Transient Detection and Frequency Estimation Based on Hilbert Transform for Testing of PMUs under Voltage Sags

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***Abstract*—The perspective of a widespread use of real-time instrumentation such as Phasor Measurement Units (PMUs) in evolving electrical grids demands the metrological characterization of the measurement devices under more challenging environments. Besides, the advent of power grids with significant presence of low inertia renewable generation and rapid power quality events such as voltage sags and swells are leading to concerns about the limits for measurement errors that can cause serious operation failures due to misleading actuation of protection systems. Their performance are also affected directly by conflicting definitions for power system frequency under transient conditions. Therefore, we developed a calibrator capable of reproducing voltage sag representative signals. In order to assess the performance of our calibrator, we present in this work the performance of transient detectors and frequency estimators based on the Hilbert Transform suitable for noisy sampled signals with magnitude or phase steps of a few cycles duration.**

***Index Terms*—transient, PMU, voltage sag, frequency, estimation**

I. Introduction

PMUs (Phasor Measurement Units) are devices intended to estimate power system phasors, frequency and rate of change of frequency (ROCOF) in a specific time and phase-related to a common based reference function, as defined in the last IEEE Standard C37-118.1 [1] (hereafter called IEEE Std). A phasor is a well defined quantity that can represent a purely sinusoidal functions with constant parameters. Strictly speaking, a sinusoidal function has no beggining and no end, but as phasors can represent the behavior of a power system operating under steady-state conditions, they may be estimated from a finite set of digitally sampled signals.

The traditional approach for transient situations is to detect their occurrence and consider the measurements not reliable as long as the transient lasts [2]. This procedure can be inappropriate for devices intended to have fast response in electric grids subjected to frequently low-quality signals, such as those with intensive low-inertia generators (e.g. solar and wind). Recent works point out the occurrences of catastrophic failures due to low-inertia renewables power plants submitted to voltage sags with the duration of a few cycles [3], [4].

We have recently developed a system to reproduce the magnitude tests and step tests prescribed in the IEEE Std, which metrological assessment was presented in [5]. We have also pointed out that the current standardized tests and definitions for reference synchrophasors are not suitable for the proper characterization of PMUs submitted to voltage sags with a few cycles duration. An issue to be addressed is that the tests account for only one step, whereas voltage sags often last a few cycles, what means that one observation window can contain two or more steps inside it. Also, real world voltage sags contain simultaneous magnitude and phase steps, whereas the standard prescribe separated tests.

The parametric models used to estimate the parameters account for the representation of signals with only one step inside the observation interval, either a magnitude or a phase step. The models need as an input a reliable estimation of the instant of the step occurrence, which can be achieved in the majority of situations by an estimator based on the instantaneous frequency, as provided by the discrete Hilbert transform of the sampled signal [6]. However, as the IEEE Std does not specify the initial phase for the step occurrence, we verified that the estimator proposed in [5] could fail if the magnitude step happens to occur near to the zero crossing.

This work proposes a new system to test the behavior of PMUs submitted to voltage sags, comprising the algorithms for the detection of transients and their instants of occurrence, comprising:

1) a hybrid detector based on instantaneous magnitude and instantaneous frequency, capable of estimating the instant of occurrence of steps within the performance limits reported in section III;
2) a procedure to improve the performance of the detector under noisy conditions based on polynomial fit with step discontinuities;
3) a procedure for smooth grid frequency estimation.

II. Mathematical Background

Recent works discuss the suitability of frequency and ROCOF measurements, specially during transient events such as voltage sags, which cause magnitude and phase jumps in the measured voltage signals [4], [7]. One fundamental issue is the definition of phase and frequency for power systems [8]. We present here a brief discussion about definitions for grid frequency measurements in power systems; a proposed hybrid detector of transients with the instantaneous frequency and magnitude as provided by the Hilbert Transform; and a procedure to diminish the noise influence in the estimation.

