

# Detection and Location of Power Quality Disturbances Based on Mathematical Morphology and Hilbert-Huang Transform

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**Abstract** – With the power system development, the application of non-linear power components is becoming more and more extensive, and the power quality situation of power system has become increasingly severe. This paper proposes a novel method based on mathematical morphology (MM) and Hilbert-Huang transform (HHT), which is used to detect and analyze power quality disturbances. To effectively suppress noises, the traditional (Maragos) mathematical morphological filter is improved, so the main characteristics of original signal are reserved. And, the de-noising signal is processed by empirical mode decomposition (EMD) to obtain a set of intrinsic mode function (IMF) components. Then, using Hilbert transform to every IMF component, the instantaneous amplitude and frequency of the components can be accurately computed. Simulation results show that this method is feasible and effective and can accurately determine time, amplitude and frequency of the power quality disturbances.

**Keywords** – mathematical morphology, Hilbert-Huang transform, morphological filter, empirical mode decomposition, power quality disturbances.

## I. INTRODUCTION

The main problem of power quality includes harmonic distortion and transient phenomena while power system fault and switching operations, such as voltage sag, voltage swell, voltage interruption, transient oscillation, transient pulse, etc.

The commonly used methods include Fourier transform (FT), wavelet transform (WT), neural network and so on. FT has a range of advantages, for example, easy calculation, wide application and reliable computing. So it is widely used, but there is spectral leakage and fence effect. The windows and interpolation can effectively suppress leakage phenomenon and fence effect, but it greatly reduced the resolution, and also practical applications can not use unlimited windows. WT can accurately detect a signal of singular points and the location of the disturbance, but its calculation is very complex. When using WT, the type of the basic wavelet function employed directly affects the effectiveness in identifying transient elements hidden within the dynamic signal [1].

Hilbert-Huang transform (HHT) recently is a new method to analyze the stationary and non-stationary signal [2]. HHT represents the signal being analyzed in

the time-frequency domain by combining the empirical mode decomposition (EMD) with Hilbert transform. Comparison with the above-mentioned two methods, this method is based on the instantaneous frequencies resulting from the intrinsic mode function components of the signal being analyzed; thus, it is not constrained by the uncertainty limitations with respect to the time and frequency resolutions to which other techniques are subject.

Mathematical Morphology (MM) is developed from set theory and integral geometry, and is concerned with the shape of a signal waveform in the complete time domain rather than the frequency domain. MM can be developed as an efficient non-linear filter. Reference [3] has presented a morphological filter with the average combination of opening-closing and closing-opening filtering, to filter white noise and impulse noise, respectively. Using morphological operators, the radical shape of the disturbed signal can be recognized, even if the original signal is mixed with strong noise or serious distortion. In addition, MM has the feature of easy calculation and implementation.

In actual processing, the sampling signal is often mixed with different kinds of noises due to sampling errors, random disturbances and other unstable factors. When there exists great deal of noise mixed with disturbance singular points in the sampling signal, it may bring large errors while distinguishing the singular points caused by power quality disturbances. How to extract transient information from sampling signals with random noise has vital role in analyzing power quality disturbances [4].

This paper proposes a novel method, which is based on mathematical morphology (MM) and Hilbert-Huang transform (HHT), to detect and analyze power quality disturbances. To effectively suppress kinds of noises, the traditional (Maragos) mathematical morphological filter is improved, so the main characteristics of original signal are reserved. And, the de-noising signal is processed by empirical mode decomposition (EMD) to obtain a set of intrinsic mode function (IMF) components. Then, using the Hilbert transform to every IMF component, the instantaneous amplitude and frequency of the component can be accurately determined.

## II. MATHEMATICAL MORPHOLOGY

### A. Mathematical morphology theory

Based on the research achievement of integral geometry, G. Matheron and J. Serra founded MM theory in 1964. Its basic idea is to use a probe, which is named as structuring element (SE), to collect information of the signals. As the SE moving in the signal constantly, it can review the interrelation among every part, and pick up useful information to analyze and describe the signals. Because the power system signal analysis commonly deals with one-dimensional signal, this paper only introduces erosion, dilation, opening, closing and the compound opening and closing, under the one-dimensional discrete condition [5] [6].

Suppose that the definition domain of the input signal  $f(n)$  and the structuring element  $g(n)$  is  $F=\{0,1,\dots,N-1\}$  and  $G=\{0,1,\dots,M-1\}$ , respectively, and  $N \geq M$ .

