

Bipedal Locomotion Planning Based on Divergent Component of Motion(DCM)

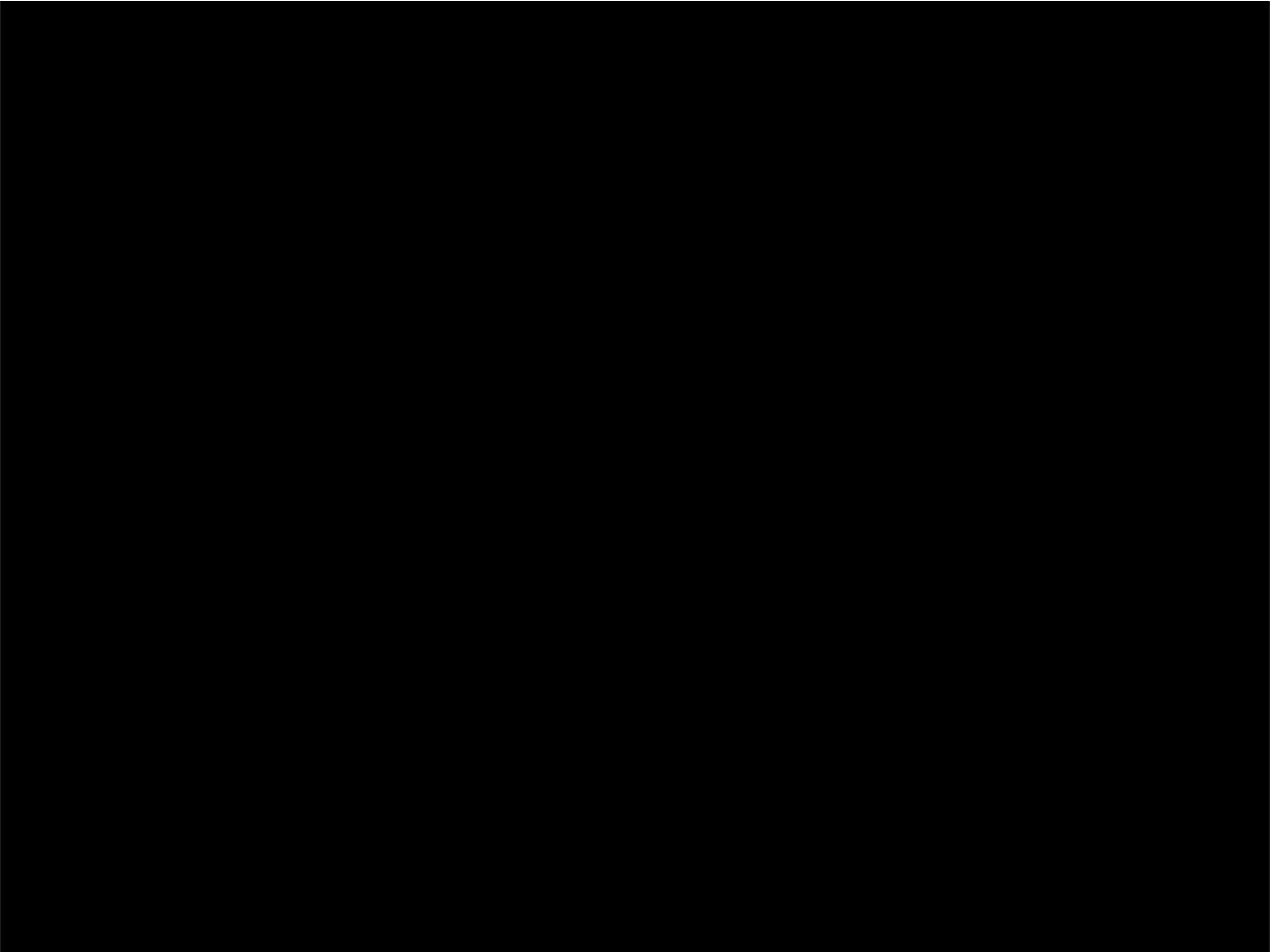
Legged Robots Course

Professor: Auke Ijspeert

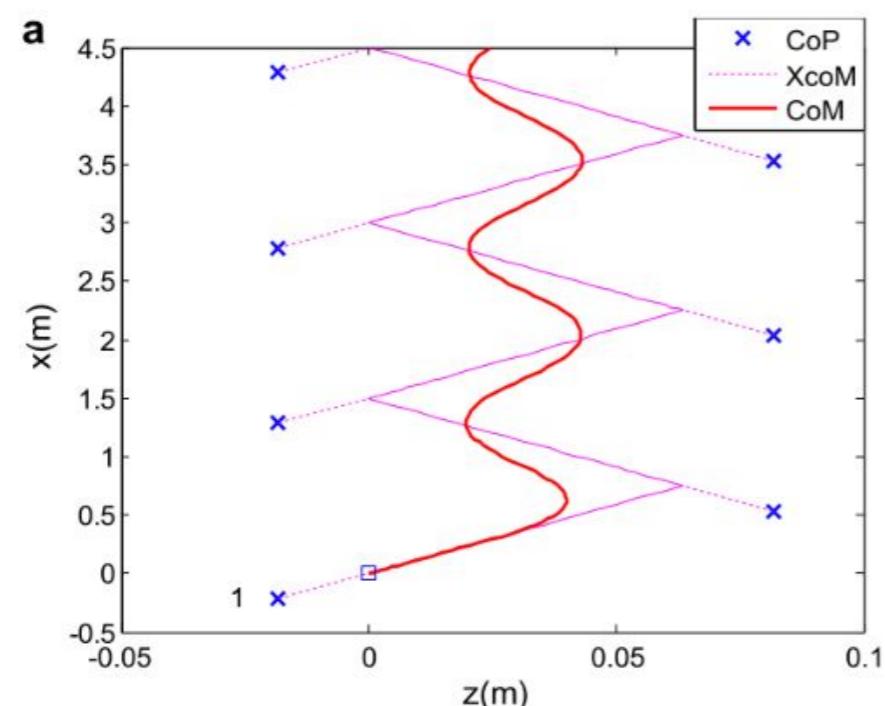
Presented by: Milad Shafiee

EPFL

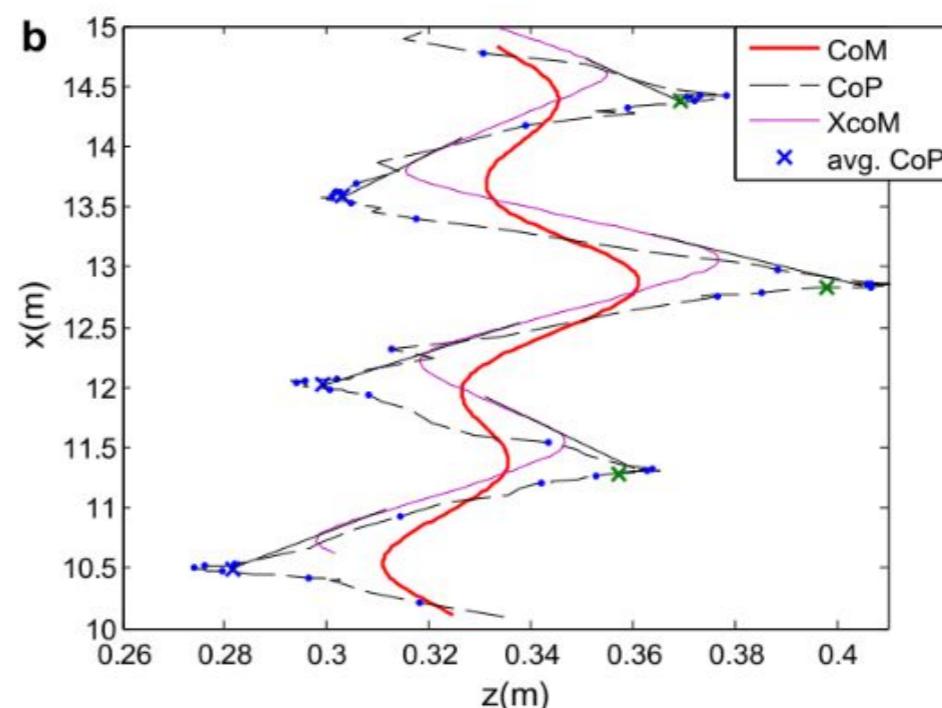
Asimo runs at 9 Km/h by using the DCM concept for locomotion planning!



Human locomotion data is well-fitted with the Extrapolated Center of Mass(XCoM) Concept



□ XCoM(DCM) based motion planning

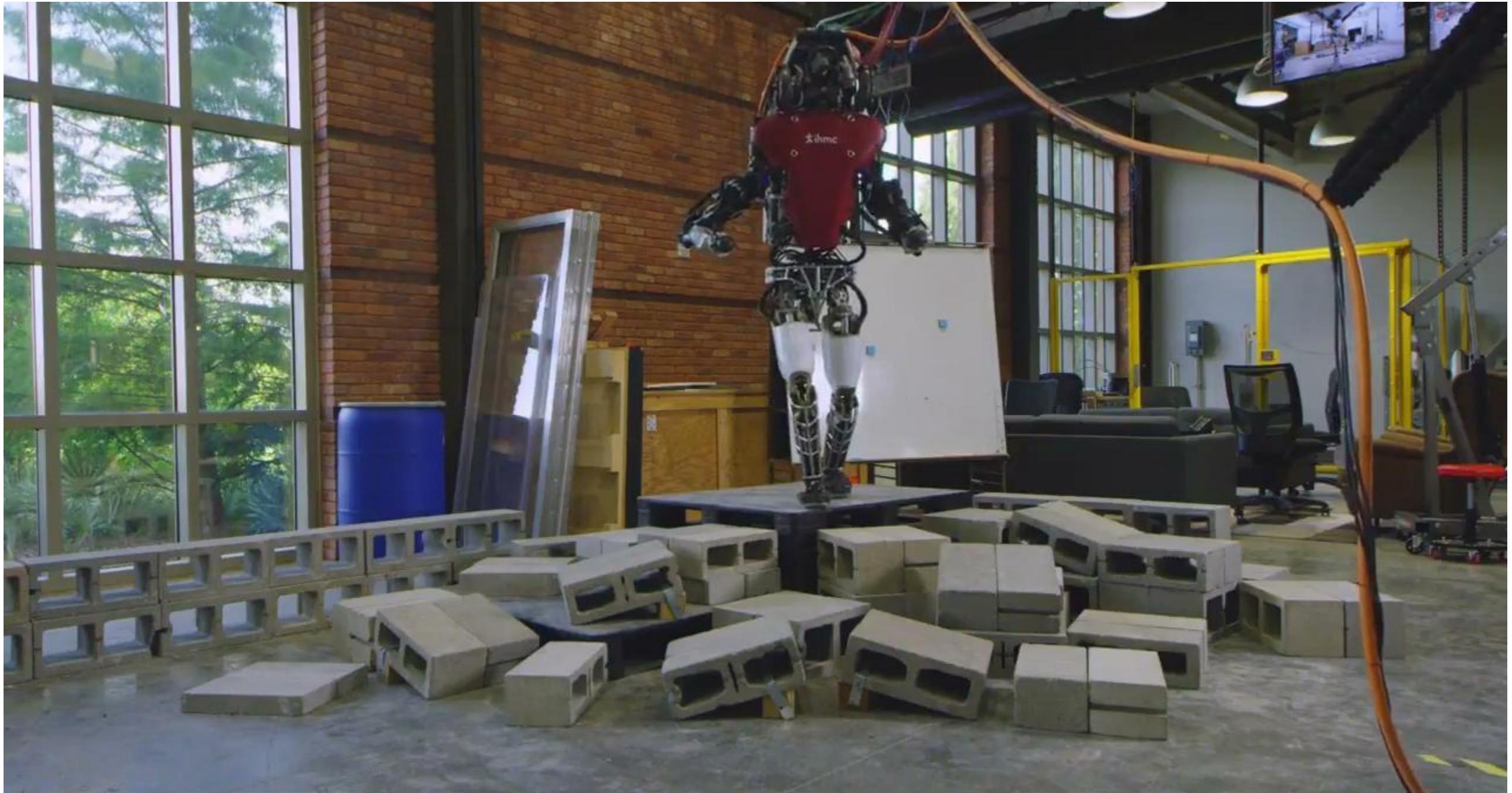


□ Human experimental data of locomotion

Hof, A. L. "The 'extrapolated center of mass' concept suggests a simple control of balance in walking." *Human movement science* 27.1 (2008): 112-125. (Citation: 456)

Hof, A. L., M. G. J. Gazendam, and W. E. Sinke. "The condition for dynamic stability." *Journal of biomechanics* 38.1 (2005): 1-8. (Citation: 1258)

IHMC came in 2nd place in Darpa Robotics Challenge(DRC) by having the DCM planner that you will implement in the project!



