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Uncertainty on Asynchronous Time Event Prediction

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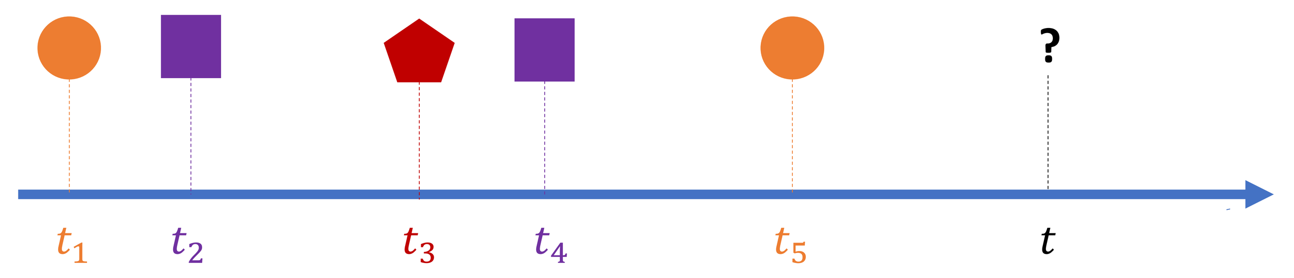
tl;dr

- **Complex evolution of prediction** for the next event
- Real time **uncertainty estimation** in asynchronous event prediction

Asynchronous events

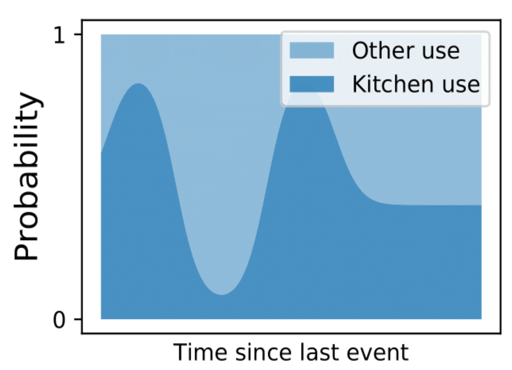
- Discrete events occurring irregularly over time
- At a given time, the most probable event can change



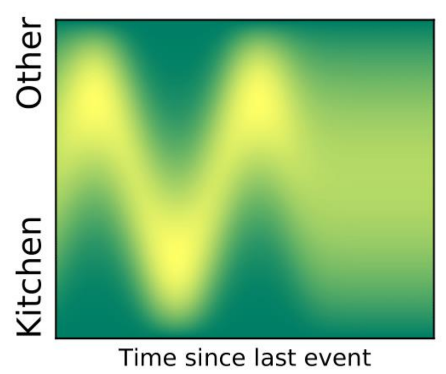
Research questions

(1) Prediction: At a given time, what event is likely to occur?

(2) Uncertainty estimation: How sure are we in our prediction?



