Models for synchrophasor with step discontinuities in magnitude and phase: estimation and performance

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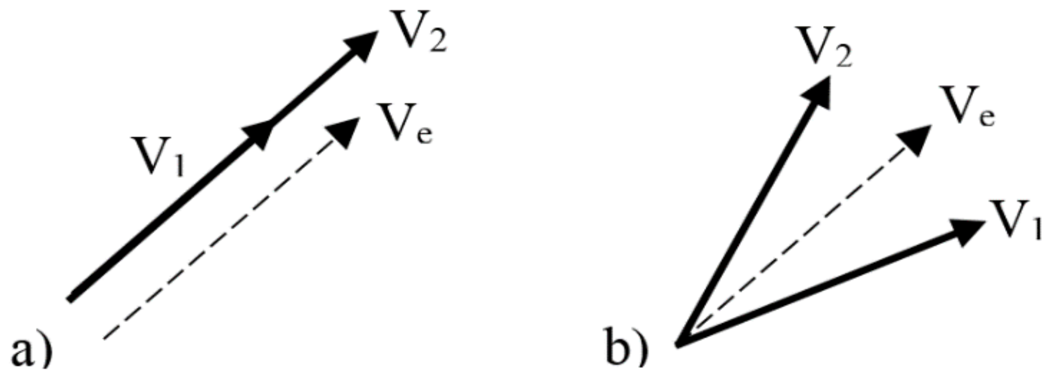
Summary

- Motivation
- Proposed mathematical models
  - Magnitude Step
  - Phase Step
- Estimation of model parameters from discrete-time signals
  - Hilbert transform
  - Non-linear least-squares estimator
- Numerical simulations: computation errors
- Lab prototype system assessment

Motivation

PMU calibrators do need to perform magnitude or phase tests.



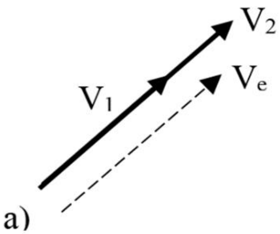
a) We should be able to measure frequency and phase
b) We should be able to measure frequency and magnitude

Moreover, V_e can be used as a reference value as an intermediate phasor.

Mathematical models

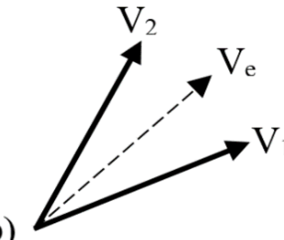
Magnitude Step

$$y(t) = x_1[1 + x_2 u(t-\tau)]\, cos(\omega t + \varphi) + \eta(t)$$

a)


Phase Step

$$y(t) = x_1 \cos\big(\omega t + \varphi + x_3\, u(t-\tau)\big) + \eta(t)$$

b)


Additive white gaussian noise

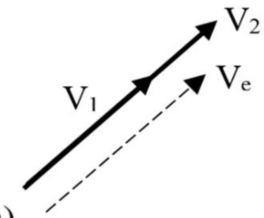
Noise variance

$$\eta_0 = \left(\frac{\sigma_y}{10^{\frac{SNR}{20}}}\right)^2,$$

Mathematical models

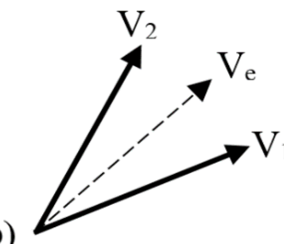
Magnitude Step

$$y(t) = x_1[1 + x_2 u(t-\tau)]\, cos(\omega t + \varphi) + \eta(t)$$

a) 

Phase Step

$$y(t) = x_1 \cos\big(\omega t + \varphi + x_3\, u(t-\tau)\big) + \eta(t)$$

b) 