The dilation and erosion of  $f(n)$  by  $g(n)$  are defined as

$$(f \oplus g)(n) = \max[f(n-m) + g(m)] \quad (1)$$

$$(m = 0 \sim M-1)$$

$$(f \ominus g)(n) = \min[f(n+m) - g(m)] \quad (2)$$

$$(m = 0 \sim M-1)$$

Under normal circumstances, the expansion and corrosion are not reversible each other. So the conjugation of them can form new morphological operations, named by opening and closing, defined as follows.

The opening and closing of  $f(n)$  by  $g(n)$  are defined as

$$(f \circ g)(n) = (f \ominus g \oplus g)(n) \quad (3)$$

$$(f \bullet g)(n) = (f \oplus g \ominus g)(n) \quad (4)$$

Generally speaking, opening operation is used to eliminate scrap points, sparks and "little bridges", or say smoothing the image. And closing operator is used to stuff "little holes" or connect the contiguous two regions. In order to smooth and restrain positive and negative impulse noise, the compound opening and closing is used; opening-closing and closing-opening are defined as

$$f_{oc}(n) = (f \circ g \bullet g)(n) \quad (5)$$

$$f_{co}(n) = (f \bullet g \circ g)(n) \quad (6)$$

### B. Construction of morphological filter

The opening-closing and closing-opening filters have all the features of opening and closing operation. So

Maragos utilized the average value of these two filters. It can approach the original signal very well. This kind of combination filter is widely used in image process and signal process, defined as

$$f_{nm} = [f_{oc}(n) + f_{co}(n)]/2 \quad (7)$$

## III. HILBERT-HUANG TRANSFORM THEORY

Hilbert-Huang Transform (HHT) is introduced by Dr. Norden Huang in the GSFC Laboratory in 1998. There are basically two processes in the algorithm of Hilbert-Huang Transform: the empirical mode decomposition and the Hilbert Transform.

### A. Empirical Mode Decomposition

The empirical mode composition (EMD) method will generate a collection of intrinsic mode function (IMF) components, which are data series with different time scales. The IMF components must satisfy the following two conditions [7]:

1. In the entire length of the data, the number of the extreme point and zero point must be equal or differ by one at most.

2. In any data points, mean of the envelope of local maximum value and the envelope of local minimum value must be zero.

The specific decomposition process of EMD is [8]:

(1) Figure out all the local extreme points of the signal  $s(t)$ .

(2) Made the upper envelope  $v_1(t)$  of all maximum values and the lower envelope  $v_2(t)$  of all minimum values.

(3) Then, get the mean of upper envelope and lower one:  $m_{11}(t) = [v_1(t) + v_2(t)]/2$

Following, a new data series can be got through subtracting the average envelope from the original data series:  $h_{11}(t) = s(t) - m_{11}(t)$ .

Judge whether  $h_{11}(t)$  is IMF, if not, regard  $h_{11}(t)$  as the new  $s(t)$ , repeat the process listed above, until  $h_{11}(t)$  meets IMF conditions, write,  $c_1(t) = h_{11}(t)$ , look it as IMF1.

(4) Look  $r_1(t) = s(t) - c_1(t)$  as a new  $s(t)$ , repeat step (1) ~ (4) listed above, find IMF2, denoted as  $c_2(t)$ , then, look  $r_2(t) = r_1(t) - c_2(t)$ , repeat the steps above until  $r_n(t)$  is becoming a monotonous signal or has only one pole.

Finally, the original data series  $s(t)$  can be expressed by these IMF components and a trends term  $r_n(t)$ :

$$s(t) = \sum_{i=1}^n c_i(t) + r_n(t) \quad (8)$$

### B. Hilbert transform

Suppose that the signal  $X(t)$ , its Hilbert transform can be defined as

$$Y(t) = \frac{1}{\pi} \int_{-\infty}^{+\infty} \frac{X(\tau)}{t - \tau} d\tau \quad (9)$$

Its anti-transform is:

$$X(t) = \frac{1}{\pi} \int_{-\infty}^{+\infty} \frac{Y(\tau)}{\tau - t} d\tau \quad (10)$$

