Models for synchrophasor with step discontinuities in magnitude and phase, their parameter estimation and performance

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***Abstract* — This work proposes an alternative method to assess the calibration of phasor measurements units (PMUs) under conditions of step discontinuities in magnitude or phase. Two parametric mathematical models are used to represent the signal and an iterative numerical method is used to best fit the model parameters to the samples. This approach avoids the adjustment of time windows around the instant of the discontinuity. The estimated parameters can be used to calculate a reference phasor with an appropriate definition. Expand a little more**

Frequency estimation has an important role

***Index Terms* — Calibration, dynamic tests, phasor measurement units, synchrophasor, uncertainty.**

I. Introduction

The dynamic behavior of modern electric grid demands testing the performance of phasor measurement units (PMUs) under magnitude steps and phase steps [1]. The accuracy of those measurements depends on the reference values provided by PMU calibration systems, obtained by synchronously generating and sampling standard test signals. Recent developments towards the calibration of PMUs for distribution grids demand even lower uncertainty levels [2]. The calibration process depends on generating and sampling synchronized waveforms and estimating phasors from the acquired waveform, which are used as reference values to be compared to the values provided by the PMU under test. The phasor which phase is related to an ideal cosine function with maximum centered in the Universal Time Code (UTC) second is called a synchrophasor. PMUs also provide estimations of frequency and rate of change of frequency (ROCOF) at a given report rate.

For the estimation of synchrophasors, a stationary phasor waveform can be curve fitted with a steady state sinusoidal function with good accuracy. Methods to estimate parameters of low variating signals for PMU calibrations are also presented in [2] and [3]. Variations in parameters or nonlinearities are modeled by low order Taylor series expansion, followed by iterative procedures.

However, in the specific case of an observed phasor disturbed by a step discontinuity in magnitude or phase, the estimation using an underlying steady state model is inappropriate and does not guarantee convergence nor accuracy. Besides, there is a lack of definition of what the reference phasor should be. To overcome this difficulty, the method used in [3] adjusts the timestamp and position of the analysis window to skip the discontinuity and set the phasor estimates where the discontinuity occurs with those of obtained from the previous or following window. That way, it avoids the mathematical modelling of a step discontinuity and considers the reference value coming from the steady state.

The mentioned procedures are designed to calibrate PMUs, and may be not detailed enough to evaluate the performance of calibration systems. Methods for a more detailed analysis of calibration systems under step conditions are proposed in [4], using a pointwise root mean squared error for the performance evaluation of estimation algorithms.

Although not trying to break the stationary paradigm of the phasor representation of a signal, we propose an alternative approach to evaluate the measurements of PMU calibration systems. For that, this extended version of [5] offers the following contributions:

1) Proposition of single intermediary phasor for transient situations, which can be used as reference values for calibrations and easily implemented in the existing systems, in place of traditional estimation schemes. It depends on the estimation of parameters of an underlying model. Thus, 2 is proposed.

2) Synchrophasor models that account for step discontinuities in magnitude or phase. Signals with similar behavior can digitally sampled and submitted to numerical methods to estimate the parameters which best fit the signal to the appropriate model. That can be done by 3) and 4).

This paragraph of the first footnote will contain the date on which you submitted your paper for review. It will also contain support information, including sponsor and financial support acknowledgment. For example, "This work was supported in part by the U.S. Department of Commerce under Grant BS123456."

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3) A method to estimate the instant of step discontinuity using the instantaneous frequency estimation provided by the Hilbert transform of a signal;

4) A method to estimate the other parameters of the models proposed in 2), provided the step instant estimation obtained in 3), by means of a nonlinear least-square method (NL-LS).

5) A preliminary evaluation of a laboratory system intended to be a PMU calibrator based on the methods exposed.

In order to analyze the contribution to the uncertainty of calibration systems, we made simulations to obtain the numerical errors of each numerical method. The paper is organized in the following sections: II) mathematical background, where are presented the proposed models for dynamic signals with magnitude or phase steps, the intermediary phasor definitions, the basic concepts of the Hilbert transform and of the NL-LS method; III) Monte Carlo simulations to analyze the numerical errors of each method; IV) Laboratory measurements to evaluate the use of a proposed system for the calibration of PMUs; V) Discussion of results and conclusion.

