CS 688 **Graphic modeling** **spring 2023**

Homework Set 4 Group Members: ByungKwon Moon

1.

$$p(y_i = 1|y_{-i}, b, w) = \frac{p(y_i = 1, y_{-i}|b, w)}{p(y_{-i}|b, w)}$$
$$= \frac{p(y|b, w)}{p(y_{-i}|b, w)}$$
$$= \frac{\frac{1}{Z}\prod_{l=1}^{d}\exp(b_l y_l)\prod_{(l,j)\in\text{pairs}}\exp(w_{lj}y_l y_j)}{\sum_{y_i=\pm 1}\frac{1}{Z}\prod_{l=1}^{d}\exp(b_l y_l)\prod_{(l,j)\in\text{pairs}}\exp(w_{lj}y_l y_j)}$$

since we are conditioning on y_{-i}. for the denominator we can sums over all possible assignments of y_i,

We can simplify the numerator by removing the factors that do not depend on y_i:

$$p(y_i = 1|y_{-i}, b, w) = \frac{\exp(b_i)\prod_{(j)\in\text{nb(i)}}\exp(w_{ij}y_j)}{\exp(b_i)\prod_{(j)\in\text{nb(i)}}\exp(w_{ij}y_j) + \exp(-b_i)\prod_{(j)\in\text{nb(i)}}\exp(-w_{ij}y_j)}$$

$$\textbf{Let} s' = b_i + \sum_{j\in nb(i)} w_{ij}y_j \textbf{and} exp(i+j) = exp(i)exp(j)$$

