Evaluation of the Synchronous Generation and Sampling Technique

Waldemar G. Kürten Ihlenfeld, Enrico Mohns, Hans Bachmair, Günther Ramm, and Harald Moser

***Abstract*—The simultaneous synchronous generation and sampling technique allows alternating current (ac) quantities [root mean square (rms) values of voltage, voltage ratios, and power] to be determined with uncertainties of the order of a few parts in 10^6. Mathematical models for estimating measurement uncertainties and experimental comparisons of such a system against existing primary standards at the PTB were carried out.**

***Index Terms*—AC/DC transfer, ac metrology, ac power, ac voltage, effective value, phase angle, rms value.**

I. Introduction

THE synchronous generation and sampling technique of the new alternating current (ac) measurement system at the Physikalisch-Technische Bundesanstalt (PTB) is described in detail in [1]–[3]. The use of a single clock-frequency for synchronizing the generation and sampling of an ac signal is the key factor for attaining uncertainties as low as a few parts in 10^6. The clock-synchronization practically eliminates triggering errors and strongly reduces time-jitter effects. Especially at power line frequencies (50 Hz–60 Hz), effective (or rms) values of voltage, and ac power can be determined with comparable or even lower uncertainties than those obtained by thermal ac-dc transfer techniques. Extensive mathematical modeling and experimental investigations [2], [3], conducted at the PTB, validate the metrological capabilities of the PTB system for use as a sampling standard for ac metrology from direct current (dc) up to about 1 kHz.

II. Sources of Errors

Mathematical models for estimating measurement uncertainties firstly investigate a nonideal source operating in conjunction with an ideal sampler and secondly an ideal source with a nonideal sampler. The superposition of error contributions for the real case of both nonideal source and sampler furnishes reliable estimations of measurement uncertainties.

A. Errors Arising From the Source

In this particular case, a nonideal source [a digital-to-analog converter (DAC)], synthesizes a nearly sinusoidal voltage signal $u(t)$ synchronously with a common clock reference. This signal is synchronously sampled with an ideal integrating analog-to-digital converter (ADC). The sampler takes samples synchronously with the same clock reference over an integration time T_i at regular time intervals of length T_a (sampling time). The sampled voltage U_ν (at a time νT_a, where ν is an integer) of an integrating ADC is the mean value of the signal $u(t)$ over a time span T_i and is given by

$$U_v = \frac{1}{T_i}\int_{\nu T_a}^{\nu T_a + T_i} u(t)dt. \tag{1}$$

The sampling procedure is illustrated in Fig. 1.

The estimation of the effective value, from MN samples (N samples per period over M periods), of a signal with fundamental frequency f_o (period $T_o = 1/f_o$) is given by

$$\hat{U}_{\text{eff}} = \frac{1}{\text{Sinc}\left(\frac{\pi T_i}{T_o}\right)}\sqrt{\frac{1}{MN}\sum_{\nu=1}^{MN} U_\nu^2} \tag{2}$$

where $\text{Sinc}(x) = \sin(x)/(x)$ is the function accounting for the transfer function of the sampler in the frequency domain due to (1).

Of primary concern is the spectral purity of the synthesized signal of the source, or its total harmonic distortion (THD). This is because spurious signals (or harmonics) superimposed on the fundamental signal increase THD and produce systematic errors (aliasing errors) for harmonic components located beyond the Nyquist frequency $1/(2T_a)$ of the sampler. The main sources of errors that can be listed are side-band harmonics (due to the step approximation of a sinusoidal voltage signal), DAC quantization, glitches or spikes, clock-feedthrough, differential and integral nonlinearity of the DAC, clock delay, clock jitter, and DACs thermal and flicker noise.