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Estimation of Parameters

Magnitude Step

$$y(t) = x_1[1 + x_2 u(t - \tau)]\, cos(\omega t + \varphi) + \ \eta(t)$$

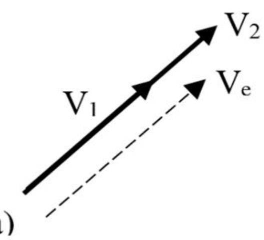
Phase Step

$$y(t) = x_1 \cos\big(\omega t + \varphi + x_3\, u(t - \tau)\big) + \ \eta(t)$$

Intermediate phasors

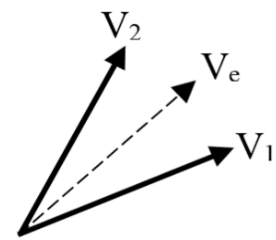
$$\hat{V}_e = \widehat{X_e}\angle\hat{\varphi} = \frac{\hat{x}_1\hat{\tau}+\hat{x}_1(1+\hat{x}_2)(T-\hat{\tau})}{T}\angle\hat{\varphi}$$

a)



$$\hat{V}_e = \hat{X}\angle\hat{\varphi}_e = \hat{X}\angle\frac{\hat{\varphi}\hat{\tau}+(\hat{\varphi}+\hat{x}_3)(T-\hat{\tau})}{T}$$

b)



Detection of Steps

Magnitude Step

$$y(t) = x_1[1 + x_2 u(t - \tau)]\, cos(\omega t + \varphi) + \eta(t)$$

Phase Step

$$y(t) = x_1 \cos\big(\omega t + \varphi + x_3\, u(t - \tau)\big) + \eta(t)$$

Hilbert Transform

Given a real narrowband monocomponent signal $x(t)$, $-\infty < t < \infty$, let $z(t)$ be called the analytic signal associated to $x(t)$, defined as

$$z(t) = x(t) + j\tilde{x}(t), \tag{7}$$

where

$$\tilde{x}(t) = H\big(x(t)\big) = \int_{-\infty}^{\infty} \frac{x(u)}{\pi(t-u)} du, \tag{8}$$