II. Mathematical Background

A. Mathematical models for dynamic signals

A pure sinusoidal waveform with one magnitude step, located at $t = \tau$ can be modeled in continuous time

$$\hat{y}(t) = x_1[1 + x_2 u(t-\tau)]\cos(\omega t + \varphi) + \eta(t), \quad (1)$$

where $u(t)$ is the step function. A similar model for the phasor waveform with one phase step is

$$\hat{y}(t) = x_1 \cos\big(\omega t + \varphi + x_3 u(t-\tau)\big) + \eta(t). \quad (2)$$

The step function $u(t-\tau)$ is used as an idealization of a fast transient in magnitude or phase occurring at the instant τ. Provided a sufficiently accurate estimate of τ, the set of parameters $\mathcal{P} = \{x_1, x_2, x_3, \omega, \varphi\}$ can then be adjusted to obtain a waveform that best fits the data received by the calibration system sampler, where x_1 is the signal nominal magnitude, x_2 is a decimal value representing the magnitude change, x_3 is the amplitude of the phase step, ω is the angular frequency, φ is the initial phase, and $\eta(t)$ represents interfering noise. Given a prescribed signal to noise ratio (SNR) in dB, the amplitude of noise is

$$\eta_0 = (x_1 \;/\; 10^{SNR/20}). \quad (3)$$

B. Reference values

After one estimates the model parameters, the problem of obtaining one phasor that represents the waveform arises. Instead of considering the values estimated from the adjacent windows, one alternative proposal could be an intermediate value for magnitude or phase. The concept is illustrated in Fig. 1, where the phasor V_1 represents the waveform during an initial steady state, V_e is a phasor representative of an intermediate state during the occurrence of a magnitude or phase step, and V_2 represents the signal in the final steady state condition.

Intermediate phasors can be obtained, for example, using a weighted means out of the model parameters. For any $\tau \in [0, T]$, for a waveform with magnitude step described by (1),

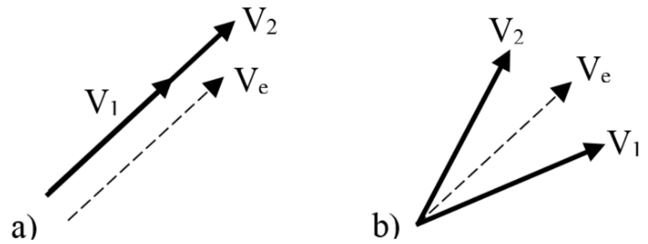


Fig. 1 - Transitioning phasors for a) magnitude step, b) phase step

the intermediate phasor would be

$$V_e = X_e \angle\varphi = \frac{\hat{x}_1\hat{\tau} + \hat{x}_1(1+\hat{x}_2)(T-\hat{\tau})}{T}\angle\hat{\varphi}; \quad (5)$$

and for a waveform with a phase step test described by (2), the intermediate phasor would be

$$V_e = X\angle\varphi_e = \hat{X}\angle\frac{\hat{\varphi}\hat{\tau} + (\hat{\varphi}+\hat{x}_3)(T-\hat{\tau})}{T}. \quad (6)$$

C. Hilbert transform

Hilbert transform has been used to estimate instantaneous frequency (IF) of narrowband monocomponent signals, which is the case of electric network phasor components. There are various applications of IF estimation reported in the literature, e.g., characterization of electric disturbances [6] and detection of edits in audio signals that bear the electric network frequency [7]. Anomalous perturbations on the IF can flag the occurrence of discontinuities in the signal. The time instant they happened can be estimated via appropriate amplitude threshold schemes. The concept is shown in Fig. 2 and Fig. 3.

Given a real signal $x(t)$, $-\infty < t < \infty$, let $z(t)$ be called the analytic signal associated to $x(t)$, defined as

$$z(t) = x(t) + j\tilde{x}(t), \quad (7)$$

where

$$\tilde{x}(t) = H[x(t)] = \int_{-\infty}^{\infty}\frac{x(u)}{\pi(t-u)}dt, \quad (8)$$

is the Hilbert transform of $x(t)$. If $z(t)$ is expressed in the polar form

$$z(t) = A(t)e^{j\theta(t)}, \quad (9)$$

$$A(t) = \sqrt{x^2(t) + \tilde{x}^2(t)}, \quad (10)$$

$$\theta(t) = \tan^{-1}(x(t)/\tilde{x}(t)), \quad (11)$$

the instantaneous frequency (IF) can be defined as

$$f_i = \frac{1}{2\pi}\left(\frac{d\theta(t)}{dt}\right). \quad (12)$$

The discrete implementation of $H[x(t))]$ can be done with fast fourier transform (FFT) … explain how