A. Frequency and phase definitions for power systems

In section 5.2 of the IEEE Std [1], the frequency of a signal

$$x(t) = X_m \cos(\Psi(t)) \tag{1}$$

is defined by

$$f(t) = \frac{1}{2\pi}\frac{d\Psi(t)}{dt}. \tag{2}$$

Let $\Psi(t)$ be described by

$$\Psi(t) = 2\pi g(t)t + \phi(t). \tag{3}$$

If $g(t) = f_1$ and $\phi(t) = \phi_0$ are constant, $g'(t) = 0$ and $\phi'(t) = 0$, then applying (2) to (3) gives

$$f(t) = g'(t)t + g(t) + \phi'(t)/(2\pi) = g(t). \tag{4}$$

The synchrophasor of $x(t)$ for system nominal frequency f_0 is defined considering the cosine argument as

$$\Psi_s(t) = 2\pi f_0 t + \phi_s(t). \tag{5}$$

Applying (2) to (5) gives

$$f(t) = f_0 + \frac{d}{dt}\frac{\phi_s(t)}{2\pi} = f_0 + \Delta f(t), \tag{6}$$

Hence, from (4) and (6),

$$g(t) = f_0 + \Delta f(t), \tag{7}$$

what implies that all time variation in $g(t)$ is incorporated in the function $\phi_s(t) = \Delta f(t)$, and one can estimate the frequency $g(t)$ by (7) with the measurement of the synchrophasor phase $\phi_s(t)$.

However, if $g(t)$ or $\phi(t)$ are not constant, this procedure will not provide suitable independent estimations for $g(t)$ and $\phi(t)$, because (4) does not hold. In the case of fast transients like the ones prescribed in the IEEE Std or voltage sags, Roscoe et al. [4] propose an alternative model for the cosine argument

$$\Psi_R(t) = \theta(t) + \psi(t), \tag{8}$$

where the phase argument has two independent parameters, one ($\theta(t)$) to take account of slow deviations and other ($\psi(t)$) containing information of the fast transients (like phase steps). They also introduced the concept of ”underlying frequency” and ”underlying ROCOF”, both related to slow varying phase $\theta(t)$.

We consider that the frequency to be monitored during transient events containing magnitude or phase steps, at least for protection purposes, should be related somehow to the angular speed of the generators, what implies that the high frequencies inserted in the signals by fast transients should not interfere significantly in the frequency estimation. Hence, the ”underlying frequency” is a more appropriate representation of the grid frequency. Nonetheless, the practical use of this model requires an effective way to disaggregate $\theta(t)$ and $\psi(t)$, what can be achieved, e.g., with the analysis of the instantaneous frequency and magnitude, as provided by the Hilbert Transform.

B. Instantaneous magnitude, phase and frequency via Hilbert Transform

Given a narrowband monocomponent signal $x(t)$, $t \in \mathbb{R}$ let its associated analytical signal be

$$z(t) = x(t) + jH\{x(t)\} = a_i(t)e^{j\theta_i(t)}, \tag{9}$$

where

$$H\{x(t)\} = \int_{-\infty}^{+\infty} \frac{x(u)}{\pi(t-u)}du \tag{10}$$

is the Hilbert Transform of $x(t)$. The instantaneous frequency of $z(t)$ is

$$f_i(t) = \frac{1}{2\pi}\frac{d\theta_i(t)}{dt}. \tag{11}$$

A discrete version of $H(x[n])$ can be obtained via fast Fourier transform [6], from which discrete estimations of the instantaneous magnitude $a_i(t)$, phase $\theta_i(t)$ and frequency $f_i(t)$ can be obtained. Discontinuities in the signal can be detected by monitoring anomalies in a_i and f_i [5], [9], [10], by defining appropriate detection signals and threshold levels.

C. Transient detection signals

Let $x(t)$ a signal described by the parametric model

$$x(t) = X_m[1 + k_x\nu(\tau_1,\tau_2)]\cos\left((2\pi f_u t + \phi_0 + k_a\nu(\tau_1,\tau_2)\right), \tag{12}$$

where X_m is the initial magnitude, k_x is the relative amplitude of the magnitude step, f_u is the underlying frequency, ϕ_0 is the initial phase, k_a is the amplitude of the phase step, and

$$\nu(\tau_1,\tau_2) = u(t-\tau_1) - u(t-\tau_2) \tag{13}$$

is a function that represents step transitions in two different instants, $\tau_1 < \tau_2$.