is the Hilbert transform of $x(t)$. If $z(t)$ is expressed in the polar form

$$z(t) = A(t)e^{j\theta(t)}, \tag{9}$$

$$A(t) = \sqrt{x^2(t) + \tilde{x}^2(t)}, \tag{10}$$

$$\theta(t) = \tan^{-1}(x(t)/\tilde{x}(t)), \tag{11}$$

the instantaneous frequency (IF) can be defined as

$$f_i(t) = \frac{1}{2\pi}\left(\frac{d\theta(t)}{dt}\right). \tag{12}$$

Detection of Transients

$$z[n] = x[n] + jH(x[n]) \qquad d[n] = |f_i[n]| - m(f_i[n])$$

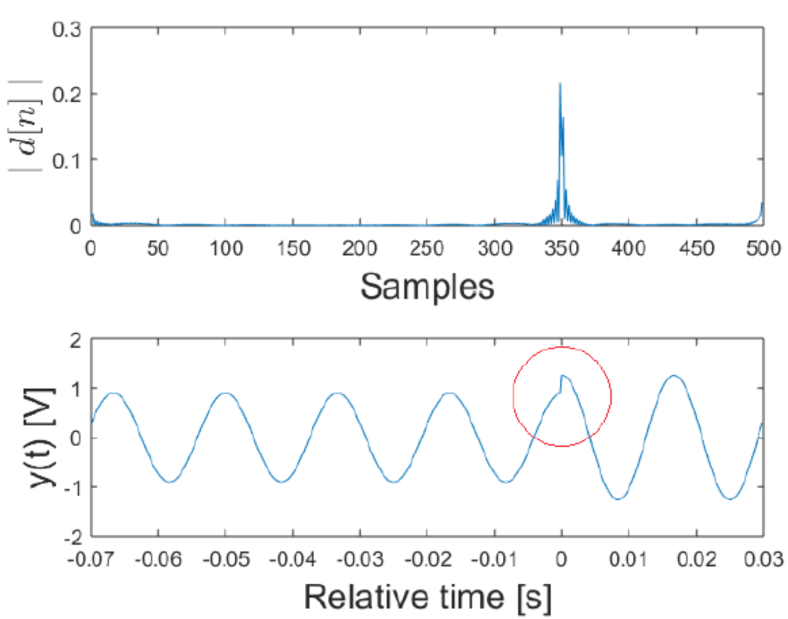


Fig. 2 Detection signal $d[n]$ (top plot) associated with a phasor waveform with magnitude step (bottom plot). (τ = 70% of window duration).

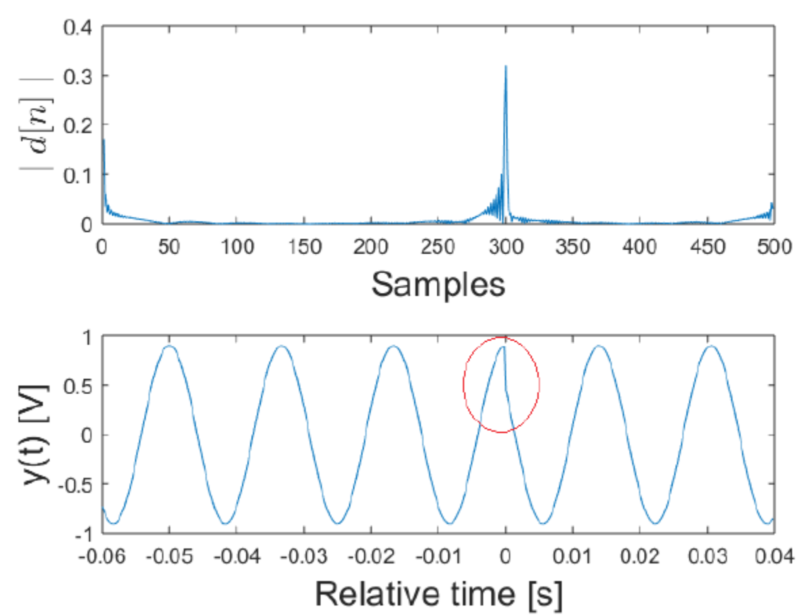


Fig. 3 Detection signal $d[n]$ (top plot) associated with a phasor waveform with phase step (bottom plot). (τ = 60% of window duration).

Estimation of Parameters

Magnitude Step

$$y(t) = x_1[1 + x_2 u(t-\tau)]\, cos(\omega t + \varphi) + \eta(t)$$

Phase Step

$$y(t) = x_1 \cos\left(\omega t + \varphi + x_3\, u(t-\tau)\right) + \eta(t)$$

$$\varepsilon(\mathcal{P}) = \frac{1}{2}\sum_{k=1}^{N}\left(y(k) - \hat{y}(k\Delta t)\right)^2$$

$$\min_{\mathcal{P}} \varepsilon(\mathcal{P})$$

- **Non-linear least-squares estimator: Levenberg-Marquardt**

Levenberg-Marquardt

- **Non-linear functions w.r.t parameters**
- **Iterative**
- **Gauss-Newton + steepest descent**
- **Numerical approximation of Jacobian**
- **Local minima, needs convex function**

Computation errors - HT

TABLE I

NOMINAL VALUES AND UNCERTAINTIES USED IN THE SIMULATIONS

Parameter	x_1	x_2	x_3	$\omega/2\pi$	φ
Nominal	1 V_p	± 0.1	± 10°	60 Hz	360°, ±120°
U[%]	1	1	1	0.05	1