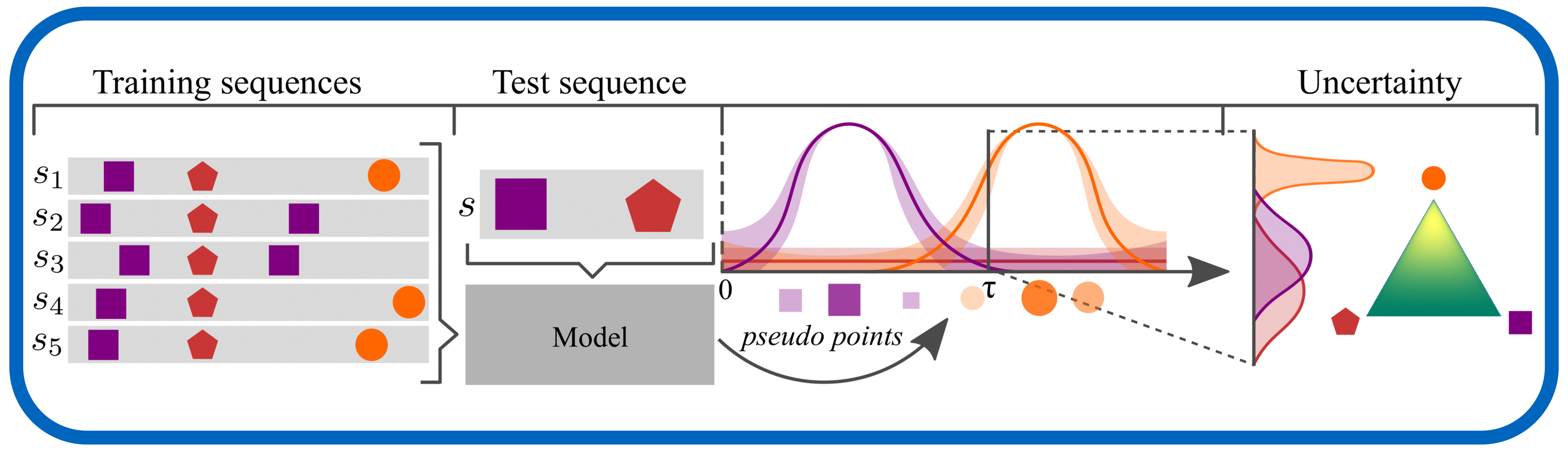
(1) Prediction



(2) Uncertainty estimation

Model overview

1. For each class c, generate M pseudo points $(w_j^{(c)}, \tau_j^{(c)}, y_j^{(c)})$ based on a hidden state of an RNN whose input is a sequence.
2. Fit a complex evolution of the predicted categorical distribution $\boldsymbol{p}(\tau) \sim P(\theta(\tau))$. The distribution over the simplex $P(\theta(\tau))$ gives certainty localized around pseudo points.



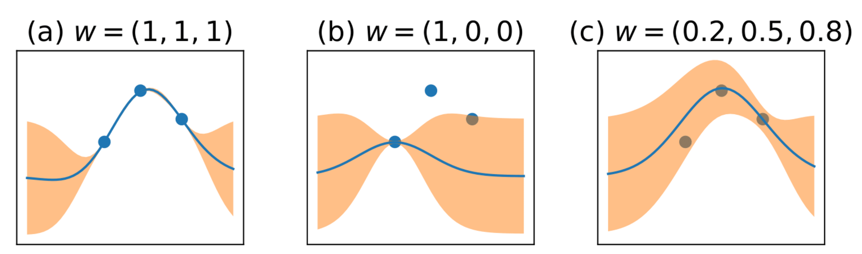
*Equal contribution

Model 1: Logistic Normal via Weighted Gaussian Process

- The pseudo points $(\tau_j^{(c)}, y_j^{(c)})$ weighted with $w_j^{(c)}$ are used to generate a Weighted Gaussian Process in the logit-space. The weigths $w_j^{(c)}$ allow to discard unnecessary training points in Gaussian Process.

(Kernel)

$$k'(\tau_1, \tau_2) = min(w_1, w_2) k(\tau_1, \tau_2)$$



- The distribution $\boldsymbol{p}_c(\tau) \sim logistic\ N(\tau_c(\tau), \sigma_c^2(\tau))$ follows a Logistic Normal distribution.

Model 2: Dirichlet via Function Decomposition

- The pseudo points $(w_j^{(c)}, \tau_j^{(c)}, \sigma_j^{(c)})$ are used to describe the evolution of the Dirichlet parameters over time via basis function decomposition.

$$\log \alpha_c(\tau) = \underbrace{\sum_{j=1}^{M} w_j^{(c)} \cdot N(\tau | \tau_j^{(c)}, \sigma_j^{(c)})}_{\text{Basis function decomposition}} + \vartheta \leftarrow \text{Prior}$$

Effective number of observations of class c at time τ

- The distribution $\boldsymbol{p}\ (\tau) \sim Dir(\alpha_c(\tau))$ follows a Dirichlet dsitribution.

