

1. $p(z, x) = p(z)p(x|z)$

$$p(z) = \mathcal{N}(z; 0, I)$$

$$p(x|z) = \text{Bernoulli}(x; f(z)) \quad (f : \mathbb{R}^2 \rightarrow [0, 1]^{784}).$$

$$q(z|x) = \mathcal{N}(z; g(x), 0.01I) \quad (g : [0, 1]^{784} \rightarrow \mathbb{R}^2).$$

$$\text{ELBO}(x^{(i)}) = \mathbb{E}_{z^{(i)} \sim q(z^{(i)}|x^{(i)})} [\log p(z^{(i)}, x^{(i)}) - \log q(z^{(i)}|x^{(i)})]$$

$$= \mathbb{E}_{z^{(i)} \sim q(z^{(i)}|x^{(i)})} [\log p(z^{(i)}) + \log p(x^{(i)}|z^{(i)}) - \log q(z^{(i)}|x^{(i)})]$$

$$= \mathbb{E}_{z^{(i)} \sim q(z^{(i)}|x^{(i)})} [\log \mathcal{N}(z^{(i)}; 0, I) + \log \text{Bernoulli}(x^{(i)}; f(z^{(i)})) - \log \mathcal{N}(z^{(i)}; g(x^{(i)}), 0.01I)]$$

then lets look each of the value in the bracket.

$$\mathcal{N}(z^{(i)}; 0, I) = (\det(2\pi I))^{-\frac{1}{2}} \exp\left(-\frac{1}{2}(z^{(i)} - 0)^T I^{-1}(z^{(i)} - 0)\right).$$

$$= (2\pi)^{-\frac{1}{2}d} \exp\left(-\frac{1}{2}z^{(i)} \cdot z^{(i)}\right)$$

$$\log \mathcal{N}(z^{(i)}; 0, I) = -0.5(z^{(i)} \cdot z^{(i)}) - 0.5d \log(2\pi)$$

$$\log \text{Bernoulli}(x^{(i)}; f(z^{(i)})) = \sum_{j=1}^d x_j^{(i)} \log f_j(z^{(i)}) + (1 - x_j^{(i)}) \log(1 - f_j(z^{(i)}))$$

$$\log \mathcal{N}(z^{(i)}; g(x^{(i)}), 0.01I) = -0.5(z^{(i)} - g(x^{(i)})) \cdot (z^{(i)} - g(x^{(i)})) \cdot (0.01I)^{-1} \cdot (z^{(i)} - g(x^{(i)})) - 0.5d \log(2\pi) - 0.5d \log(0.01)$$

now plug back into it

$$\mathbb{E}_{z^{(i)} \sim q(z^{(i)}|x^{(i)})} [-0.5(z^{(i)} \cdot z^{(i)}) + \sum_{j=1}^d (x_j^{(i)} \log f_j(z^{(i)}) + (1 - x_j^{(i)}) \log(1 - f_j(z^{(i)}))) + 50((z^{(i)} - g(x^{(i)})) \cdot (z^{(i)} - g(x^{(i)}))) + 0.5d \log(0.01)]$$

$$= \mathbb{E}_{z^{(i)} \sim q(z^{(i)}|x^{(i)})} [-0.5(z^{(i)} \cdot z^{(i)}) + \sum_{j=1}^d (x_j^{(i)} \log f_j(z^{(i)}) + (1 - x_j^{(i)}) \log(1 - f_j(z^{(i)}))) + 50((z^{(i)} - g(x^{(i)})) \cdot (z^{(i)} - g(x^{(i)})))] + C$$

