HW3

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This is my first time writing up a document with Latex using R Markdown. Please excuse the terrible formatting.

1.a)

$\int_{-\infty}^{\infty} \frac{\partial f(v,t)}{\partial t} = \int_{-\infty}^{\infty} \frac{\partial}{\partial v} \left[\frac{v-V}{\tau_m}\right] f(v,t) + \frac{2e}{3\tau_m} \frac{\partial^2 f(v,t)}{\partial v^2}$

$c_1 = \frac{v-V}{\tau_m} f + \frac{2e}{3\tau_m} \frac{\partial}{\partial v} f$

Both terms on the right go to zero as $v \to \infty \therefore c_1 = 0$.

$0 = \frac{v-V}{\tau_m} f + \frac{2e}{3\tau_m} \frac{\partial}{\partial v}$

Multiplying both sides by $\frac{\tau_m}{f}$ and integrating:

$\int_{-\infty}^{\infty}(v-V)dv + \frac{2e}{3} \int_{-\infty}^{\infty} \frac{1}{f} \frac{\partial}{\partial v} dv$

$0 = \frac{1}{2}(v-V)^2 + \frac{2e}{3} ln(f) + c_2$

$-\frac{1}{2}(v-V)^2 - c_2 = \frac{2e}{3} ln(f)$

$\frac{3e}{2}(-\frac{1}{2}(v-V)^2 - c_2) = ln(f)$

$e^{\frac{3e}{2}(-\frac{1}{2}(v-V)^2 - c_2)} = f(v)$

$e^{\frac{-3(v-V)^2}{4e} - \frac{3}{2e}c_2} = f(v)$

$\frac{e^{\frac{-3(v-V)^2}{4e}}}{e^{\frac{3}{2e}c_2}} = f(v)$

$\frac{1}{e^{\frac{3}{2e}c_2}} e^{\frac{-3(v-V)^2}{4e}} = f(v)$

Notice that this is similar to the Normal PDF. Therefore, the normalization factor for the Normal (which guarantees the integral of the PDF = 1), should equal the normalization factor for our PDF.

$\frac{1}{\sqrt{(2\pi\frac{2}{3e})}} = \frac{1}{e^{\frac{3}{2e}c_2}}$

$\sqrt{(2\pi\frac{2}{3e})} = e^{\frac{3}{2e}c_2}$

Solving in Wolfram Alpha: $c_2 = ln(\sqrt{(3\pi)}\sqrt{(\frac{1}{e})})\frac{2}{3}e$

The final PDF should be:

$f(v) = \frac{1}{\sqrt{(2\pi\frac{2}{3e})}} e^{\frac{-3(v-V)^2}{4e}}$

b) V is equal to the mean and $\frac{2e}{3}$ is equal to the variance. The stationary PDF f(v) does not depend on τ_m because our PDF is independent of time and τ_m is time-dependent.

2.a)

Let F = 0, G = -1/τ, $D^{(1)}$=-(p-C)/τ, $D^{(2)}$= D.

Using equation 8.49a on pg. 308:

$\frac{d\langle P\rangle}{dt} = \left\langle D^{(1)} \right\rangle = \langle -(p-C)/\tau \rangle$
$\langle P\rangle = c_1 e^{\frac{-t}{\tau}} + C$

Letting the initial population = p_0:

p_0 = ke^0+C
p_0= k+C
k = p_0-C

$\therefore \langle P\rangle$= ($p_0$-C)$e^{\frac{-t}{\tau}}$+C

Comparing to equation 8.79a from pg. 315 since, asymptotically, $\alpha = \langle P\rangle$:

$\frac{d}{dt}(\alpha - \langle P\rangle) = \frac{-1}{\tau}(\alpha - \langle P\rangle)$

$\alpha = \langle P\rangle + (p_0 - \langle P\rangle)e^{\frac{-t}{\tau}}$

For $\left\langle \tilde{P^2} \right\rangle$:

$\frac{d}{dt}\left\langle \tilde{P^2} \right\rangle = \frac{-2}{\tau}\left\langle \tilde{P^2} \right\rangle + 2D$

Using equation 8.78b from pg. 315 since asymptotically $\beta = \left\langle \tilde{P^2} \right\rangle$:

$\frac{d}{dt}(\beta - D) = \frac{-2}{\tau}(\beta - D)$

$\beta = D - De^{\frac{-2t}{\tau}}$

b)

$\alpha = \langle P\rangle + (p' - \langle P'\rangle)\int_0^t e^{\frac{-t}{\tau}} ds$

$\beta = \left\langle \tilde{P^2} \right\rangle - (\left\langle \tilde{P'^2} \right\rangle)2\int_0^t e^{\frac{-t}{\tau}} ds$

$f(p,t|p',0) = \frac{1}{\sqrt(2\pi(\left\langle \tilde{P^2} \right\rangle - (\left\langle \tilde{P'^2} \right\rangle)2\int_0^t e^{\frac{-t}{\tau}} ds))} e^{\frac{-\langle P\rangle + (p' - \langle P'\rangle)\int_0^t e^{\frac{-t}{\tau}} ds}{2(\left\langle \tilde{P^2} \right\rangle - (\left\langle \tilde{P'^2} \right\rangle)2\int_0^t e^{\frac{-t}{\tau}} ds)}}$

c) $f(p,t) = \frac{\int f(p',0)dp'}{\sqrt(2\pi\left\langle \tilde{P^2} \right\rangle)} e^{-\frac{(p-\langle P\rangle)^2}{2\left\langle \tilde{P^2} \right\rangle}}$

If t $\rightarrow \infty$ the PDF f(p,t) relaxes asymptotically to Normal PDF.

$f(p,t) = \frac{1}{\sqrt(2\pi\left\langle \tilde{P^2} \right\rangle)} e^{-\frac{(p-\langle P\rangle)^2}{2\left\langle \tilde{P^2} \right\rangle}}$

d)

If D$\rightarrow$ 0 then $\left\langle \tilde{P^2} \right\rangle \rightarrow 0$, thus the asymptotic PDF becomes f(p,t)=$\delta(p - C)$