Let $x[n]$ be a discrete signal obtained by sampling $x(t)$. The discrete version of the analytic signal associated to $x[n]$ is given by

$$z[n] = x[n] + jH\{x[n]\} = a_i[n]e^{j\theta_i[n]}. \tag{14}$$

1) Instantaneous magnitude detection: From the first order differences signal $g_i[n] = a_i[n+1] - a_i[n]$, let the detection signal for the instantaneous magnitude be

$$d_g[n] = |g_i[n] - m(g_i[n])|. \tag{15}$$

Given a threshold value $\Lambda_m = k_m m(d_m[n])$, where k_m is a given constant value, and $m(d_m[n])$ is the median of the detection signal $d_m[n]$. If the maximum value of the detection signal $d_{mmax} \geq \Lambda_m$, one can estimate the instant of occurrence τ_m by taking the sample index of the maximum value.

2) Instantaneous frequency detection: With the discrete signal $f_i[n]$ obtained by numerical differentiation of $\theta_i[n]$, we can define a detection signal for the instantaneous frequency

$$d_f[n] = |f_i[n] - m(f_i[n])|. \tag{16}$$

Given a threshold value $\Lambda_f = k_f m_f(d_f[n])$, where k_f is a given constant value, and $m_f(d_f[n])$ is the median of the detection signal $d_f[n]$, If the maximum value of the detection signal $d_{fmax} \geq \Lambda_f$, one can estimate the instant of occurrence τ_f by taking the sample index of the maximum value.

3) Hybrid estimator: We propose a hybrid estimator (HE), based on choosing between τ_m or τ_f estimations. The choice criteria is the highest relation d_{mmax}/Λ_m or d_{fmax}/Λ_f.

D. Detection signals with noise

For a measurement environment, a more realistic modelling should consider the presence of noise. For that, we can consider a signal

$$x_\eta(t) = x(t) + \eta(t), \tag{17}$$

with $x(t)$ from (12) added to a random function $\eta(t)$ representing noise. Given σ_x, the standard deviation of the signal $x(t)$ and a prescribed SNR level in decibels, for a zero-mean Gaussian white noise, the noise variance is

$$\eta_0 = \left(\frac{\sigma_x}{10^{\frac{SNR}{20}}} \right)^2 \tag{18}$$

As signal noise conditions can deteriorate the performance of estimators [5], we propose submitting the discrete signals $a_i[n]$ and $\theta_i[n]$ to the procedure PATV (Polynomial Approximation and Total Variation) from Selesnick [11] before performing the differentiation.

1) PATV procedure: For time series that can be modelled by

$$y[n] = s[n] + r[n] + \eta[n], \quad n = 1, \ldots, N, \tag{19}$$

where $s[n]$ is a low-order polynomial, $r[n]$ is approximately piecewise constant and $\eta[n]$ is zero-mean Gaussian white noise, the PATV procedure simultaneously estimate $s[n]$ and $r[n]$ by finding jointly the coefficients $\mathbf{a}$ and the signal $r[n]$ solving the minization problem

$$\min_{\mathbf{a},\mathbf{y}} \lambda \sum_{n=2}^{N} |r[n] - r[n-1]| + \sum_{n=1}^{N} |y[n] - p[n] - r[n]|^2 \quad , \tag{20}$$

where

$$p[n] = a_1\, n + \cdots + a_d\, n^d, \tag{21}$$

and λ is a user-defined positive regularization factor that controls the trade-off between noise reduction and signal distortion. The advantage of this approach is that one can estimate directly the approximated polynomials $p[n]$ with a continuous derivative shape, rejecting simultaneously noise and step discontinuities.

2) Noiseless Detection signals: If we define the signal $w[n] = p[n] + r[n]$, the median extraction is not needed because it is implicitly done in the PATV procedure. Hence, the noiseless magnitude detection signal is given directly by the first order differences

$$d_{gNL}[n] = w_{a_i}[n+1] - w_{a_i}[n]. \tag{22}$$

Given a threshold value Λ_g, the transient detection valid if $d_{gNL} \geq \Lambda_g$. Likewise, the noiseless phase detection signal is

$$d_{fNL}[n] = w_{\theta_i}[n+1] - w_{\theta_i}[n]. \tag{23}$$

Given a threshold value Λ_f, the transient detection is valid if $d_{fNL} \geq \Lambda_f$.

3) Noiseless Hybrid Estimator: The noiseless hybrid estimator (NLHE) chooses the estimation which has the greater difference between maximum values of the detection signals (d_{gNLmax} or d_{fNLmax}) and their respective threshold values.