Side-band harmonics are always present and are so called because they are located at the side-bands of the clock frequency or update frequency of the DAC [2] and at multiples thereof (i.e., at $f_{\text{clk}} - f_o$ and $f_{\text{clk}} + f_o$; $2 \cdot f_{\text{clk}} - f_o$ and $2 \cdot f_{\text{clk}} + f_o$ and so forth). They are dependent on the number of time steps used for synthesizing the signal $u(t)$. Some harmonics cannot be removed by an antialiasing filter, since they fold back into the Nyquist bandwidth of the DAC. Furthermore, a true rms device (a thermal converter) would detect higher frequencies harmonics (due to its higher bandwidth), which cannot be detected by the integrating sampling system. Therefore, it is mandatory to use an antialiasing filter and to choose the clock frequency much higher than the signal fundamental frequency. In this way, coherent higher order harmonics (spurious spectral lines) and correlated effects arising from finite quantization can be strongly attenuated [2].

Spikes or glitches are always present on the signal and are originated from charge injection and time delays in analog

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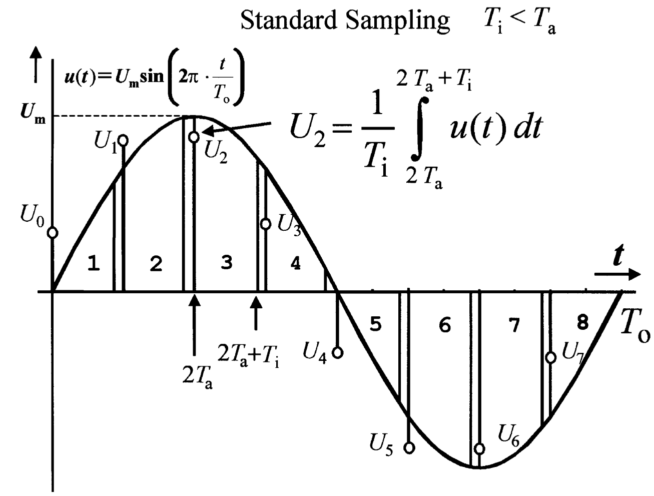

Fig. 1. Illustration of the sampling procedure. An integrating ADC samples the signal at sampling intervals T_a over an integration time T_i. The sampled value is represented by a circle located at vT_a and corresponds to the mean value over a time span $\{vT_a; vT_a + T_i\}$.

switches internal to the DAC. Larger spikes occur at the zero crossings of the synthesized voltage when the code of the DAC changes the state of all bits at the same time. Spikes with an amplitude of 12 mV and 1.25 μs duration may produce systematic deviations (in the application described in [2]) as high as 0.2 parts in 10^6 (for a synthesized sinusoidal fundamental at 62.5 Hz and 5 V rms). These deviations depend on N, signal amplitude, and the sampling parameters, T_a and T_i chosen. Spikes of smaller amplitudes, however, occur on each step of a synthesized signal. Their effect on $\hat{U}_{\mathrm{eff}}$ in respect of the true rms value of the signal can be estimated from their spectrum, which depends on their time duration and time integral. Small spikes produce higher systematic deviations as f_o increases. A low-pass filtering for small spikes results in a disturbing square wave with the same frequency f_o superimposed on the fundamental [2].

Clock-feedthrough on the fundamental may be represented by an exponential-decaying sinusoidal superimposed on each step. Due to its shape, however, the associated deleterious effects on the effective value can be almost entirely eliminated by an anti-aliasing low-pass filter.

Differential and integral nonlinearities of the DAC produce a wide spectrum of odd-harmonics of the fundamental. Part of this spectrum is eliminated by an anti-aliasing filter. Nevertheless, an estimation of their effect on $\hat{U}_{\mathrm{eff}}$ can be made on a statistical basis by assuming that the power density spectrum is distributed over the entire Nyquist bandwidth of the DAC (dc to $f_{\mathrm{clk}}/2$). This conservative approximation simplifies further calculations enormously. Thus, as for quantization, a variance and a corresponding signal-to-noise-ratio (SNR) may be estimated and a noise like treatment can be applied.