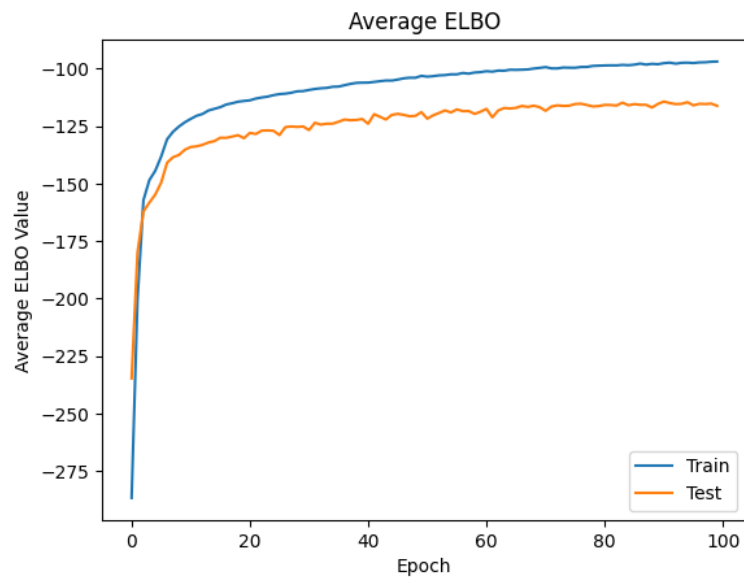
$$C = 0.5d \log(0.01)$$

2. we can plug into the q1

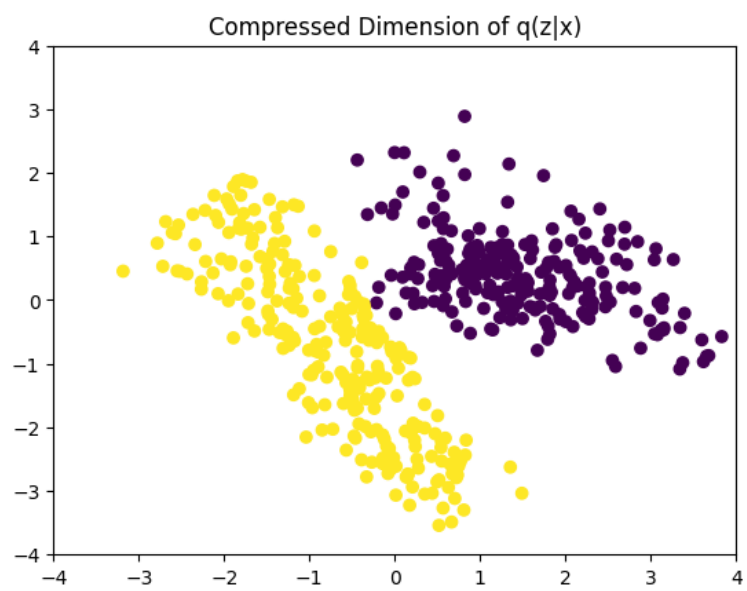
$$\mathbb{E}_{\epsilon^{(i)} \sim \mathcal{N}(0, I)} [-0.5((0.1\epsilon^{(i)} + g(x^{(i)})) \cdot (0.1\epsilon^{(i)} + g(x^{(i)}))) + \sum_{j=1}^d (x_j^{(i)} \log f_j(0.1\epsilon^{(i)} + g(x^{(i)})) + (1 - x_j^{(i)}) \log(1 - f_j(0.1\epsilon^{(i)} + g(x^{(i)})))) + 50(0.1\epsilon^{(i)} \cdot 0.1\epsilon^{(i)})] + C$$

$$\text{where } C = 0.5d \log(0.01)$$

3. If for the elbo first datapoint in the training set? (report)
[-0.10000038 , -0.10000038]

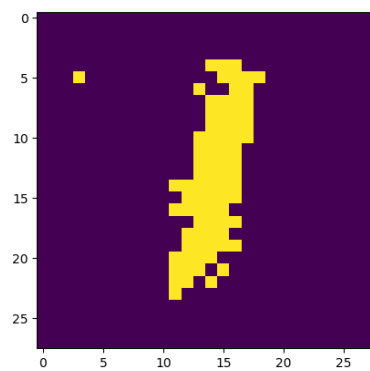


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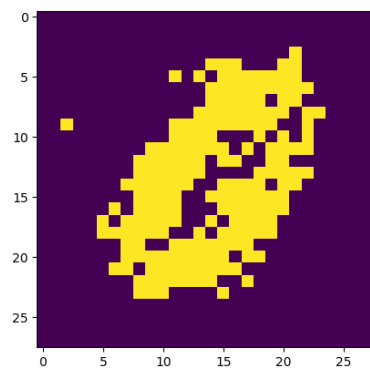