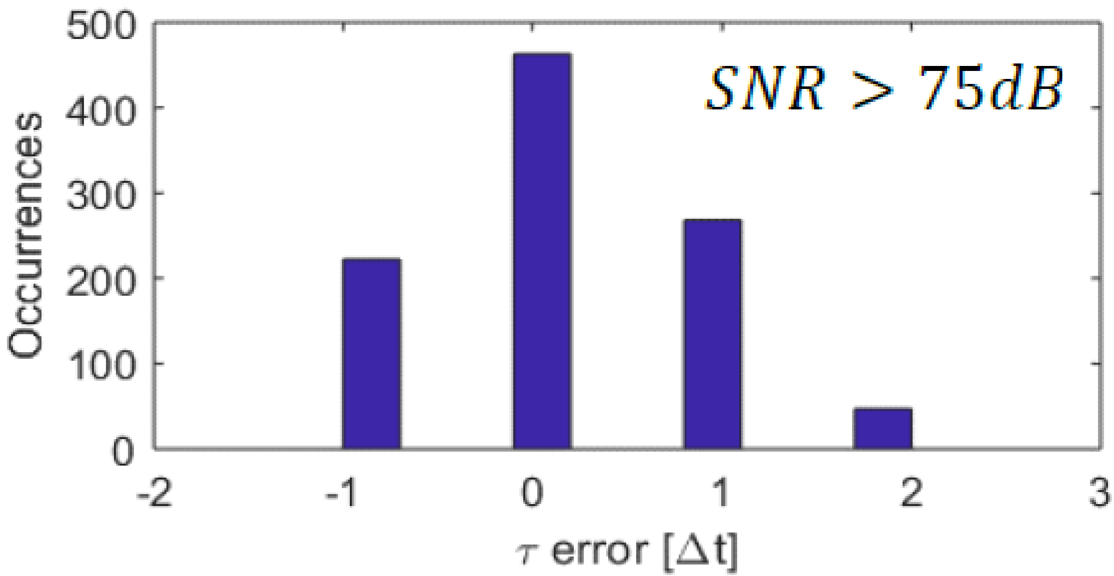


Fig. 4 Histogram of errors in step instant estimation.

Computation errors LM

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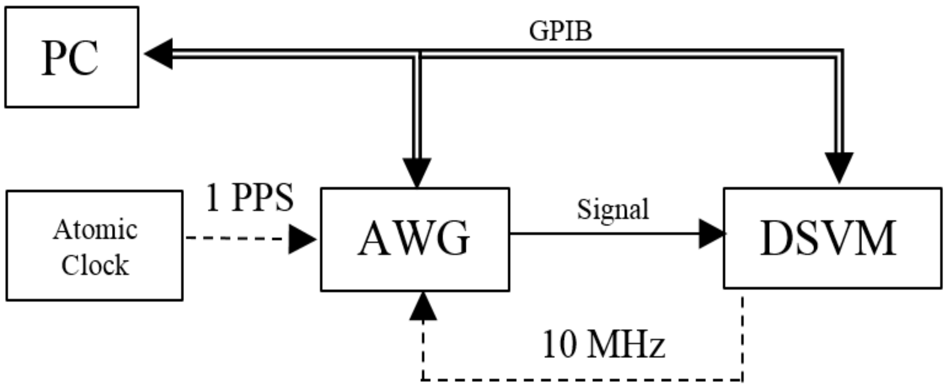
TABLE II
STANDARD DEVIATION OF NUMERICAL ERRORS FOR MAGNITUDE STEPS