$X(t)$  and  $Y(t)$  form a plural conjugate right, its analytical signal is:

$$Z(t) = X(t) + jY(t) = a(t)e^{j\theta(t)} \quad (11)$$

In this formula:

$$a(t) = \sqrt{X^2(t) + Y^2(t)} \quad (12)$$

$$\theta(t) = \arctan[Y(t)/X(t)] \quad (13)$$

The instantaneous frequency can be defined as:

$$f(t) = \frac{1}{2\pi} \frac{d\theta(t)}{dt} \quad (14)$$

#### IV. IMPROVED MORPHOLOGICAL FILTER

##### A. Selection of structuring element

The selection of structuring element (SE) is the key of morphological filter, because SE is used to extract the similar geometric characters in the signal, and at the same time ignore the irrelevant features. The structuring elements have line, curve (such as sine, cosine, second and third, etc.), triangle, circle, cross and other polygon (such as rectangular, diamond, hexagonal, etc.). In the power system, SE which is often used is line, flat, semi-circular, triangle as well as square [4].

##### B. Improved morphological filter

The opening-closing and closing-opening filters can filter positive and negative impulsive noise together, but statistic bias is existed. So it is difficult to filter out all kinds of noises. To some extent, Maragos filter can approach the original signal very well, but it only uses the same type of SE to carry out morphological transform. Only adopting one SE will not get the best filter effect in cases where there is more than one kind of noise in the signal.

In order to improve existing methods, this paper constructs a kind of compound filter with two structuring elements. Instead of formula (5), (6), the new formula is defined as

$$F_{oc}(n) = (f \circ g_1 \bullet g_2)(n) \quad (15)$$

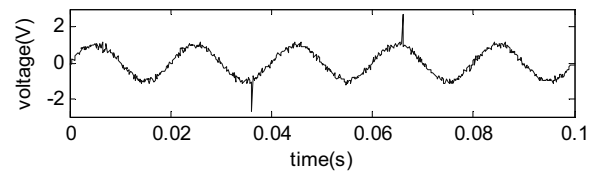
$$F_{co}(n) = (f \bullet g_1 \circ g_2)(n) \quad (16)$$

In this formula,  $g_1$  and  $g_2$  is different form of structuring element respectively. The improved morphological filter is:

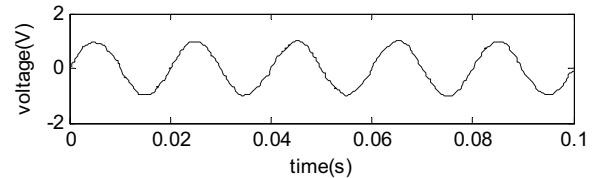
$$F_{mm} = [F_{oc}(n) + F_{co}(n)]/2 \quad (17)$$

##### C. Filter simulation

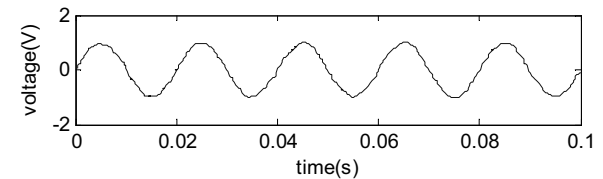
To inspect the de-noising capacity of the improved morphological filter, this paper will use white noise and impulse noise to simulate. The original signal is:  $s(t) = \sin(2\pi \times 50 \times t)$ , and construct a signal with white noise of variance 0.1 and positive and negative impulse noise with the magnitude 2pu. Now sampling frequency is 6400Hz (128 sampling points per period). The structuring elements are semi-circular with the length 9 and radius 0.05 and triangle with the length 13. The results of the simulation are shown in Fig.1 and Table 1. It is seen that the improved morphological filter is more effective than the Maragos morphological filter.



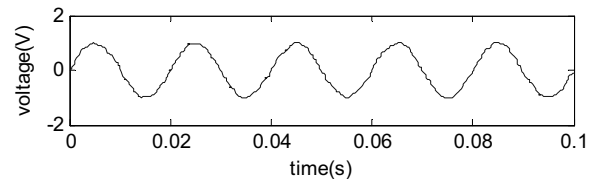
(a) Noisy signal.



(b) De-noising signal (improved morphological filter).



(c) De-noising signal (Maragos filter using triangle SE).



(d) De-noising signal (Maragos filter using semi-circular SE).

Fig.1 Results of the simulation.