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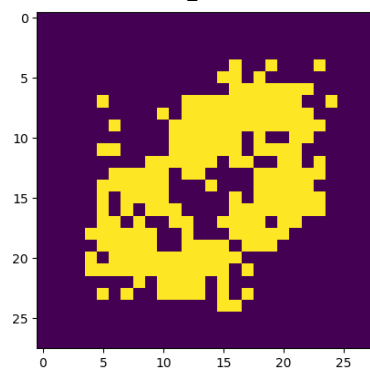
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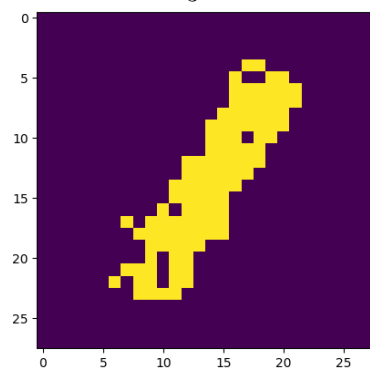
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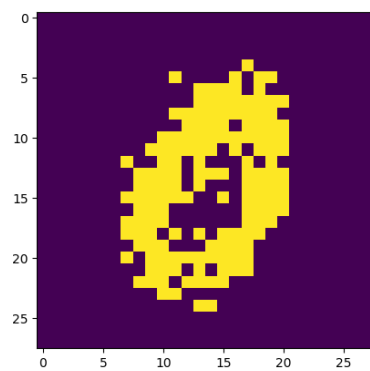
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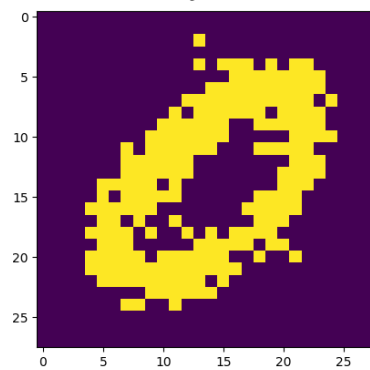
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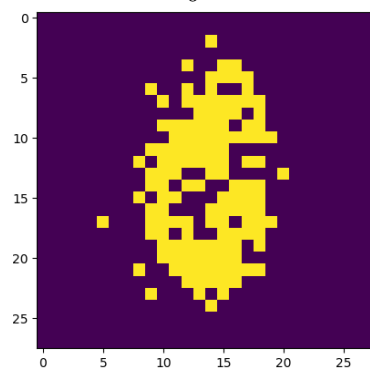
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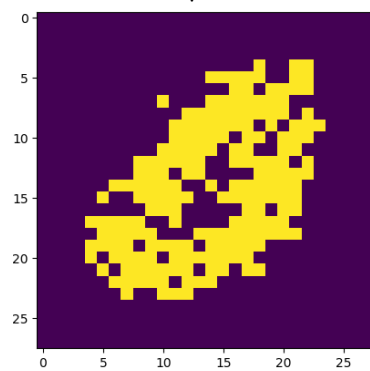
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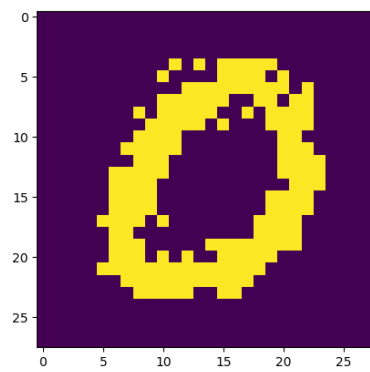
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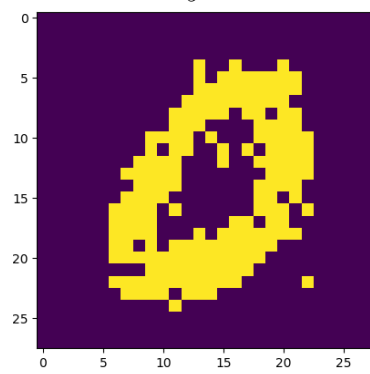
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