculate

$$\sum_{k=32}^{80} 2r^k.$$

Please upload a scan of your handwritten answers on the assignments of the course Teams page.