Table 1 SNR of De-noising signal.

| Index  | Noisy signal | semi-circular SE | triangle SE | this method |
|--------|--------------|------------------|-------------|-------------|
| SNR/dB | 13.7957      | 25.1502          | 25.1987     | 25.9088     |

## V. ANALYSIS OF POWER QUALITY DISTURBANCES

### A. Analysis of Voltage swell, Voltage sag and Voltage interruption using HHT

Voltage sag is a decrease to between 0.1 to 0.9pu in rms voltage at the power frequency for duration of 0.5 cycles to 1 minute [9]. In the simulation, construct voltage sag signal with white noise of variance 0.1 and positive and negative impulse noise with the magnitude 2pu. Now sampling frequency is 6400Hz. The setting beginning time and ending time that the disturbance occurs is 0.12 second and 0.32 second, the magnitude is 0.5pu. The structuring elements are semi-circular with the length 9 and radius 0.05 and triangle with the length 13. Noisy signal and de-noising signal are shown in Fig.2 and Fig.3 respectively. With HHT, the amplitude and frequency of the de-noising signal is shown in Fig.4.

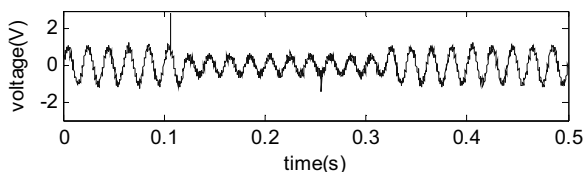


Fig.2 Noisy signal (voltage sag).

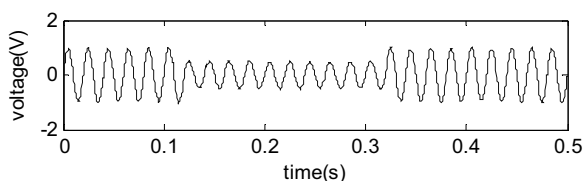


Fig.3 De-noising signal (voltage sag).

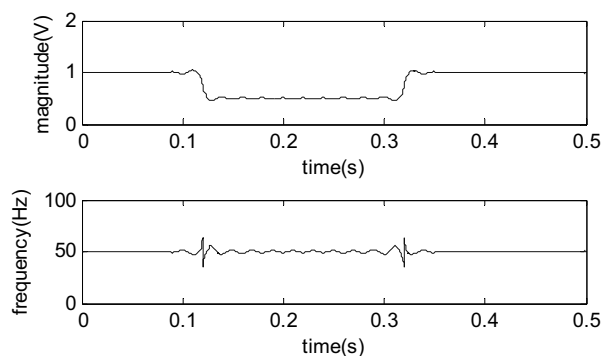


Fig.4 Amplitude and frequency of signal using HHT.

Voltage swell is defined as an increase in rms voltage at the power frequency for durations from 0.5 cycles to 1 minute. Typical magnitudes are between 1.1 and 1.8pu. Voltage interruption occurs when the supply voltage or current decreases to less than 0.1pu for a period of time not exceeding 1 minute [9]. Using the same method, the time-frequency characteristics of Voltage swell and Voltage interruption can be detected.

Noisy Voltage swell signal and de-noising signal are shown in Fig.5 and Fig.6 respectively. With HHT, the amplitude and frequency of the de-noising signal is shown in Fig.7. Noisy Voltage interruption signal and de-noising signal are shown in Fig.8 and Fig.9 respectively. With HHT, the amplitude and frequency of the de-noising signal is shown in Fig.10.

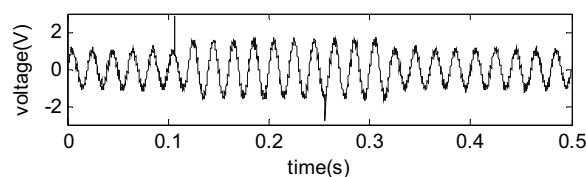


Fig.5 Noisy signal (voltage swell).

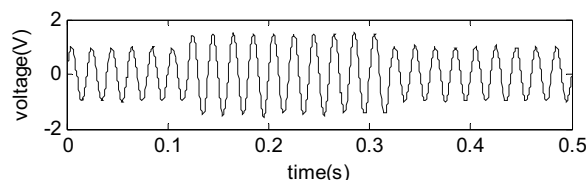


Fig.6 De-noising signal (voltage swell).