Additionally, from the first order polynomial approximation w_{θ_i} of $\theta_i[n]$, one can have a smooth estimation for the "underlying frequency" of the grid.

III. Numeric Simulations

In order to assess the performance of the estimators, we ran Monte Carlo simulations. For each Monte Carlo run, we generate a new signal by digitally generating samples from equation (17), with a discrete time interval of $\Delta t = 1/4800$ s and fundamental frequency $f_u = 60$ Hz. As related in [5], the worst case for a transient detector happens when the step is magnitude only, occurring near to a zero crossing of the signal ($\phi_0 = 90°$). Figure 1 shows an example of a sampled signal. Simulations were done considering this case, which means the performance figures are conservative values for the uncertainty of the estimator.

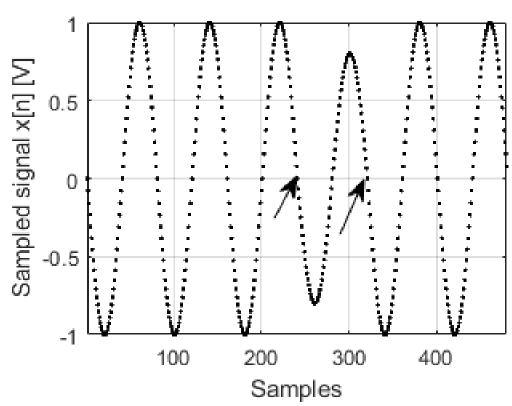

Fig. 1. Sampled signal with magnitude step ($k_x = 0.2, \tau_1 = 240\Delta t, \tau_2 = 320\Delta t$). The arrows indicate the magnitude transitions near to the zero crossings.

The τ estimation error is given by

$$\epsilon_\tau = |\hat{\tau} - \tau|, \tag{24}$$

where $\hat{\tau}$ is the estimated value and τ is the reference value. Let N_e be the number of $\hat{\tau}$ for which $\epsilon_\tau > 2\Delta t$. For N_{MC}

Monte Carlo runs, we report the relative number of estimation errors, given by

$$\mathcal{E}_{>2} = \frac{N_e}{N_{MC}}. \tag{25}$$

A. Transient detection

Comparison of τ detection under noisy conditions using the hybrid detector 1)HE; 2) NLHE.
figure/table
comments

B. Frequency estimation

figure - histogram
comments

C. Limitations

Describe limitations: $5\% \leq \tau_i \leq 95\%$ of the observation window, and duration ($\tau_2 - \tau_1$ of at least half cycle. Magnitude step of at least 5%. Phase step of at least 5°.

IV. Measurements

A. Laboratory setup

Brief Description
setup photo
setup block diagram
brief comment on noise conditions

B. Performed tests

1) Detection of magnitude only: figure 2
Discussion on tau and frequency estimation.

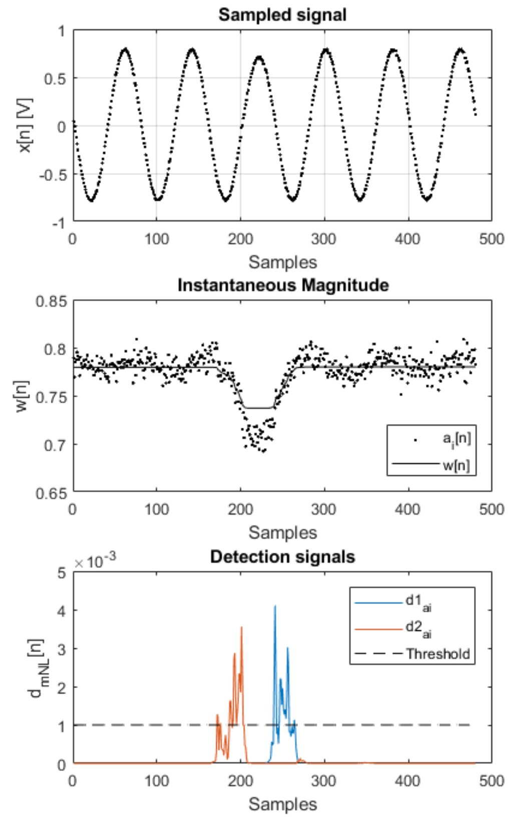

Fig. 2. Estimation of τ_1 and τ_2 of a sampled signal from the calibration system.

V. Conclusion

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