Linear Inverted Pendulum (LIP) model (2D)

- Solving the equation of motion:

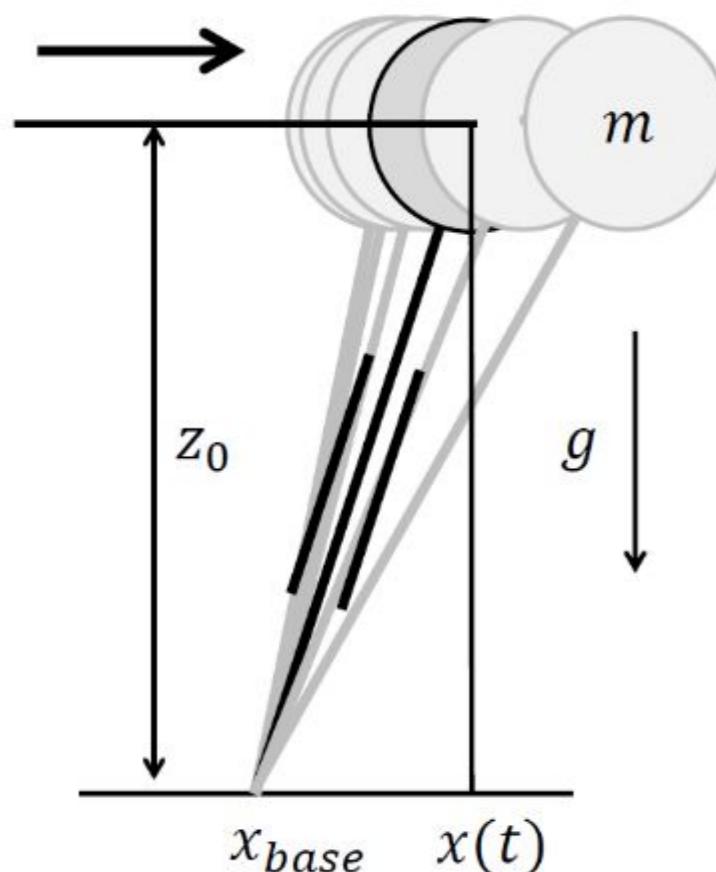
$$\ddot{x}(t) = \frac{g}{z_0} (x(t) - x_{base}), \omega = \sqrt{g/z_0}$$

$$x(t) = Ae^{-\omega t} + Be^{\omega t} + x_{base}$$

$$A = (-\dot{x}(0)/\omega + x(0) - x_{base})/2$$

$$B = (\dot{x}(0)/\omega + x(0) - x_{base})/2$$

- We thus have a closed form (i.e. analytical solution) that allows us to predict the forward movement of the center of mass



Divergent component of motion (DCM)

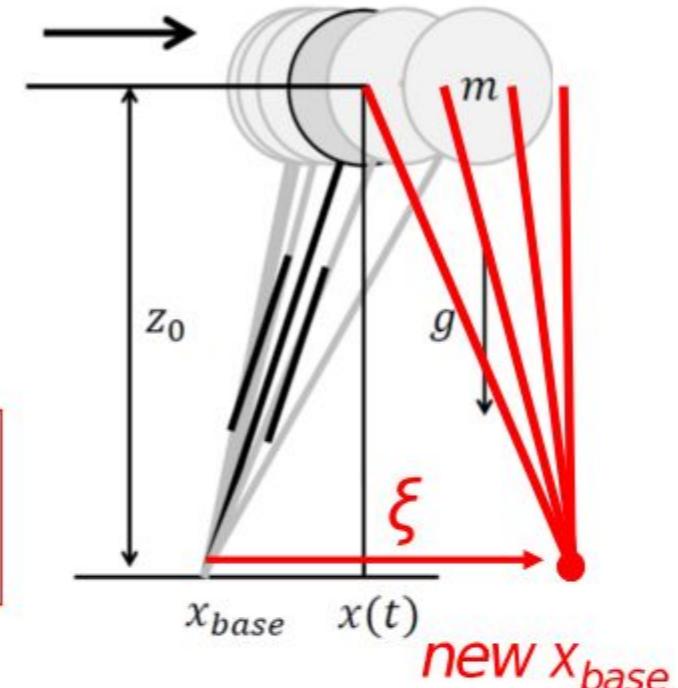
- It is therefore important to monitor the DCM, and to plan the foot steps (i.e. x_{base}) to bring the robot (the CoM) where you want and to prevent falling.

$$\xi = x + \frac{\dot{x}}{\omega}$$

- The DCM corresponds to the ***instantaneous capture point***.

- Indeed by setting the next foot step (i.e. x_{base}) to ξ at the beginning of a step, the time evolution of x goes to zero (i.e. the pendulum stops).

- Takenaka, Toru, Takashi Matsumoto, and Takahide Yoshiike. 2009. "Real Time Motion Generation and Control for Biped Robot: 1st Report: Walking Gait Pattern Generation." In *Proceedings of IROS 2009*.
- Englsberger et al (2014). Trajectory generation for continuous leg forces during double support and heel-to-toe shift based on divergent component of motion. In *2014 IEEE/RSJ International Conference on Intelligent Robots and Systems* (pp. 4022-4029). IEEE.



$$\dot{\xi} = \omega(\xi - x_{base})$$

$$\dot{x} = -\omega(x - \xi)$$

This will be tested in
the mini project with
the Atlas simulation!

