Homework #2

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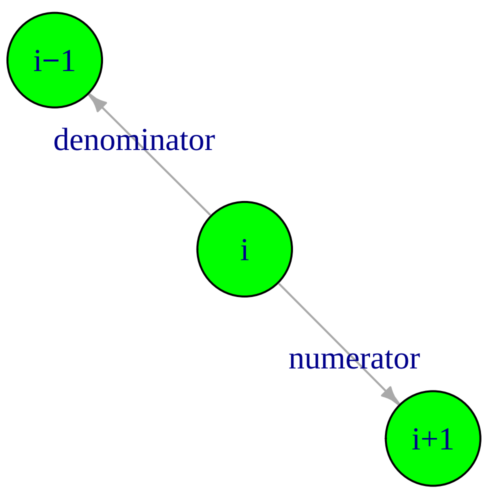
```r
library(markovchain)
library(igraph)
library(knitr)
```

1. $p_{01} = 1$; 100% probability of going from 0 to 1. Therefore, $p_{00} = 0$ for all n.
   $p_{1,i+1} = 1$; 100% probability of going to either i+1 or i-1 from i. No other possibilities!
   $\frac{p\ i,i+1}{p\ i,i-1} = (\frac{i+1}{i})^2$ Since the numerator is greater than the denominator, the chain has a tendency to move to the right.

```r
# Create graph using igraph.
# Set seed to ensure graph doesn't change.
set.seed(3)
g1 <- graph(c('i','i+1', 'i','i-1')) %>%
  set_edge_attr("label", value = c('numerator', 'denominator'))
plot(g1, vertex.size = 50, vertex.color = "green", edge.arrow.size = .5)
```

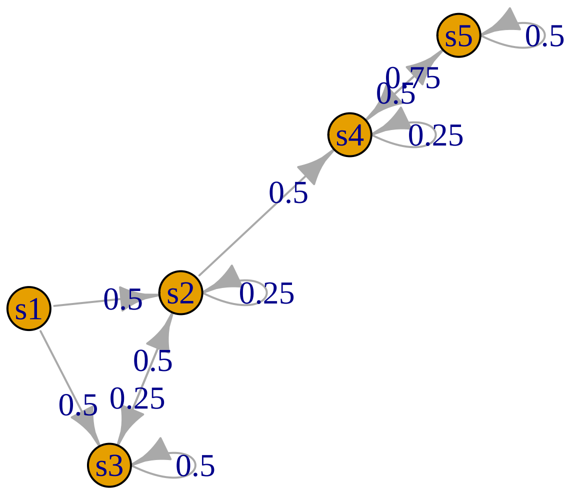


2. Setting up the problem:

```r
# Create transition matrix
m <- matrix(c(0, .5, .5, 0, 0,
              0, .25, .25, .5, 0,
              0, .5, .5, 0, 0,
              0, 0, 0, .25, .75,
              0, 0, 0, .5, .5),
            ncol = 5, byrow = T)

# Create a discrete time Markov Chain using the package "markovchain".
MC1 <- as(m, "markovchain")
```

```r
# Set seed to prevent graph from changing.
set.seed(1)
# Graph MC1.
plot(MC1, vertex.size = 20)
```



a) Classify the states.

```r
# Examine summary of MC1
summary(MC1)
```

```
## Unnamed Markov chain  Markov chain that is composed by:
## Closed classes:
## s4 s5
## Recurrent classes:
## {s4,s5}
## Transient classes:
## {s1},{s2,s3}
## The Markov chain is not irreducible
## The absorbing states are: NONE
```

4 and 5 are both a closed and recurrent class. 1 is a transient class that can never be returned to once left. 2 and 3 are transient as well.

b) Determine all the invariant probabilities.

```r
steadyStates(MC1)
```

```
##      s1 s2 s3  s4  s5
## [1,]  0  0  0 0.4 0.6
```