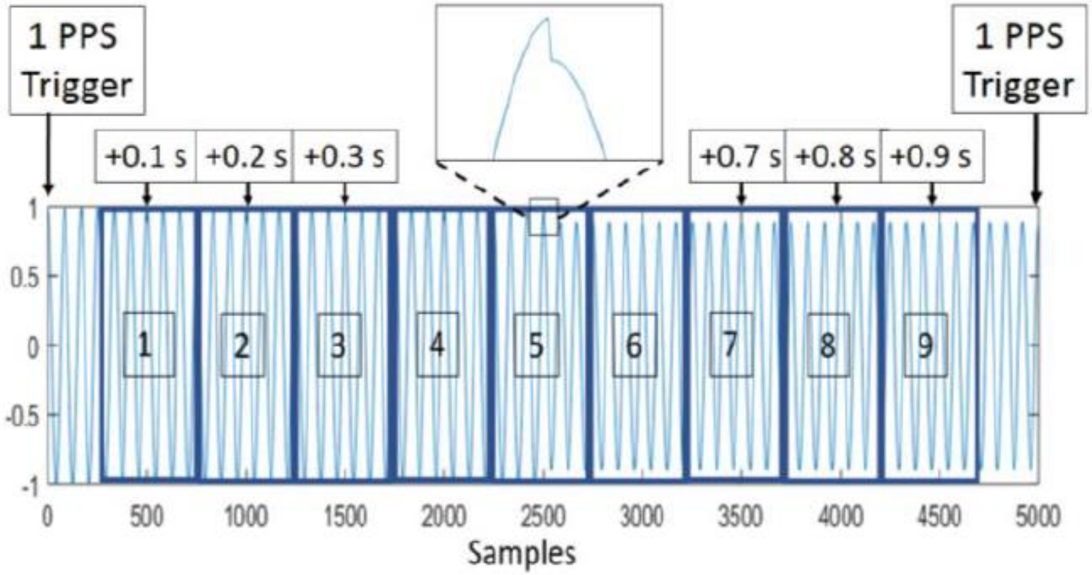
SNR [dB]	90	93	97
Frequency [$\mu Hz/Hz$]	0.14	0.1	0.06
Magnitude [$\mu V/V$]	1.5	1.0	0.6
Phase [m °]	0.4	0.1	0.06

TABLE III
STANDARD DEVIATION OF NUMERICAL ERRORS FOR PHASE STEPS

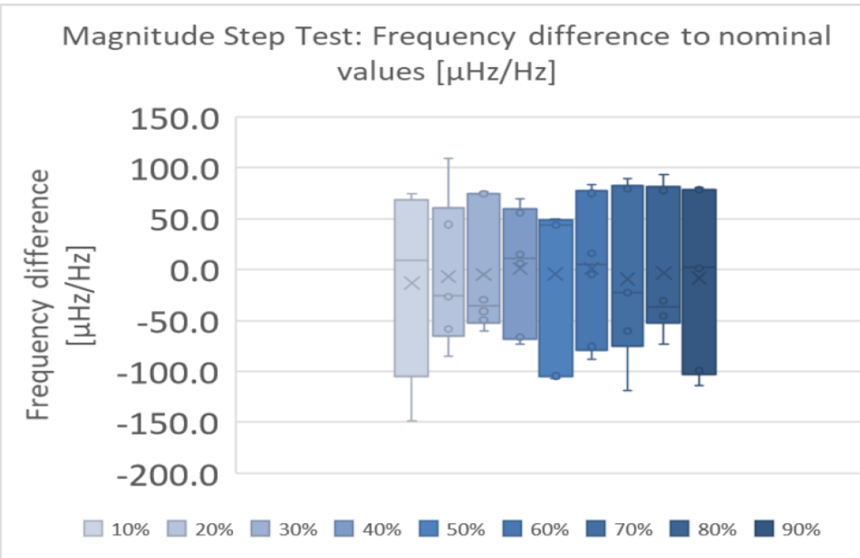
SNR [dB]	90	93	97
Frequency [$\mu Hz/Hz$]	0.26	0.19	0.1
Magnitude [$\mu V/V$]	1.5	1.0	0.7
Phase [m °]	0.17	0.11	0.07