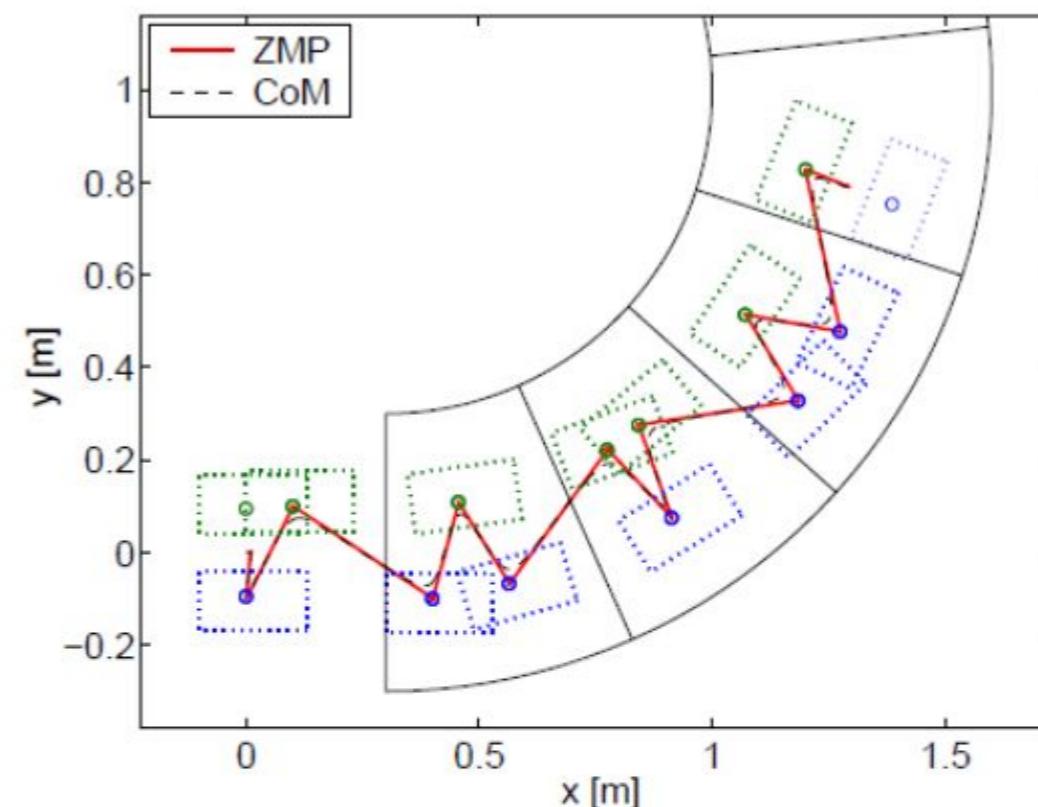
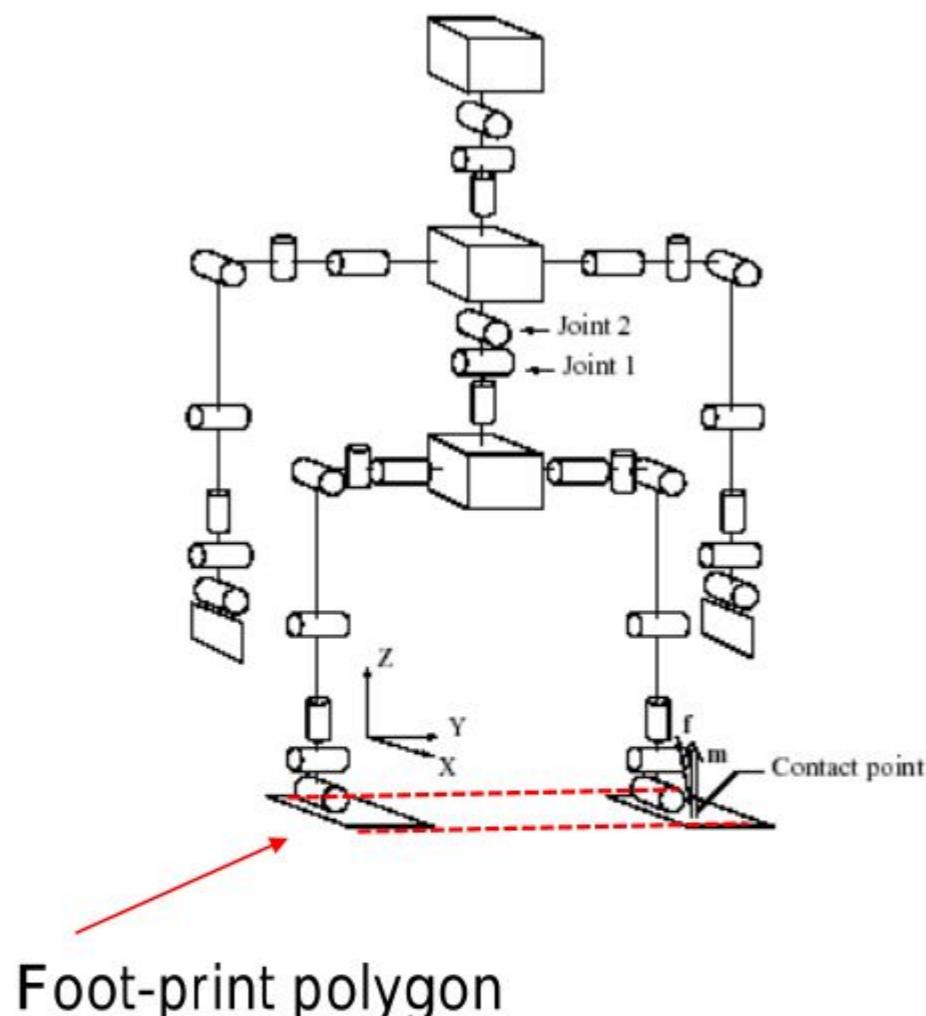
Examples of model-based approaches

Model-based control:

1. **trajectory based methods (ZMP)**
2. Virtual leg control (Raibert)
3. Virtual model control (Pratt et al)
4. Hybrid Zero Dynamics control
5. Planning methods (Little dog project)
6. Inverse dynamics and optimization