Clock delay does not influence the effective value $\hat{U}_{\mathrm{eff}}$ estimated from samples as far as it remains stationary. Clock jitter, however, increases SNR. A theoretical analysis [2] indicates that clock jitter on the DAC may be neglected for the PTB sampling system at low frequencies but it becomes relevant for f_o near to 500 Hz and above.

Uncertainty contributions due to noise (flicker and thermal) are treated on a statistical basis using their noise power density spectrum.

Further estimations of measurement uncertainties remain valid and reliable if and only if frequency components at and above half the sampling frequency $-1/(2 \cdot T_a)$—are attenuated to a level below the dynamic range of the sampler to avoid systematic deviations produced by aliasing.

B. Errors Arising From the Sampler

The sampler used in the PTB sampling standard is a high-resolution integrating ADC that operates according to the dual-slope principle. Its mathematical model may be expressed as

$$\begin{aligned} U_\nu =& U(\nu T_a) \\ =& \frac{1}{T_i + \delta_{\mathrm{JTi}}}\left(1 + \delta_{\mathrm{REF}} + \delta_G + \frac{\delta_{\mathrm{LIN}} + \delta_{\mathrm{RES}}}{\mathrm{Abs}[U_v]} U_{\mathrm{FS}}\right) \\ & \times \int_{\nu T_a + \delta_{\mathrm{JTa}}}^{\nu T_a + \delta_{\mathrm{JTa}} + T_i + \delta_{\mathrm{JTi}}} [u(t) + u_n]\, dt + u_s, \end{aligned} \tag{3}$$

where $\mathrm{Abs}[U_\nu]$ is the absolute value of U_ν with the restraint that all deltas are zero. δ_{REF}, δ_G, δ_{LIN}, and δ_{RES} represent random variables related to deviations of the internal reference voltage (dc calibration uncertainties, drifts thereof with time, and temperature), gain, linearity, and resolution of the sampler, respectively. δ_{JTi}, and δ_{JTa} represent jitter of the integration time and sampling time and U_{FS} represent the full-scale value of the measurement range (i.e., 10 V). Noise components of the source u_n are added algebraically to the signal being sampled $u(t)$ and u_S represents the sampling noise produced by the sampler. For noise contributions and for gain deviations δ_G, Gaussian probability density functions were assumed. Rectangular probability density functions were assumed for all other remaining δ-random variables in (3) [2], [3].

Any departure from the ideal sampler with respect to its internal dc reference voltage and gain errors (δ_{REF} and δ_G respectively) will produce equal effects on each sampled value U_ν. Hence, correlations among samples related to δ_{REF} and δ_G must be considered. For the remaining random variables (δ_{LIN}, δ_{RES}, δ_{JTi}, δ_{Jta}, u_n and u_s) the samples can be considered uncorrelated.

III. Estimation of Uncertainties

It is not the scope of this paper to give full coverage of all uncertainty analyzes made. For a full treatment, refer to [2], [3].

Effective values of ac voltages may be estimated using (2) or from the contributions of each spectral line determined by a discrete Fourier transformation (DFT) or fast Fourier transform (FFT) on the data in the set of N samples taken over M periods of a sinusoidal voltage generated by the source. The spectral lines show the same dependence (systematic deviations) on aliasing frequencies as that estimated for $\hat{U}_{\mathrm{eff}}$. Under the absence of aliasing conditions, the most important parameters to consider are δ_{REF} and δ_G of the sampler, because correlation among samples makes impossible any reduction of uncertainties by averaging estimations of $\hat{U}_{\mathrm{eff}}$.

TABLE I
ESTIMATED TYPE B UNCERTAINTIES ($k = 1$)

f_o [Hz]	*u for* $\hat{U}_{\text{eff}}$ = 120 V [µV/V]	*u for* I = 5 A [µA/A]	*u for* S (600 VA) [µVA/VA]
40	0.67	1.0	2.1
55.56	0.66	1.2	2.2
62.5	0.68	1.2	2.3
83.33	0.76	1.4	2.6
125	0.95	1.7	3.2
312.5*	11	11	23
416.67*	16	16	33

(*) At these frequencies higher gain errors of the sampler are expected leading to increased uncertainties.