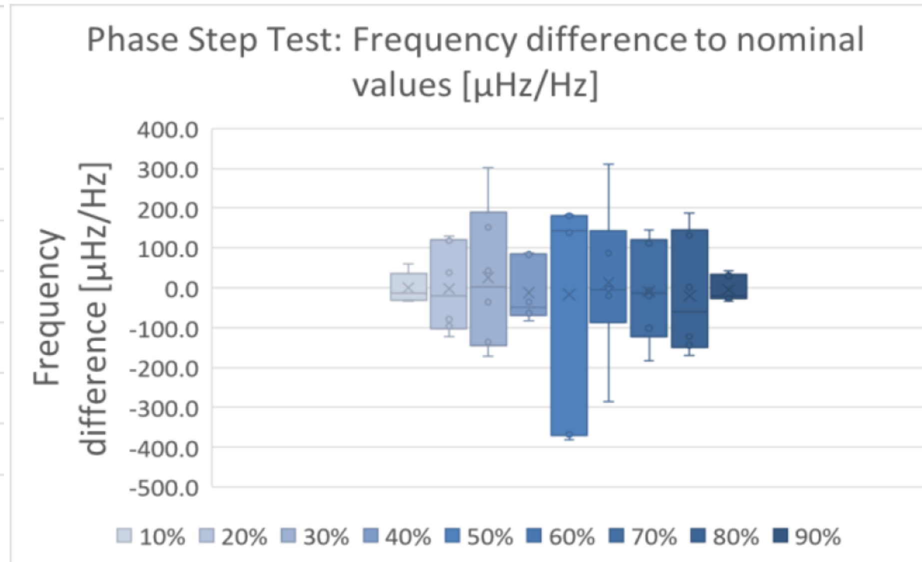
Lab measurements





Frequency measurements





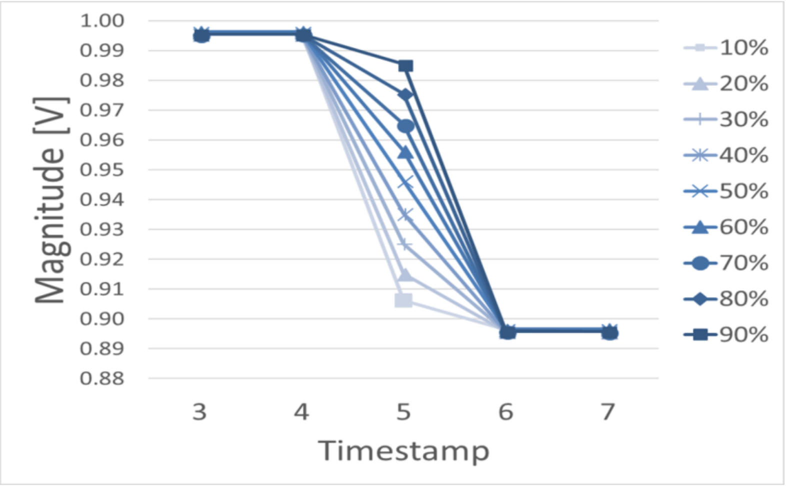
Standard deviation from $F_{nom} = 60\ Hz$

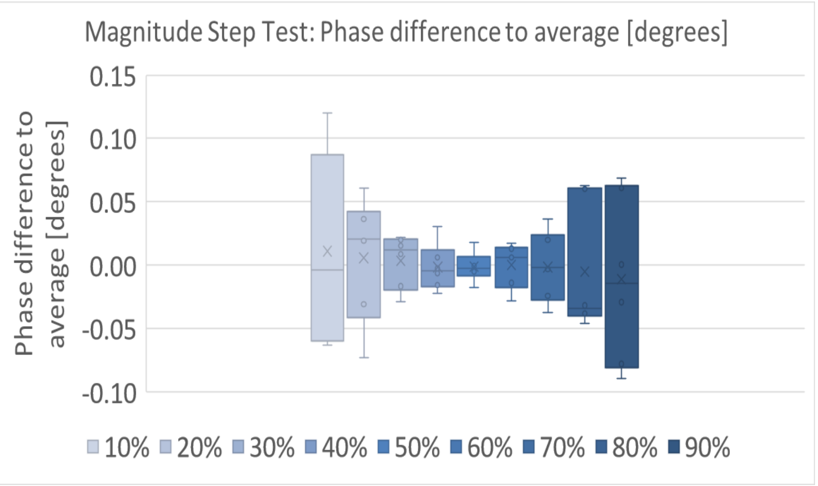
Steady State: $9\ \mu Hz/Hz\ (0.5\ mHz)$