Reviewing Previous Lectures

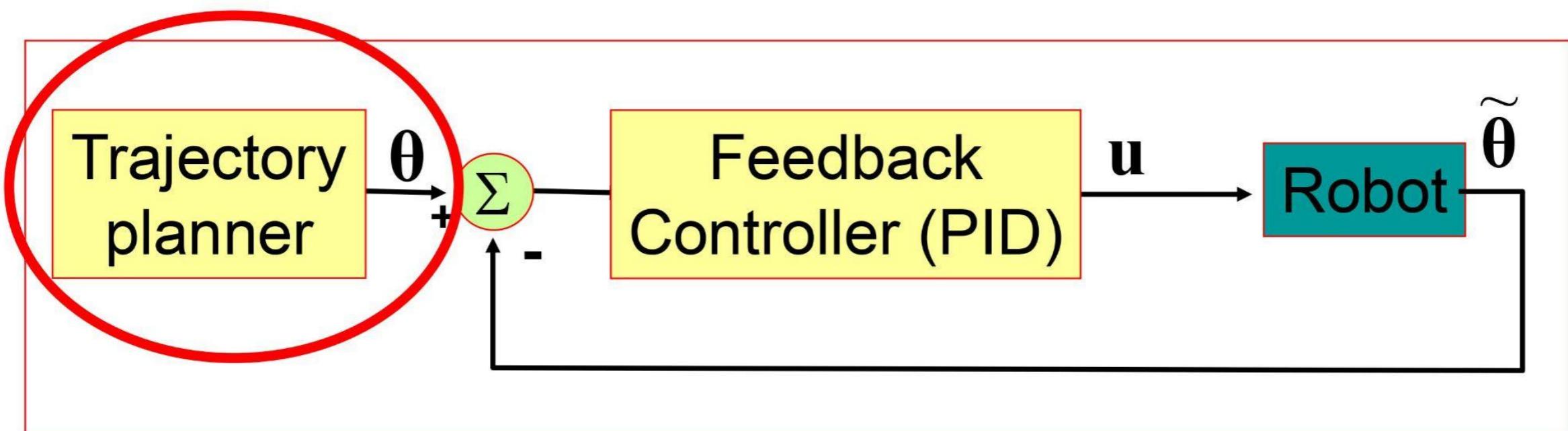
ZMP



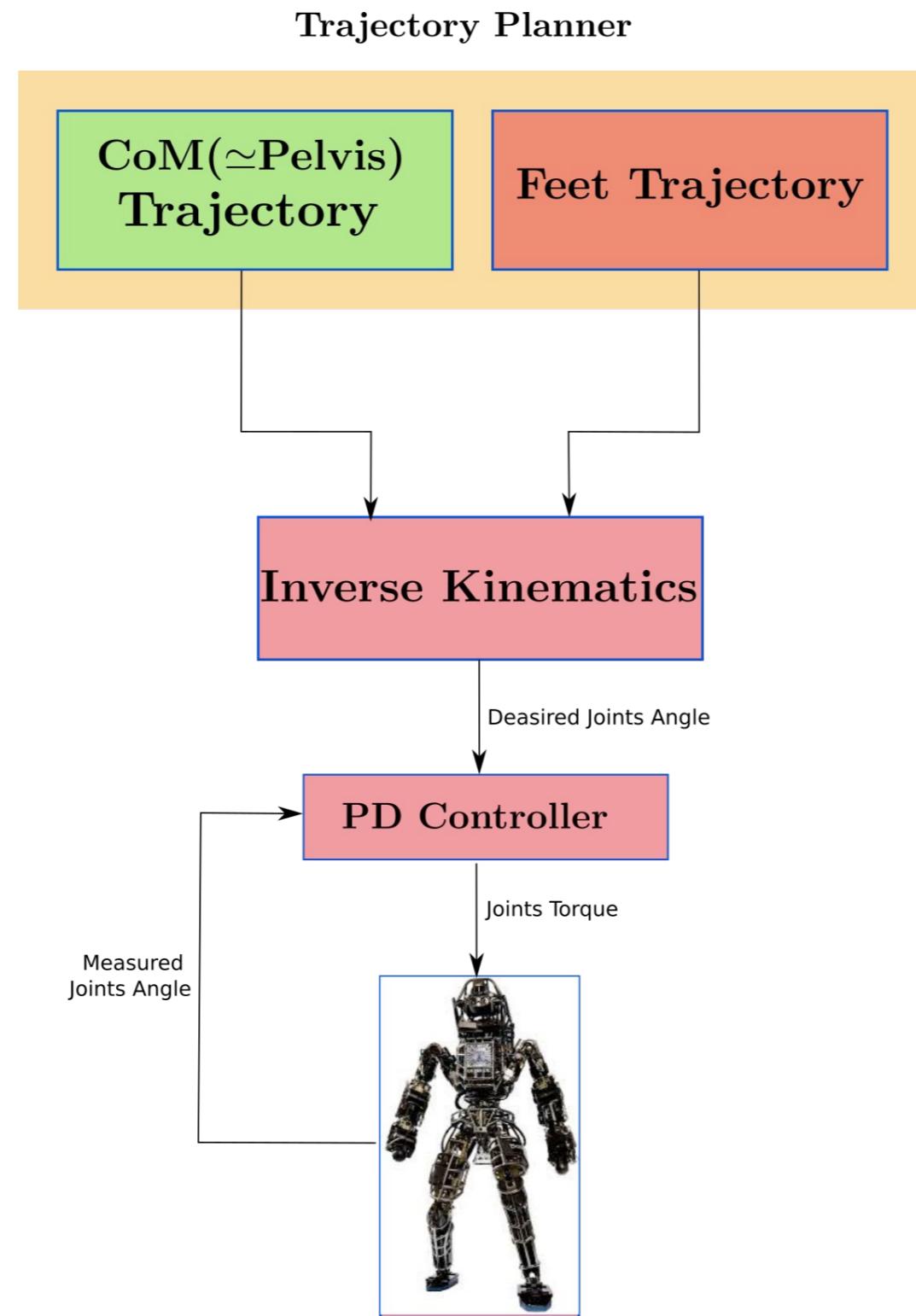
Kajita, et al. 2003. "Biped Walking Pattern Generation by Using Preview Control of Zero-Moment Point." In *ICRA 2003*.
<https://doi.org/10.1109/ROBOT.2003.1241826>.