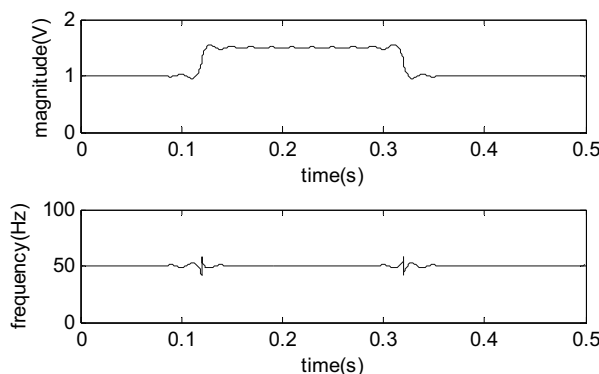


Fig.7 Amplitude and frequency of signal using HHT.

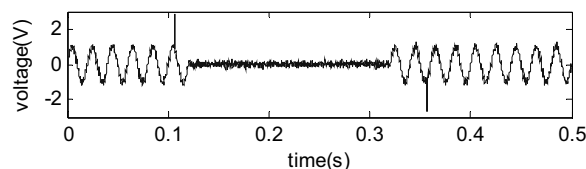


Fig.8 Noisy signal (voltage interruption).

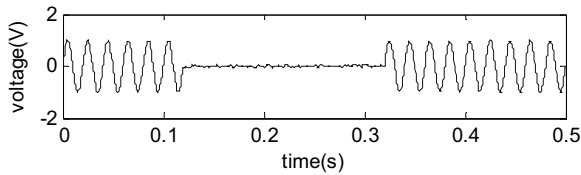


Fig.9 De-noising signal (voltage interruption).

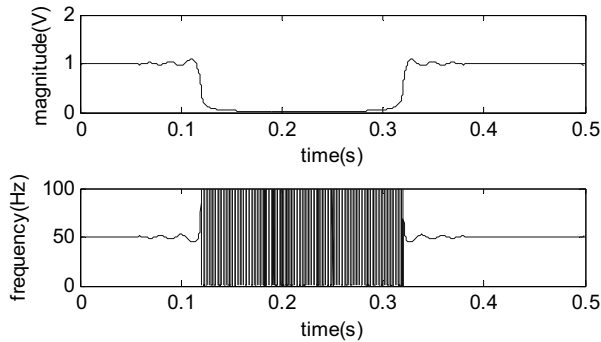


Fig.10 Amplitude and frequency of signal using HHT.

The amplitudes of the disturbances are changed at the beginning and ending time and the frequencies of the ones have high-frequency mutation at the beginning and ending time. For Figure 10, because the amplitude of Voltage interruption is 0pu, with HHT, the real and imaginary part of the obtained analytical signal is infinitesimal and the calculated frequency is infinite. The results of detection and location of disturbances are shown in Table 2.

Table 2 Time location of disturbances types.

| Disturbance signal   | Beginning time |               | Ending time  |               |
|----------------------|----------------|---------------|--------------|---------------|
|                      | Setting time   | Measured time | Setting time | Measured time |
| Voltage sag          | 0.12           | 0.1198        | 0.32         | 0.32          |
| Voltage swell        | 0.12           | 0.1203        | 0.32         | 0.3202        |
| Voltage interruption | 0.12           | 0.1202        | 0.32         | 0.3198        |

According to amplitude change, the types of disturbance signals are recognized, as well as, according to the mutation frequency, the beginning and ending time that the disturbances occur are determined.

#### B. Analysis of harmonic signal using HHT

The original harmonic signal, shown in Fig.11, is:

$$s(t) = \begin{cases} 2\sin 100\pi t + \sin 300\pi t & (0 \leq t \leq 0.2) \\ 2\sin 100\pi t + 0.5\sin 460\pi t & (0.2 \leq t \leq 0.5) \end{cases}$$

The sampling frequency is 6400Hz. In the cases, the signal, adding different frequencies of harmonic at different time deliberately, is shown in Fig.12. The structuring elements are semi-circular with the length 5

and radius 0.05 and triangle with the length 7. The de-noising signal is shown in Fig.13. Fig.14 is the results of EMD.

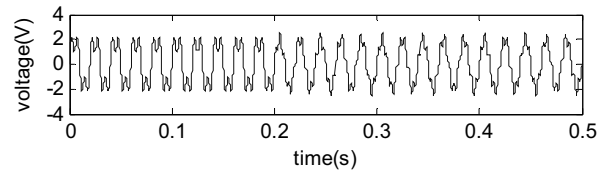


Fig.11 Original signal (harmonics).