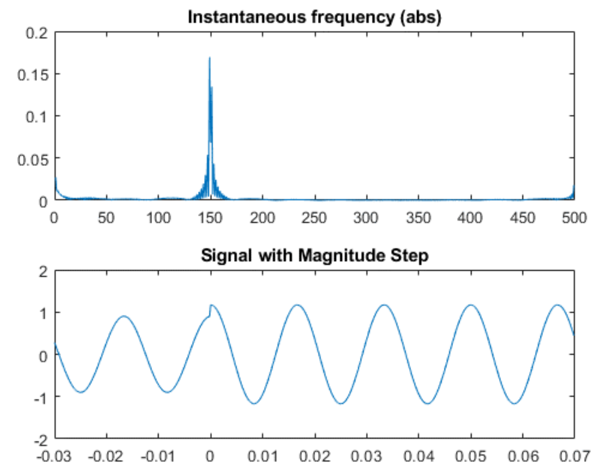


Fig. 2 - Instantaneous frequency of signal with one magnitude step

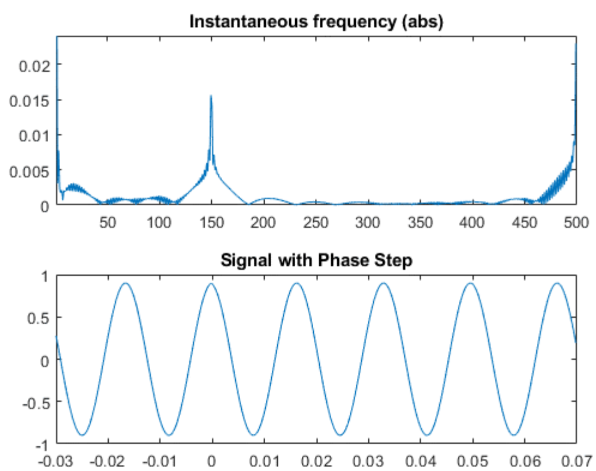


Fig. 3 - Instantaneous frequency of signal with one phase step

D. *Levenberg-Marquardt*

Consider N samples from a sequence $y(k)$, which can be either generated via computational simulation or sampled from measured real phenomenon, with uniform sampling period Δt. One wishes to fit the models (see Section II) with parameters $\mathcal{P}$ to $y(k)$. For that, one can define the error cost function

$$\varepsilon(\mathcal{P}) = \frac{1}{2}\sum_{k=1}^{N}\left(y(k) - \hat{y}(k\Delta t)\right)^2, \tag{4}$$

and try to solve the minimization problem $\min_{\mathcal{P}} \varepsilon(\mathcal{P})$.

The estimation of phasor parameters considering variations in frequency requires dealing with a non-linear function. Existing calibration systems solve this problem for steady-state signals through low order Taylor linearization [3], or using directly some non-linear minimization algorithm, e.g., Levenberg-Marquardt (LM) [4].

The Levenberg-Marquardt (LM) algorithm is an iterative technique for nonlinear minimization problems. It combines the Gauss-Newton method and the steepest descent and is very useful when the size of the algorithm step cannot be obtained in a closed form. Such NL-LS methods can reach local minima and need a convex cost function.

III. Numerical Simulations

We performed Monte Carlo simulations with 1000 iterations to estimate the errors obtained with the numerical computation of the reference values. These errors will be considered as an uncertainty component in the overall process.

The input signals were digitally generated, with all nominal parameters prescribed in the standard and random values representative of expected uncertainties in each parameter. The signals were created based on equations 1 and 2 with a 5 kHz sampling frequency, with a duration of 0.1 s, containing 6 cycles of 60 Hz. The magnitude was normalized to 1 V, the magnitude step size was ±10% of the nominal, the phase step size was ±10°, and the phase was set to 0°, 120°, or -120°.

A. *Step instant estimation with Hilbert transform*

The peaks were detected taking the maximum value of the IF subtracted by its median value. If we use an ideal signal without uncertainties in the parameters and nominal frequency, for a total duration of the window T, $\tau \in [0.05T, 0.95T]$, and $SNR > 40dB$, the maximum absolute errors are not greater than 1 Δt.