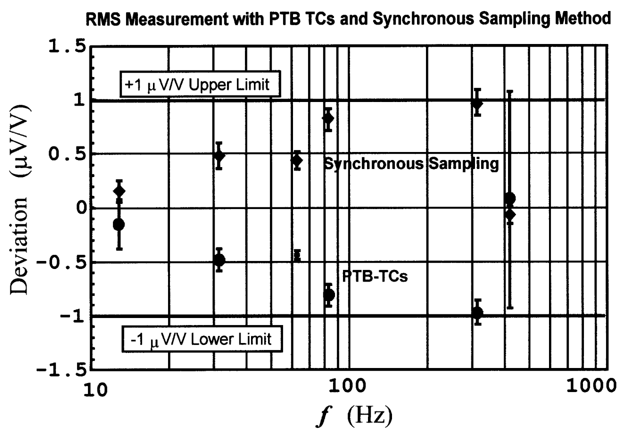

Fig. 2. Plot of the deviations when comparing the effective values at 5 V determined by the sampling system (diamonds) and thermal converters (circles). The zero line represents the mean of the estimations of both systems. Crossbars represent standard deviations (1σ) of measurements which were smaller than 0.2 μV/V for the sampling system. The agreement between the sampling system and ac-dc transfer is within ± 1 μV/V (not uncertainty limits). The values suggest the presence of small systematic differences between the two systems. This is because of drifts of the internal dc references of both systems.

For the measurement of ac power [1], two digital sources and two amplifiers are used (a voltage and a transconductance amplifier) in conjunction with a precision voltage and current transformer and its shunt.

Table I lists the estimated type B standard uncertainties u (coverage factor $k = 1$) of the primary ac sampling system of the PTB for some technical frequencies of interest. The second and third columns refer to uncertainties for determining effective values for voltage (nominal 120 V) and current (nominal 5 A), respectively. The last column states estimated uncertainties for the apparent power S (nominal 600 VA).

The following constraints must hold to attain the lowest uncertainties.

1) $T_a = 1/(N f_o)$ must hold at all times for N a multiple of two. This is the condition for synchronous sampling of a signal with frequency f_o generated from a common clock-reference.
2) The number of sampled periods M must be an integer multiple of power-line cycles in order to reduce power line interferences. Conditions 1 and 2 prevent artificial spectral components (leakage) from appearing when doing a DFT on the sampled data.

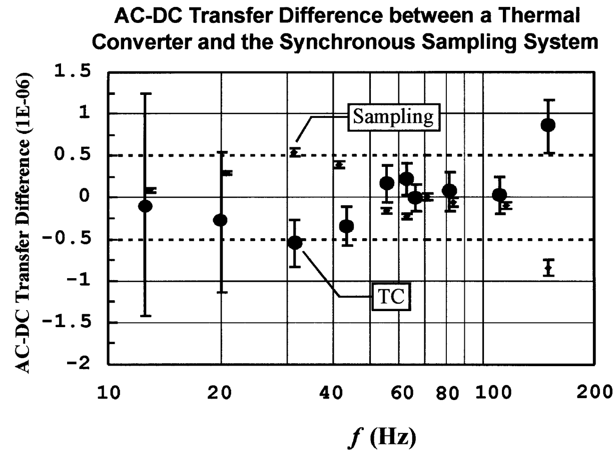

Fig. 3. Plot of the deviations when determining ac-dc transfer at 4 V determined by the sampling system (diamond) and thermal converters (circles). The zero line represents the mean of the estimations of both systems. Crossbars represent standard deviations of the mean for three measurements, which were smaller than 0.1 μV/V for the sampling system. The agreement between the sampling system and a thermal converter, from 12 Hz up to about 120 Hz, is within ± 0.5 μV/V. Since the same sampler was used simultaneously for sampling and ac-dc transfer, systematic differences are much smaller than those observed in Fig. 2. The differences between both systems are within expected limits of evaluated type B uncertainties.