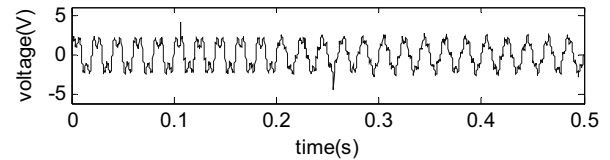


Fig.12 Noisy signal (harmonics).

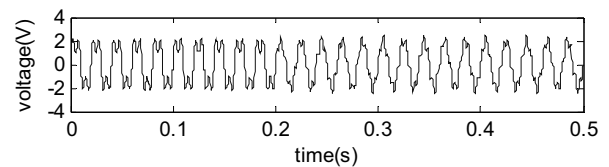


Fig.13 De-noising signal (harmonics).

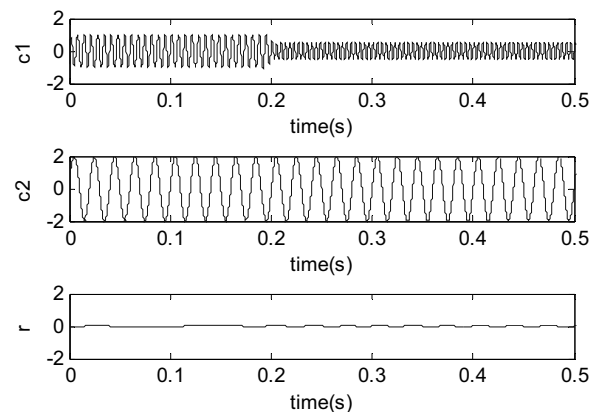


Fig.14 Results of EMD (IMF components).

According to Fig.14, c1 includes two harmonic frequencies, and c2 is fundamental. The harmonics and fundamental are separated from the signal according to the results of decomposition. Then, using Hilbert transform to every IMF component, the instantaneous amplitude and frequency of these components can be accurately obtained, shown in Fig.15 and Fig.16.

In the Fig.15, It is seen that the amplitude and frequency of every harmonic is identical to the original signal. The frequencies of c1 component include 150Hz and 230Hz, and the boundary of these two frequencies is clearly shown. The results of HHT analysis are shown in Table 3.

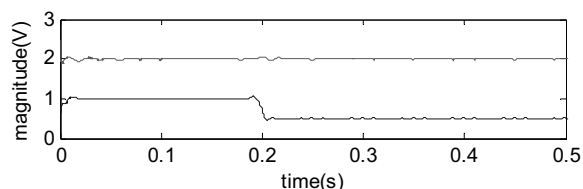


Fig.15 Amplitude of every IMF component.

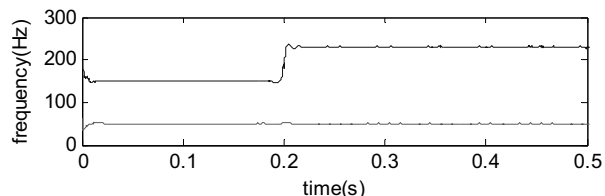


Fig.16 Frequency of every IMF component.

Table 3 Results of HHT analysis.

|              |        |          |            |
|--------------|--------|----------|------------|
| Amplitude/V  | 1.9991 | 0.9942   | 0.4987     |
| Frequency/Hz | 49.88  | 150.43   | 230.56     |
| Time/s       | 0~0.5  | 0~0.2012 | 0.2014~0.5 |

Obviously, HHT can accurately determine the amplitude and frequency of the harmonics and inter-harmonics, also the beginning, ending and mutation time that harmonics occur.

## VI. CONCLUSIONS

1) Digital simulation results show that this novel method can determine the beginning and ending time that the disturbances occur, the components of harmonics and also the beginning and ending time that harmonics occur. The improved morphological filter is better than the traditional (Maragos) filter, and it is fit to the situation of the signal with white noise and impulse noise.

2) For the detection of harmonic simulation, this paper puts forward some different frequency harmonics, and the moment of their occurrence is different. The simulation results show that HHT not only can accurately detect harmonic, but also can locate the beginning and ending time that every harmonic occurs.

3) The purpose of this paper is to solve the difficulty of filtering random noise, impulse noise and white noise in power quality disturbances detection as well as the complexity and time-consuming using the conventional methods to detect the singularity of disturbance signals and classify their types.

## ACKNOWLEDGMENT

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