Locomotion is stable if the ZMP remains within the foot-print polygons over time

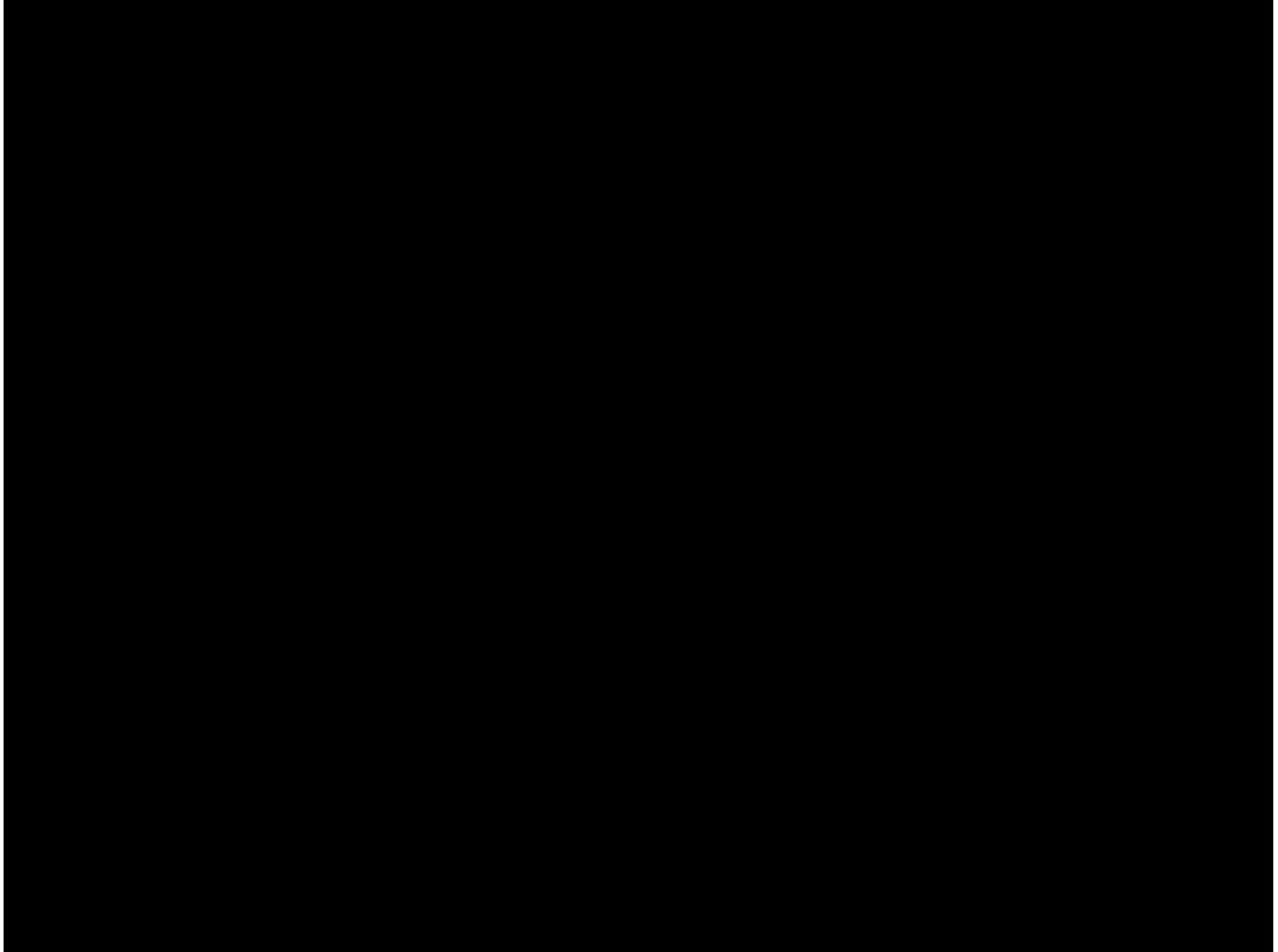
Minimalistic control diagram



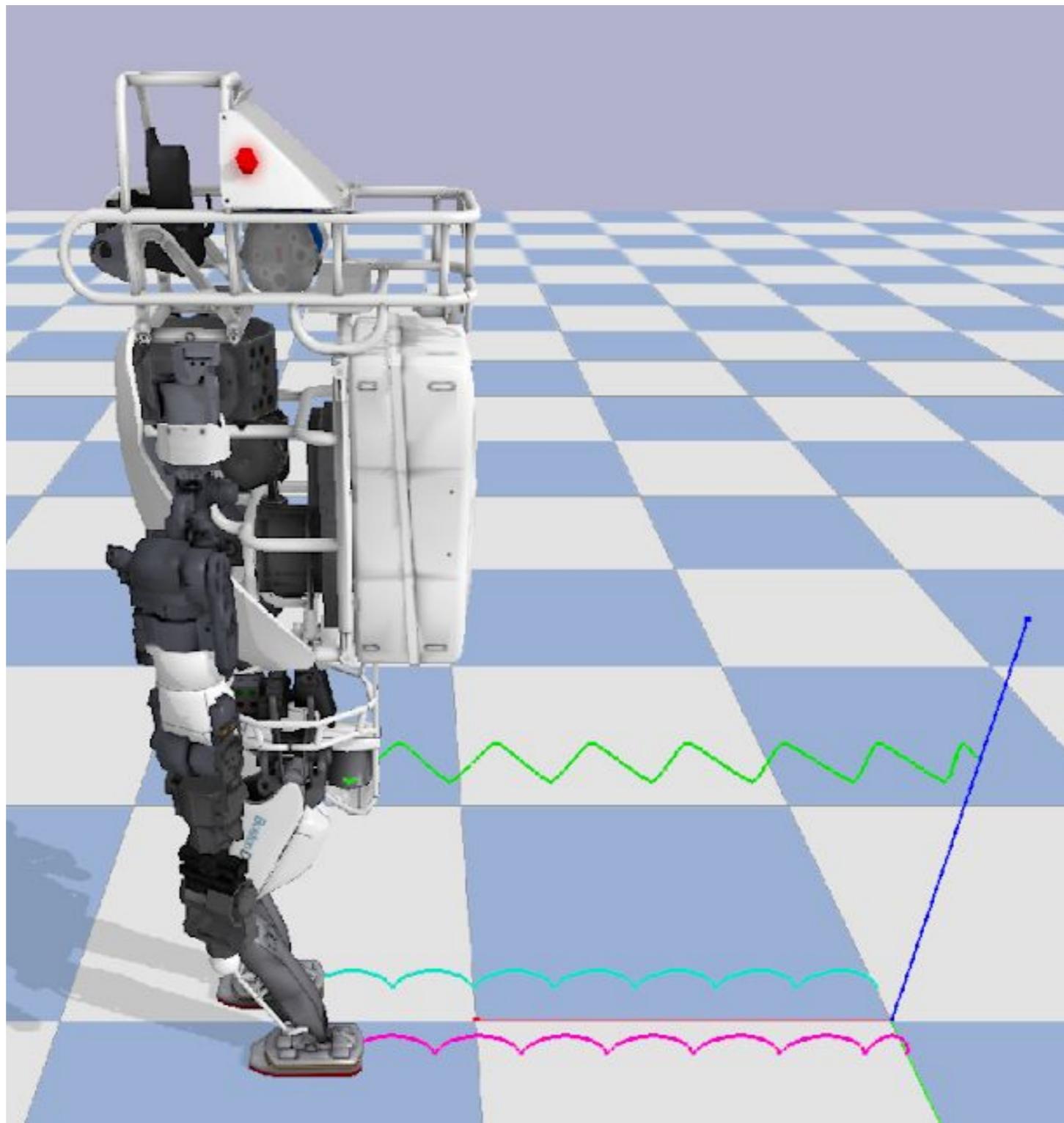
θ Desired robot posture
 $\tilde{\theta}$ Actual robot posture
 u Command (torque)



Project Definition:Atlas Walking with DCM Planner in Pybullet



Mini-Project:Atlas Walking with DCM Planner in Pybullet



Englsberger, Johannes, Christian Ott, and Alin Albu-Schäffer. "Three-dimensional bipedal walking control based on divergent component of motion." *ieee transactions on robotics* 31.2 (2015): 355-368.(254)

Linear Inverted Pendulum Model(LIPM)-Divergent Component of Motion(DCM)

$$\ddot{x}_c = \omega^2(x_c - CoP)$$

x_c Center of mass position

CoP Center of Pressure

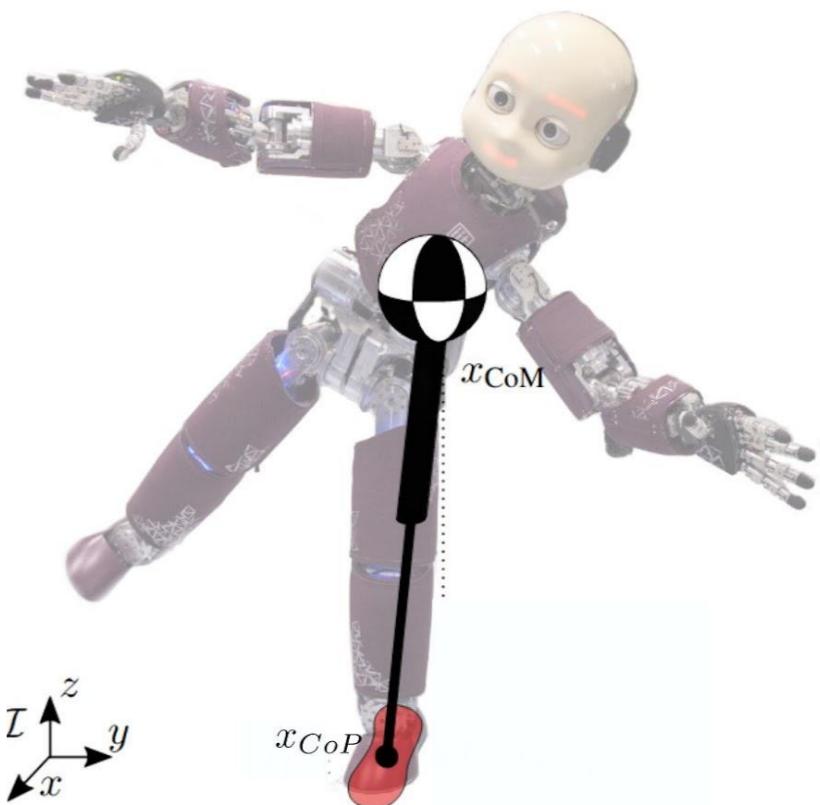
g Gravity Acceleration

Z Center of mass height(constant)