Uncertainty Cross Entropy

- The classic Cross Entropy loss looks only at the mean of the distribution $P(\theta(\tau))$:

$$\mathcal{L}_i^{UCE} = \mathrm{H}[\boldsymbol{p}_i^*, \bar{\boldsymbol{p}}_i]$$

where $\bar{\boldsymbol{p}}_i(\tau) = \mathbb{E}_{\boldsymbol{p}_i \sim P_i(\theta(\tau_i))}[\boldsymbol{p}_i]$

Not suited for uncertainty estimation

- The Uncertainty Cross entropy loss looks at the distribution $P(\theta(\tau))$:

$$\mathcal{L}_i^{UCE} = \mathbb{E}_{\boldsymbol{p}_i \sim P_i(\theta(\tau_i^*))}[\mathrm{H}[\boldsymbol{p}_i^*, \boldsymbol{p}_i]]$$

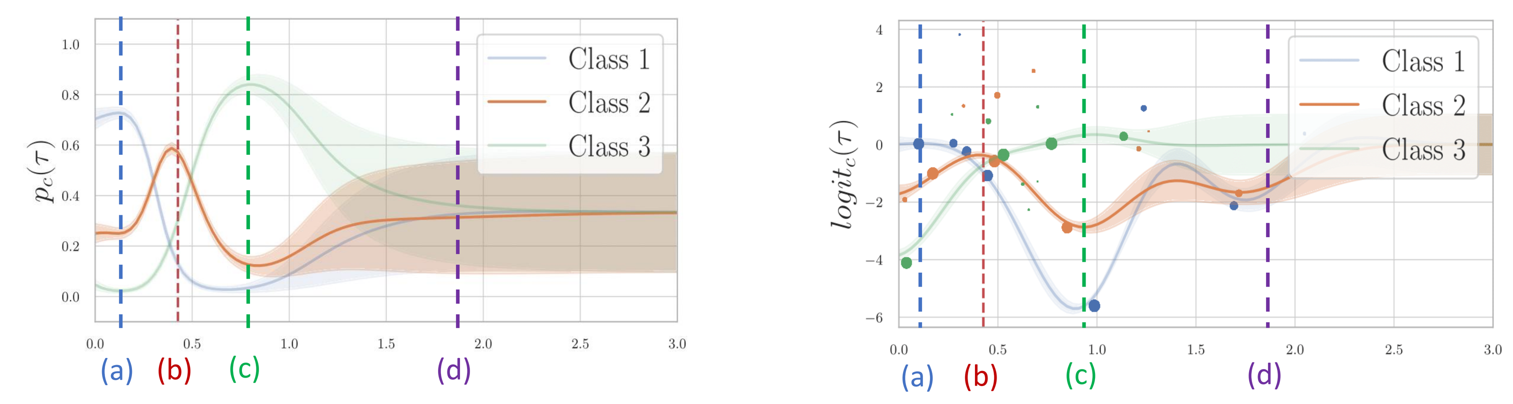
Take care of uncertainty estimation

- Regularizer adjust uncertainty estimation of out-of-distribution data:

$$r_c = \alpha \underbrace{\int_0^T (\mu_c)^2 d\tau}_{\text{Pushes mean to 0}} + \beta \underbrace{\int_0^T (\vartheta - \sigma_c^2) d\tau}_{\text{Pushes variance to } \vartheta}$$