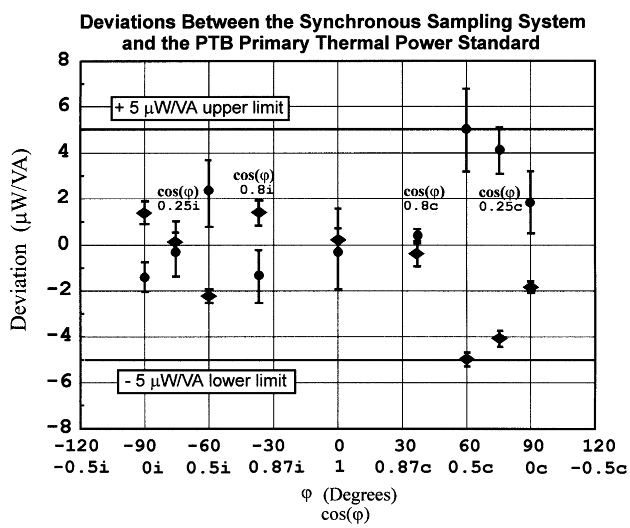

Fig. 4. Plot of the deviations when comparing active power P at 55.56 Hz, 120 V and 5 A determined by the sampling system (with $N = 16$, $T_a = 1.125$ ms, $T_i = 1.1$ ms, sinusoidal generated with 2500 steps; represented with diamonds) and the PTB thermal standard (circles) related to the apparent power S. The zero line represents the mean of the estimations of both systems, and the crossbars represent the standard deviations obtained during the measurements (five measurements for the thermal standard and five of 120 single measurements for the sampling system). The agreement between the two systems are within ± 5 μW/VA for all phase angles.

3) The suppression of harmonics of the power line frequency occurs when $1/(T_i f_o) \gg 1$ and is an integer.

IV. COMPARISON WITH THE PTB PRIMARY STANDARDS

The rms value of a synthesized sinusoidal output voltage $u(t)$ was simultaneously determined with a thermal converter and with the synchronous generation and sampling system. The results of these comparisons are outlined in Fig. 2. The zero line on the ordinate represents the mean of the estimations from both systems and the points represent the deviations of each system from the zero line.

The agreement between the two systems, when determining the effective value of a 5 V (rms value) sinusoidal voltage signal is within ± 1 μV/V in the frequency range shown.

If the sampling system is used to sample the ac and dc voltage applied to the heater resistance of a PTB primary thermal converter, the ac-dc transfer error of this system related to a PTB primary thermal converter is obtained. According to Fig. 3, the agreement between the sampling system and a thermal converter from 12 Hz up to about 120 Hz when determining ac-dc transfer differences is within ± 0.5 μV/V.

Finally, validation of the sampling system by comparing it with the PTB thermal primary power standard at 55 Hz, 120 V, and 5 A was carried out, and the results are shown in Fig. 4. Both systems agree within ± 5 μW/W for all power factors (after correcting for the small systematic deviations of the transformers and the ac shunt of both systems).

V. Conclusion

The sampling system allows ac quantities at power frequencies to be measured rapidly (seconds) and at an accuracy level comparable to or even better than those obtained by the existing highest grade instrumentation. A drawback, however, lies in its dependence on the spectral content of the signals being sampled. For the future, adaptations of the algorithms and optimization routines will also encompass the determination of ac quantities (effective values, ac ratios, ac power, and phase) under nonsinusoidal conditions.

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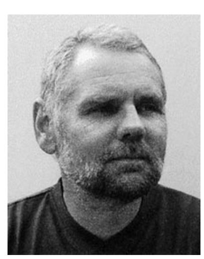

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