$$\omega = \sqrt{\frac{g}{Z}}$$

$$\xi_x = x_c + \frac{\dot{x}_c}{\omega}$$

ξ DCM

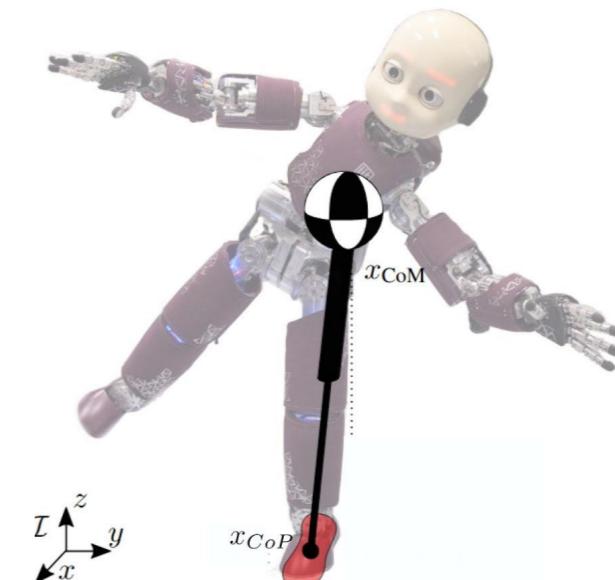


DCM Dynamics

$$\ddot{x}_c = \omega^2(x_c - CoP)$$



$$\xi_x = x_c + \frac{\dot{x}_c}{\omega}$$

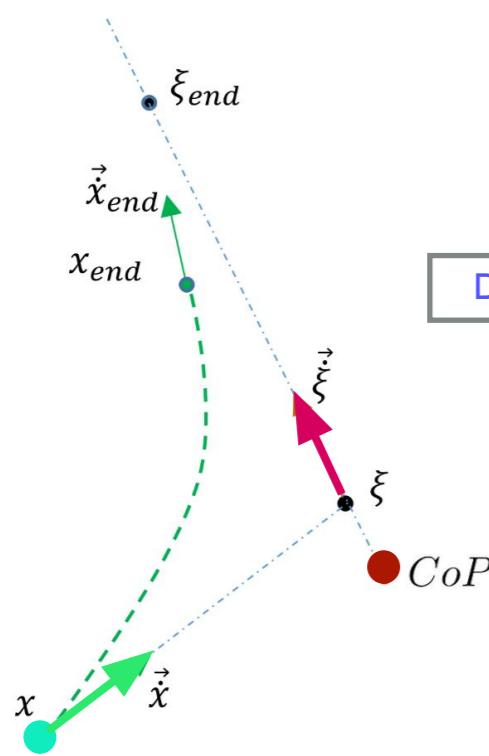


$$\dot{\xi} = \omega(\xi - CoP)$$

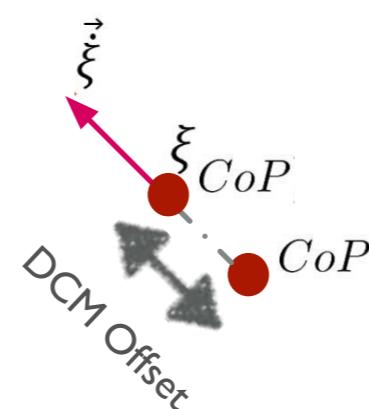
$$\dot{x} = -\omega(x - \xi)$$

Divergent Component of Motion(DCM)

Convergent Component of Motion



Why is capture point another name for DCM?



Solution of DCM ode

$$\dot{\xi} = \omega(\xi - CoP)$$



We can solve this ode by considering that CoP is a constant value (During single support phase)