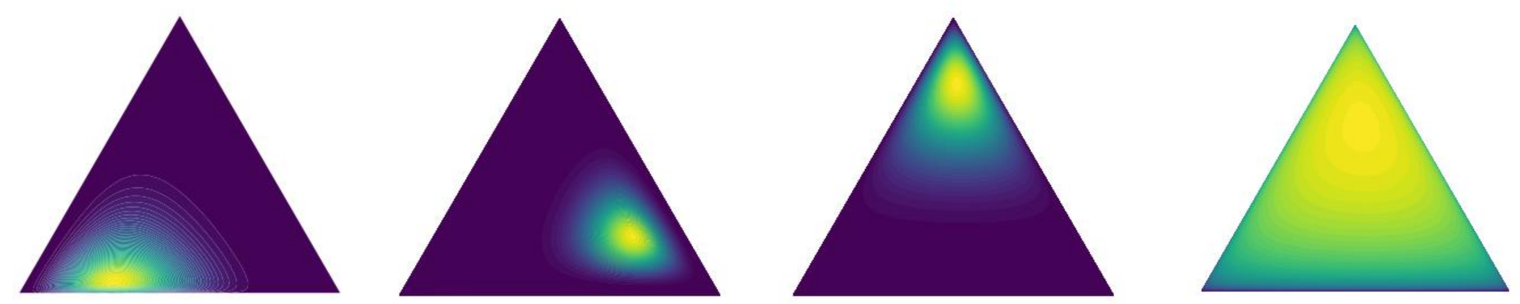
Computed by sampling

Experiment: Visualization

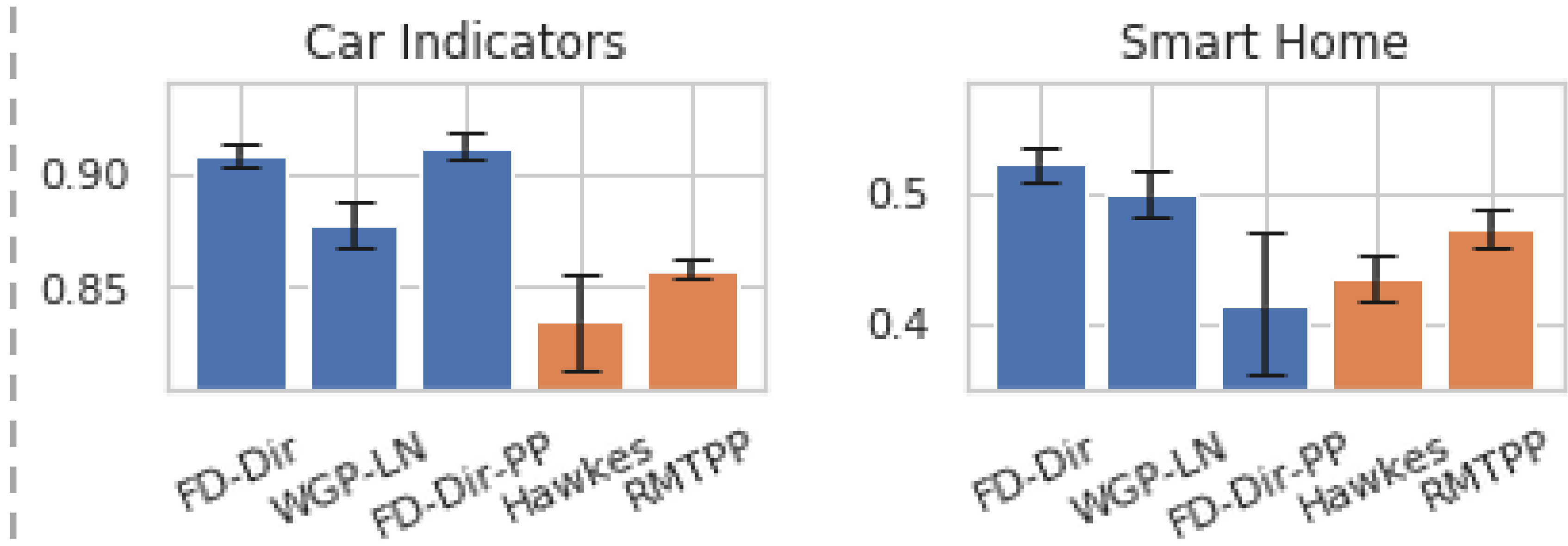


FD-Dir

LN-WGP



Experiment: Prediction



Experiment: Uncertainty