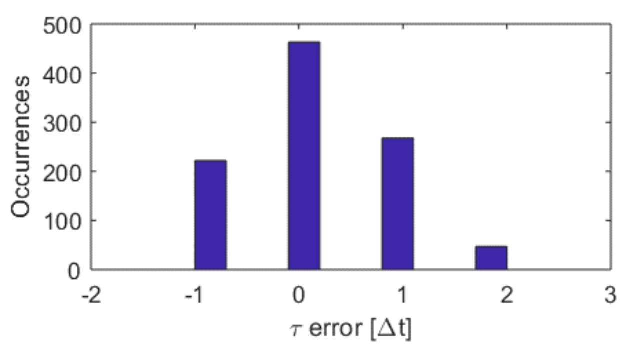


Fig. 4 - Histogram of errors in step instant estimation

In a second simulation to represent a more realistic situation, we allowed 500 ppm variation in frequency and 1% variation in the other parameters. The maximum errors obtained are not greater than 2 Δt for a $SNR > 65dB$. The distribution of occurrences is shown in a histogram in Fig. 4.

B. *Parameters estimation with non-linear least squares*

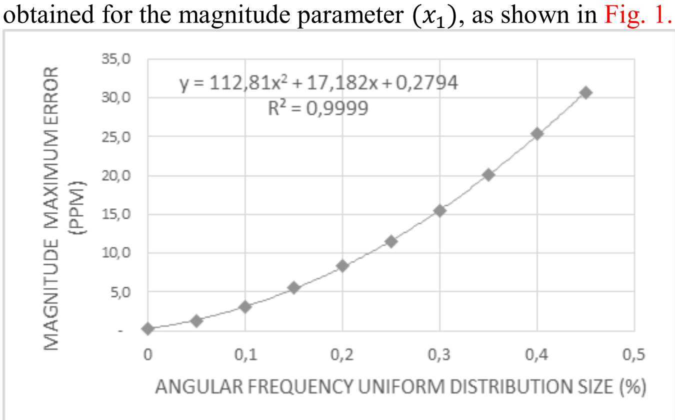
For this analysis, the step discontinuities occur in the middle of the window ($\tau = 0.5T$).

For each Monte Carlo iteration, $y(k)$ is generated with uncertainties added to the parameters, drawn from a uniform distribution centered around the nominal values, as shown in the upper part of Table I, where **d** is the interval of the uniform distribution, given in ppm. In the iterative LM algorithm, the model parameters are initiated at the nominal values, and the optimization procedure seeks for the minimum point of $\varepsilon(\mathcal{P})$, which is reached at the actual values of the parameters. In all simulations the sampling frequency was set to 4800 Hz, and the analysis window covers 6 cycles of the nominal phasor. The maximum errors obtained are shown in the lower parts of Table I. One can see that, for both tests, the estimation of the step discontinuity parameters are the most sensitive to the SNR. Moreover, they exhibit the highest maximum errors.

Table I- Parameter estimation maximum errors as a function of the SNR

Parameter	x_1	x_2	x_3	$\omega/2\pi$	φ
Nominal	100 V	0.1	10°	60 Hz	120°
d[ppm]	200	200	200	100	100
SNR	**Magnitude Step Tests maximum errors [ppm]**				
97	0.98	12	-	0.077	0.30
95	0.95	15	-	0.086	0.33
93.5	1.2	20	-	0.087	0.50
92.5	1.5	19	-	0.087	0.53
90.5	1.8	32	-	0.15	0.50
SNR	**Phase Step Tests maximum errors [ppm]**				
97	0.54	-	16	0.13	0.67
95	0.71	-	19	0.17	0.69
93.5	0.89	-	22	0.18	1.0
92.5	0.96	-	23	0.18	1.2
90.5	1.1	-	28	0.22	1.4

The uncertainties of some parameters seem to be correlated. For example, in the phase step test model, the uncertainty in the angular frequency (ω) estimation (all other parameters kept constant) has a quadratic relation with the maximum error obtained for the magnitude parameter (x_1), as shown in Fig. 1.



C. *Estimation of intermediate phasors*

With SNR of 90.5 dB, the uncertainties indicated in Table I, and also considering maximum errors of $\pm 2\Delta t$ in the estimation of τ. The maximum errors obtained were not greater than 2 ppm in every prescribed situation. Then, considering this value as the uncertainty contribution of the estimator is a conservative approach.

Histogram ?

IV. Laboratory Measurements

Aiming at validating the proposed method with real signals, several measurements were made using one digital sampling voltmeter (DSVM) and one arbitrary waveform generator (AWG), controlled by a personal computer (PC) via GPIB. The connections are shown in the block diagram of Fig. 6.