The invariant probabilities are $\pi = \{0, 0, 0, 0.4, 0.6\}$.

c) What is the probability to be in $\{4{,}5\}$ starting from 1.

```r
MC1[1, c(4,5)]
```

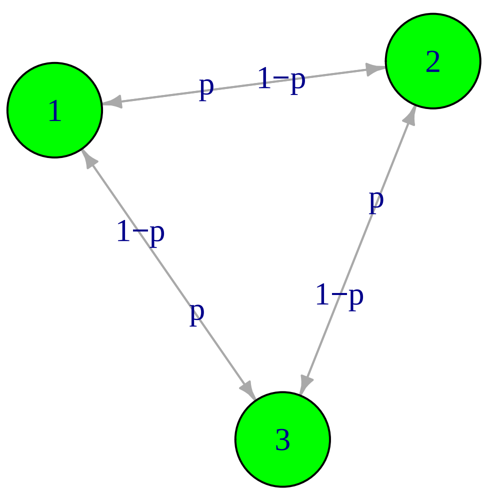
```
## s4 s5
##  0  0
```

There is a 0% probability of moving to {4,5} from 1. The chain can only move to 2 or 3 from 1. In terms of a steady state solution, however, there is a 100% of being in {4,5} since all other states are transient and {4,5} is closed.

3.

a.

```r
# Create graph using igraph.
# Set seed to ensure graph doesn't change.
set.seed(2)
g4 <- graph(c(1,2, 2,3, 3,1, 3,2, 2,1, 1,3)) %>%
  set_edge_attr("label", value = c('1-p', '1-p', '1-p', 'p', 'p', 'p'))
plot(g4, vertex.size = 50, vertex.color = "green",
edge.arrow.size = .5)
```



b.

$$P = \begin{pmatrix} 0 & 1-p & p \\ p & 0 & 1-p \\ 1-p & p & 0 \end{pmatrix}$$

c. Since the R package cannot handle variables, only numerical values, I will use 10 random values between 1 and 10 to show that the invariant probability does not depend on the value of p.

```r
# Create a vector of probabilities for p to take on.
p <- runif(10)

# Initialize an empty matrix to hold invariant probabilities for each p.
invariantProbabilities <- matrix(NA, nrow = length(p), ncol = 3)
```

```r
# Create a Markov Chain and calculate invariant probabilities for each p.
for(i in 1:length(p))
{

# Create transition matrix.
  m <- matrix(c(0, 1-p[i], p[i],
              p[i], 0, 1-p[i],
              1-p[i], p[i], 0),
            ncol = 3, byrow = T)

# Create a discrete time Markov Chain using the package "markovchain".
MC <- as(m, "markovchain")

# Determine invariant probabilities.
invariantProbabilities[i, ] <- steadyStates(MC)

}

# Assign rownames that match with p value.
rownames(invariantProbabilities) <- round(p, digits = 2)
# Assign column names that match with vertices.
colnames(invariantProbabilities) <- c(1, 2, 3)

# Show invariant probabilities for all values of p.
kable(invariantProbabilities)
```

	1	2	3
0.51	0.3333333	0.3333333	0.3333333
0.79	0.3333333	0.3333333	0.3333333
0.45	0.3333333	0.3333333	0.3333333
0.24	0.3333333	0.3333333	0.3333333
0.1	0.3333333	0.3333333	0.3333333
0.04	0.3333333	0.3333333	0.3333333
0.99	0.3333333	0.3333333	0.3333333
0.68	0.3333333	0.3333333	0.3333333
0.68	0.3333333	0.3333333	0.3333333
0.73	0.3333333	0.3333333	0.3333333

The invariant probabilities are $\pi = \{1/3, 1/3, 1/3\}$, regardless of the starting value of p.

d.

$$P(X_n = 1, X_{n+1} = 2) = \frac{1-p}{3}$$

$$P(X_n = 2, X_{n+1} = 1) = \frac{p}{3}$$