$$= \frac{exp(s')}{exp(s') + exp(-s')} = \frac{1}{1 + exp(-2s')}$$

$$1/(1 + exp(-2s')) = \frac{1}{1 + exp(-2(b_i + \sum_{j\in nb(i)} w_{ij}y_j)})$$

$$\textbf{lets say -s} = -2(b_i + \sum_{j\in nb(i)} w_{ij}y_j)$$

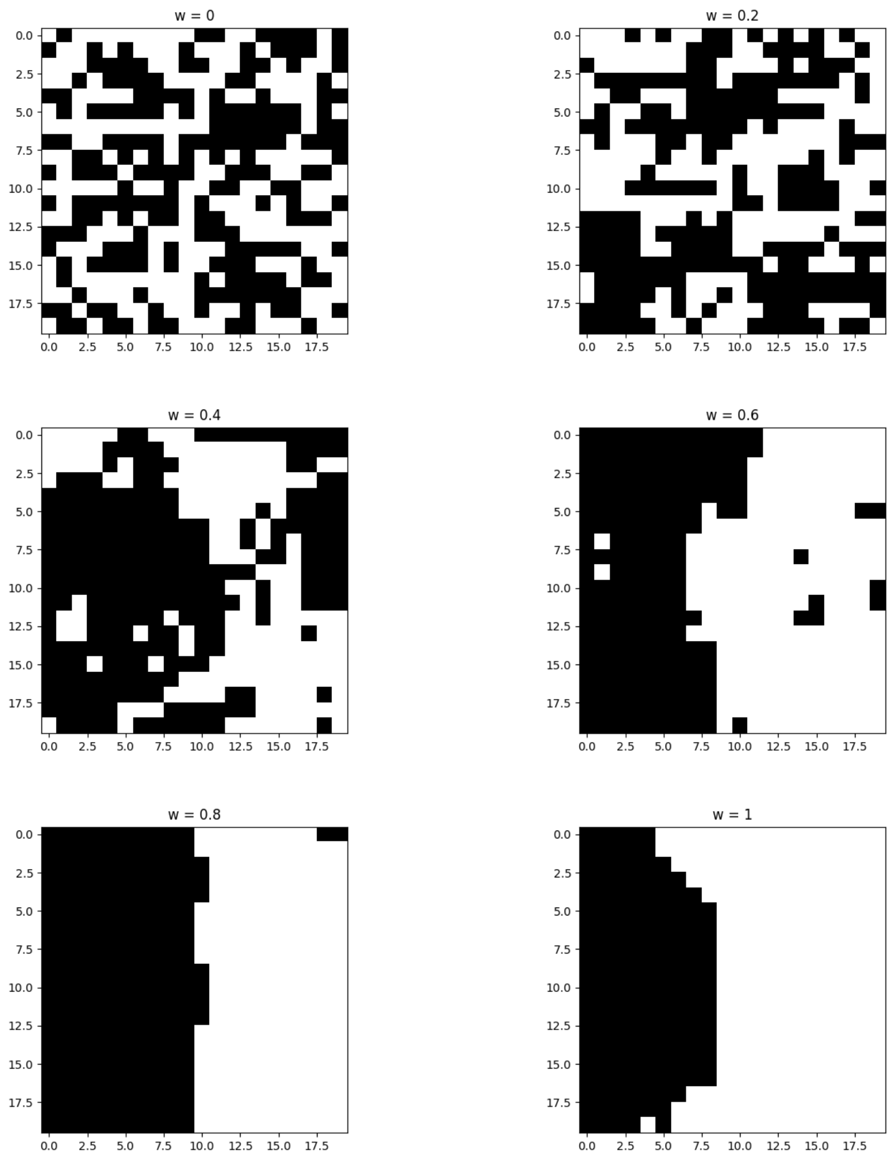
$$= \frac{1}{1 + exp(-s)} = \sigma(s)$$

$$p(y_i = 1|y_{-i}, b, w) = \sigma(s)$$

Algorithm 1 Gibbs sampling

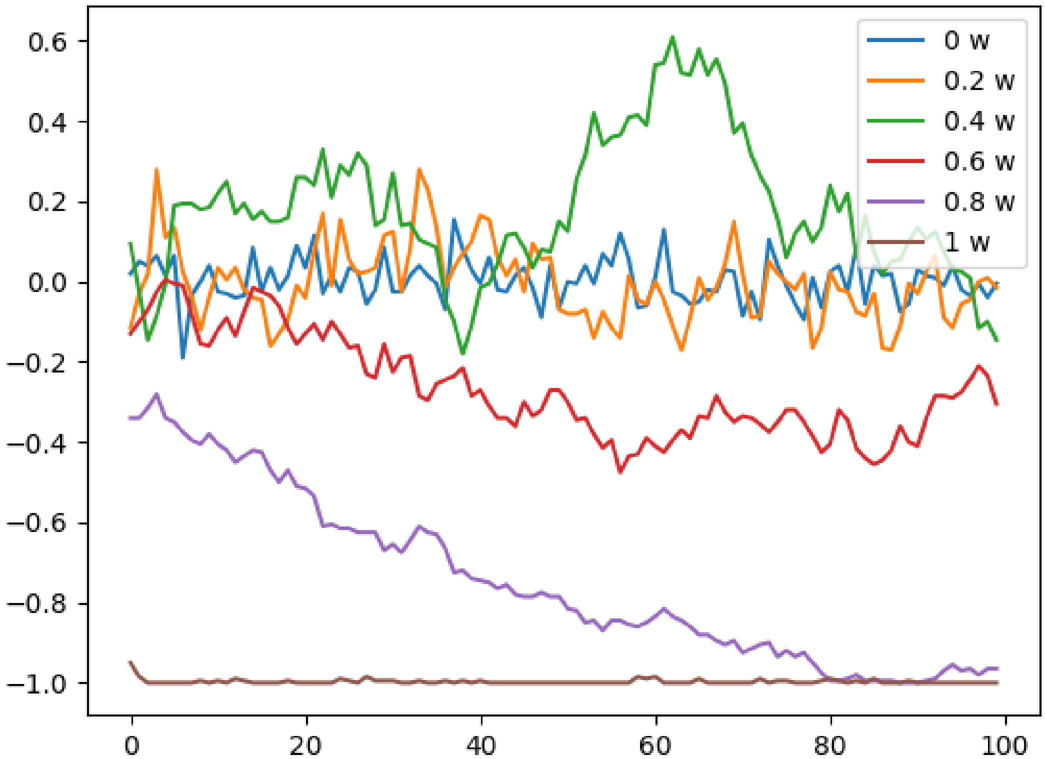
2. **Require:** $iteration, b, w$

```
initialize : samples: empty list
initialize : y^0 ∈ ℝ^d
for i from 1 to (iteration) do
    for j from 1 to d do
        pairs are 1, ..., dx1, ..., d such that wij non zero
        nb_j = [k for k in range(d) if (j,k) in pairs or (k,j) in pairs]
        p_j = 1 / (1 + exp(-2 * b[i] - 2 * sum(w[i][k] * y[k] for k in range(nb_j) )))
        if if random() < p_j then
            y[j] = 1
        else
            y[j] = -1
        end if
    end for
    samples.append(y.copy())
end for
return samples
```