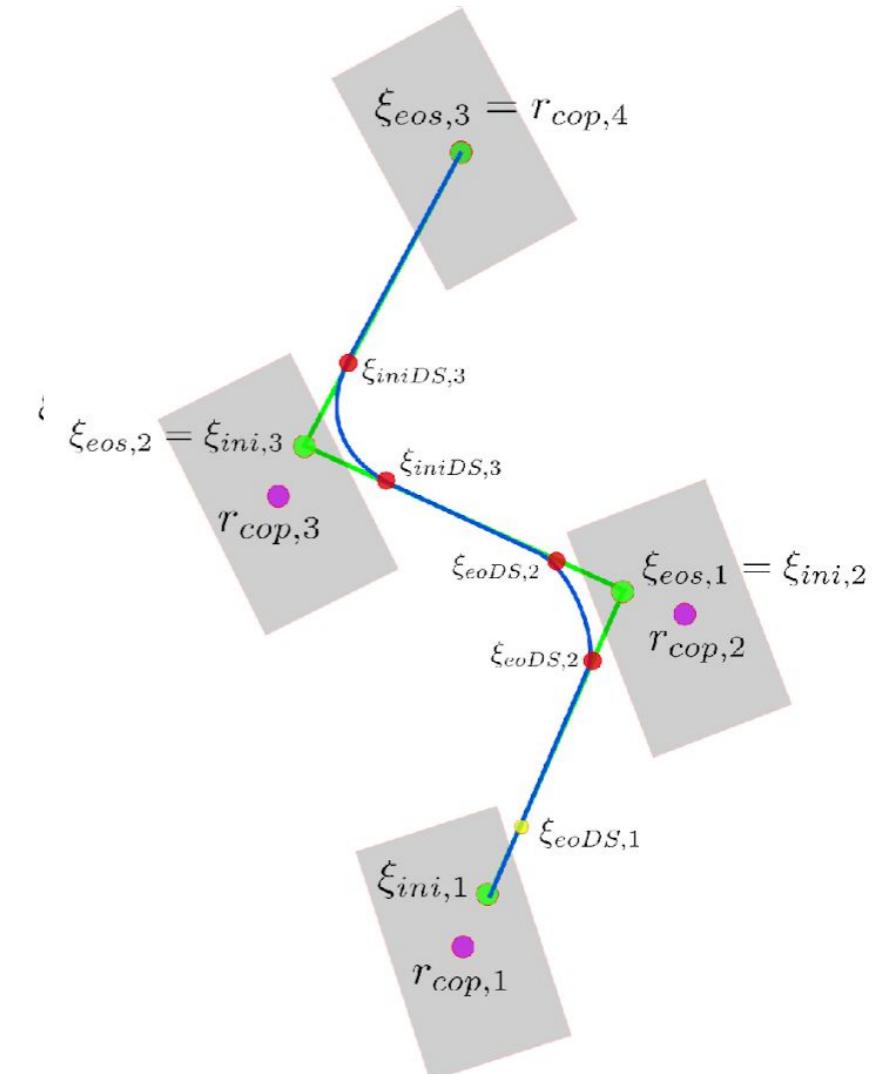


$$\xi(t) = CoP + e^{\omega t}(\xi_0 - CoP)$$

DCM Motion Planning

$$\xi(t) = CoP + e^{\omega t}(\xi_0 - CoP)$$

- Plan foot step position and step duration(T)
- Place the desired CoP in a location inside of the foot print
- Place the last DCM position on the last CoP(Capturability constraint)
- Find the initial DCM position for each step recursively based on DCM equation
- By having initial DCM and DCM equation we can find DCM Trajectory for SS
- Select the desired double support(DS) phase duration
- Based on the desired DS duration find DCM boundary conditions for DS
- Use a cubic interpolation for finding DCM trajectory in DS phase
- Replace the DCM planned trajectory for the DS with the corresponding part that was for single support



DCM Motion Planning for Single Support(SS)

$$\xi(t) = CoP + e^{\omega t}(\xi_0 - CoP)$$

- Place the last DCM position on the last CoP(Capturability constraint)

$$\xi_{eos,i} = r_{cop,i} + (\xi_{ini,i} - r_{cop,i})e^{\omega T}$$

$$\xi_{ini,i} = r_{cop,i} + (\xi_{eos,i} - r_{cop,i})e^{-\omega T}$$

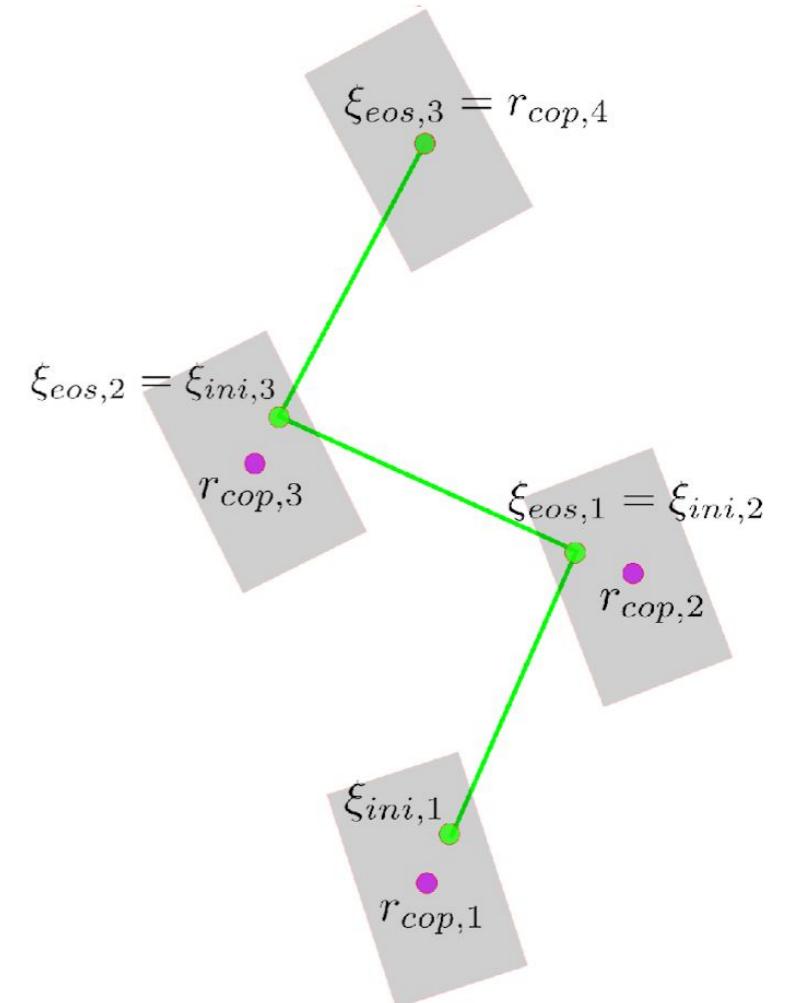
- Find the initial DCM position for each step recursively based on DCM equation

$$\xi_{eos,i-1} = \xi_{ini,i}$$

- By having initial DCM and DCM equation we can find DCM Trajectory for SS:

$$\xi_i(t) = r_{cop,i} + (\xi_{eos,i} - r_{cop,i})e^{\omega(t-T)}$$

The “internal” timestep t is reset at the beginning of each step, i.e., $t \in [0, T]$ (T is the duration of the step).



DCM Motion Planning for Double Support(DS)

- Select the desired double support(DS) phase duration

$$\Delta t_{DS,ini} = \alpha_{DS,init} t_{DS}$$

$$\Delta t_{DS,end} = (1 - \alpha_{DS,init}) t_{DS}$$

- Based on the desired DS duration find DCM boundary conditions for DS

$$\xi_{eoDS,i} = r_{cop,i} + (\xi_{ini,i} - r_{cop,i}) e^{\omega \Delta t_{DS,end}}$$

$$\xi_{iniDS,i} = r_{cop,i-1} + (\xi_{ini,i} - r_{cop,i-1}) e^{-\omega \Delta t_{DS,ini}}$$

- Use a cubic interpolation for finding DCM trajectory in DS phase

$$\begin{bmatrix} \frac{2}{T_{DS}^3} & \frac{1}{T_{DS}^2} & -\frac{2}{T_{DS}^3} & \frac{1}{T_{DS}^2} \\ -\frac{2}{T_{DS}^2} & -\frac{1}{T_{DS}} & \frac{2}{T_{DS}^2} & -\frac{1}{T_{DS}} \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \dot{\xi}_{iniDS,i} \\ \ddot{\xi}_{iniDS,i} \\ \dot{\xi}_{eosDS,i} \\ \ddot{\xi}_{eosDS,i} \end{bmatrix} = P$$

$$\begin{bmatrix} \xi(t) \\ \dot{\xi}(t) \end{bmatrix} = \begin{bmatrix} t^3 & t^2 & t & 1 \\ 3t^2 & 2t & 1 & 0 \end{bmatrix} P$$

- Replace the DCM planned trajectory for the DS