Magnitude Step: $70\ \mu Hz/Hz\ (4\ mHz)$

Phase Step: $40\ \mu Hz/Hz\ (2\ mHz)$ to $280\ \mu Hz/Hz\ (17\ mHz)$

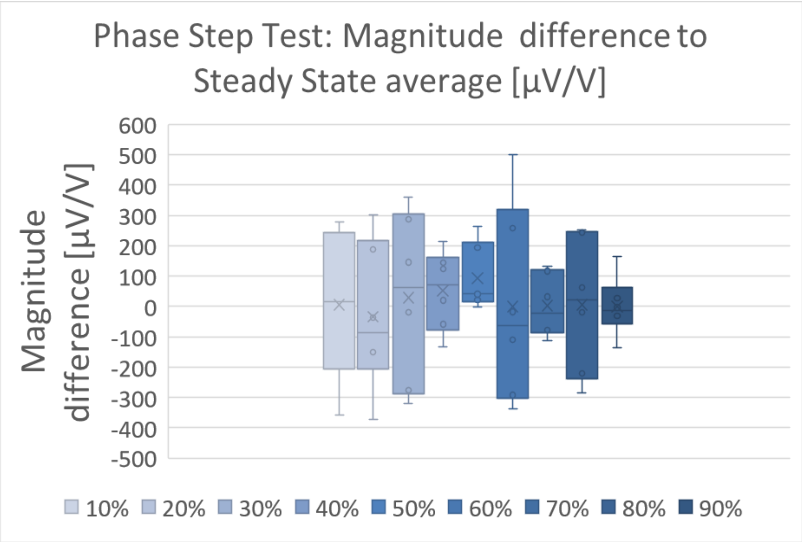
Magnitude Step Phasor

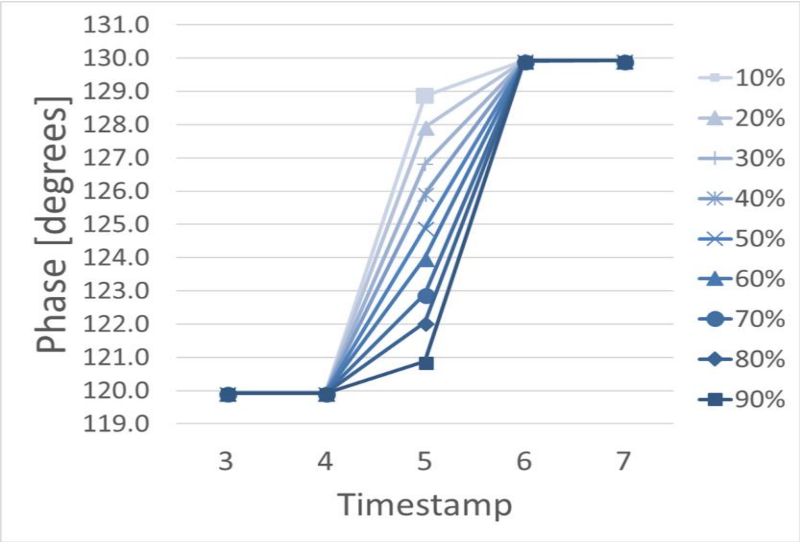




Phase (difference from the average values)
Standard deviations
Steady State: 1.7 $m°$
Magnitude Step: 10 m^o to 70 m^o

Phase Step Phasor





Magnitude (difference from the Steady State average values) Standard deviations

Steady State: $160\ \mu V/V$

Magnitude Step: $200\ \mu V/V$

Questions?

Thank you!

45 anos

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