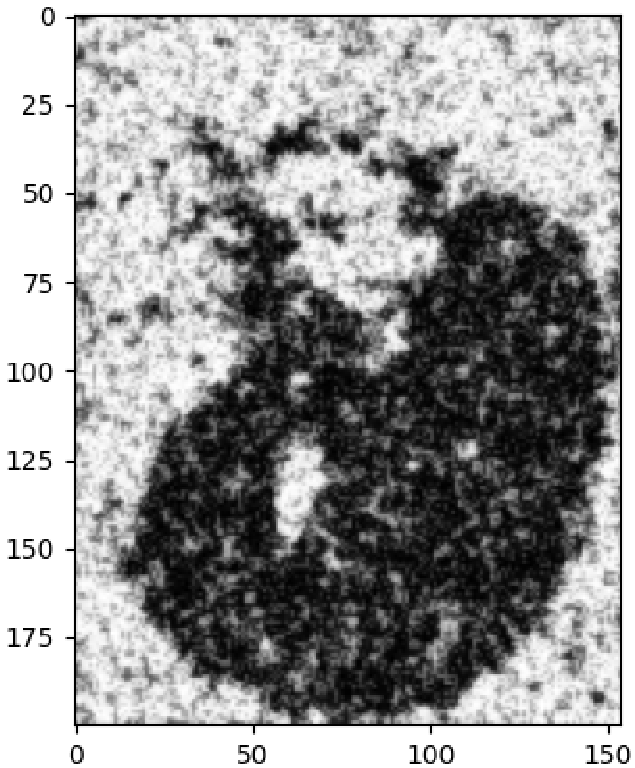
3.

4. since w_{ij} is increases, each pixel is trying to have same value with its neighbors, in other words, we can define clear regions between black(-1) and white(+1). This is because as w increases value of $\sum_{j \in nb(i)} w_{ij} y_j$ matters more. Which represent value of neighbors affect more to decide y_i



5.

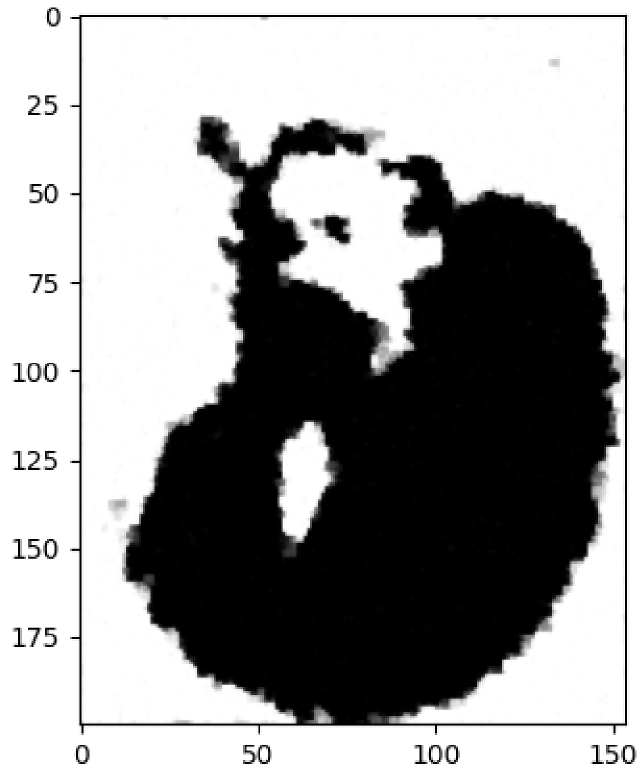
6. we know that true mean value of y is E[y] = 0. When $w >$ is greater, the mean value of y takes longer to converge to 0 because the variables are positively correlated and there is a stronger dependence between them. The curves also show that the convergence rate depends on the value of w, with higher values of w leading to slower convergence .



7. mean per-pixel squared difference of from the true output :0.32157412987012984
8. Describe (a) how you tried to find better parameters (there is no single correct answer here) (b) show an image of your final denoised image and (c) report the final error

   (a) so I run the loop that w = [0.2,0.4,0.6,0.8,1.0] and b= [0.1,0.5,1,1.5,2.0] then find the best mean value from that

```
    mse    b   w
0.8835606363636366 0.1 0.2
0.7981681818181819 0.5 0.2
1.04176861038961    1 0.2
1.2669878181818188 1.5 0.2
1.394225922077923    2 0.2
0.33841142857142864 0.1 0.4
0.37976649350649344 0.5 0.4
0.7643041168831168   1 0.4
1.2758485584415593 2 0.4
0.14346701298701292 0.1 0.6
0.1321160389610389 0.5 0.6
0.7998220519480522 1.5 0.6
1.0827960129870136 2 0.6
```




```
0.14589348051948053 0.1 0.8
0.1227434935064935 0.5 0.8
0.1499607662337662 1 0.8
0.4392784675324674 1.5 0.8
0.82739312987013 2 0.8
0.14602092207792208 0.1 1.0
0.12181210389610389 0.5 1.0
0.12121620779220778 1 1.0
0.18067115584415583 1.5 1.0
0.4873041558441558 2 1.0
```

so from this result, best vlaue is b = 1 and w =1 mse = 0.12121620779220778