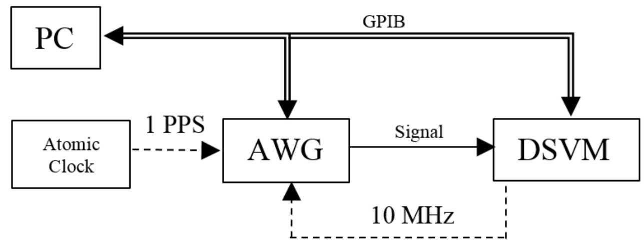


Fig. 5 - Block Diagram

The same waveforms used in simulations are reproduced by the AWG, with a nominal output of 1 V_{pp}, and sampled by the DSVM. Both are triggered with a 1 PPS (pulse per second) signal, coming from an atomic clock, so we can control the initial phase. The internal clock from the DVM is used as an external 10MHz reference signal by the generator. 5000 samples are taken during 1 s and stored in the DVM´s internal memory.

The standard [1] establishes that the synchrophasors must be obtained related to the center of a window. Setting 500 samples/window, the first complete window will happen after 250 samples, after which we have 9 windows containing 6 cycles of 60Hz. The steps of magnitude or phase occur in the 5^{th} window, as shown in . According to the procedure for equivalent sampling, the instants of occurrence of the steps are a set of equally spaced intervals $\tau_k = 0.1kT$, k = 1 ... 9.

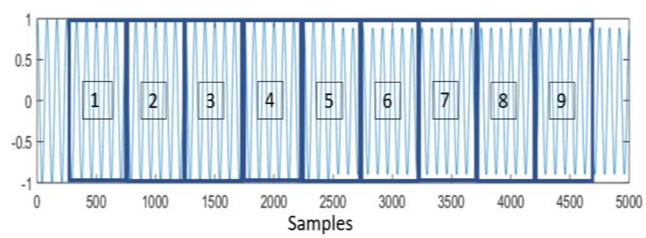


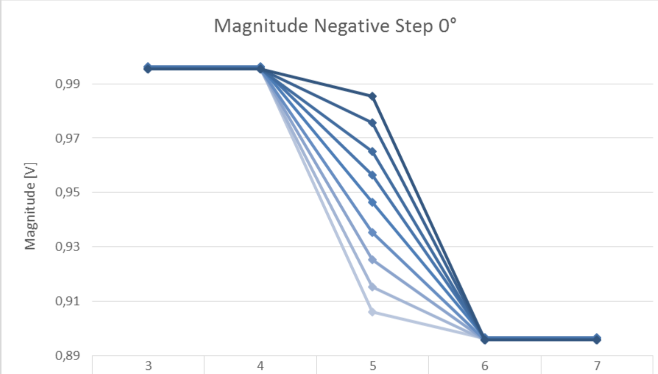
Fig. 6 - Magnitude step occurrence in the 5th window

The intermediate phasors for each waveform were calculated using (5) and (6), after obtaining the parameters using the Hilbert algorithm for the step instant, and the-LM algorithm for the others. The frequency was obtained directly from the LM estimation.

The estimates of step instant were not greater than $2\Delta t$, inside the expected uncertainty. The other parameters require a more detailed analysis.

Histogram?

A. Magnitude Steps



	3	4	5	6	7
10%	0,9962	0,9961	0,9061	0,8964	0,8965
20%	0,9956	0,9955	0,9152	0,8958	0,8957
30%	0,9956	0,9956	0,9252	0,8958	0,8958
40%	0,9955	0,9956	0,9353	0,8958	0,8960
50%	0,9962	0,9962	0,9464	0,8965	0,8965
60%	0,9963	0,9963	0,9564	0,8966	0,8966
70%	0,9954	0,9955	0,9652	0,8958	0,8957
80%	0,9955	0,9955	0,9756	0,8958	0,8958
90%	0,9955	0,9955	0,9855	0,8959	0,8958

Timestamp

V. Conclusion

Models for phasor signals disturbed by magnitude and phase step discontinuity were proposed, in the context of assessment of PMU calibration systems in transient conditions. Estimation of the model parameters via a nonlinear least-squares method was outlined. The proposed approach tackles the estimation of the step discontinuities in the phasor signal observed within an analysis window, instead of dodging the problem. Moreover, single phasor parameters are proposed for transient conditions.

The estimation accuracy of each parameter was obtained under different noise conditions and uncertainties forced upon the model used to generate the test signals. Cross-correlated uncertainties were found, which point out to the need of a deeper investigation and the possibility of further improvements in the estimation performance.

Within the limits reported, the proposed method can give reliable and accurate results to be used in PMU calibration systems, avoiding procedures to adjust windows and time-stamps around the instant of step occurrence.

Acknowledgment

I would like to thank Allen Goldstein, for the valuable comments and guidelines.

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