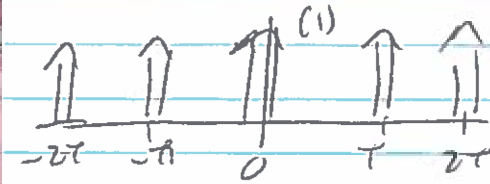
Michael Bocamazo

Sig Sys PS07 2015/03/10

1. $p(t) = \sum_{k=-\infty}^{\infty} \delta(t-kT)$

(a)



(b) Find Fourier Series Representation

$$C_k = \frac{1}{T}\left(\int_{-T/2}^{T/2} x(t)\, e^{-j\frac{2\pi}{T}kt}\, dt\right)$$

$$\frac{1}{T}\left(\underbrace{\int_{-T/2}^{\sim 0}}_{0} \cdots \underbrace{\int_{\sim 0}^{\sim 0} (1)\, e^{-j2\pi/T\, kt}\, dt}_{} + \underbrace{\int_{\sim 0}^{T/2} 0}_{0}\right)$$

By picking property and definition of impulse (Area = 1), $e^{-j2\pi/T\, k(0)} = 1$ so integral = 1

$$C_k = \frac{1}{T}$$

$$x(t) = \sum_{k=-\infty}^{\infty} \left(\frac{1}{T}\right) e^{j\frac{2\pi}{T}kt}$$

(c) $\omega_0 = \frac{2\pi}{T}$

$$X(\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt$$

$$C_k = \frac{\omega_0}{2\pi} \int_{-\infty}^{\infty} x(t)\, e^{-j\omega_0 kt}\, dt$$

$$C_k = \frac{\omega_0}{2\pi} X(\omega_0 k)$$

$$X(\omega_0 k) = C_k \frac{2\pi}{\omega_0} \qquad X(\omega) = C_k \frac{2\pi}{\omega} k \quad \frac{T}{2\pi}$$

$\omega = \omega_0 k$ ~~and~~ $\omega_0 = \frac{\omega}{k}$

(d) $P(\omega) = \int_{-\infty}^{\infty} C_k \frac{2\pi}{\omega} k$

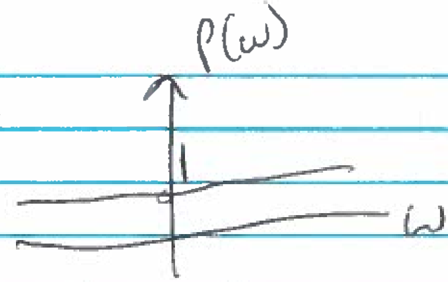
$$P(\omega) = \frac{1}{T}\, \frac{2\pi k}{\omega}$$

$$= \frac{1}{T}\left(\frac{2\pi}{\omega_0}\right)$$

$$P(\omega) = \frac{1}{T}(T)$$

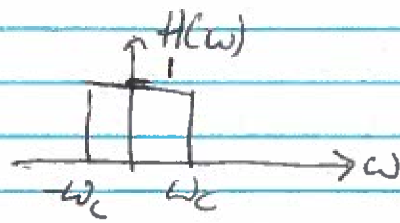
$$P(\omega) = 1$$

e $P(\omega)$



$P(\omega) = 1$ at all frequencies, ΔT does not effect either which makes sense because the impulse should have value 1 at all frequencies.
~~Changing to~~ The impulse is not dependent on the period ~~as in continuous time, it is everywhere 1~~
$P(t)$ has the same behavior regardless of the period.
This follows the definition of the impulse.
The Fourier transform is solved for a single period

2. $H(\omega)$



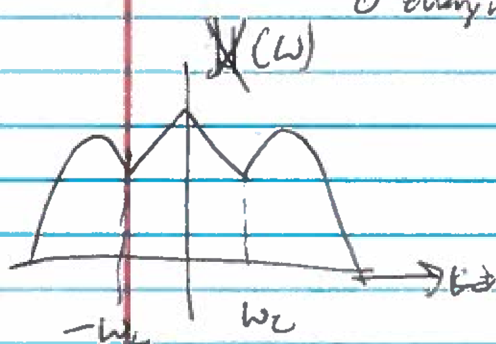
$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(\omega)e^{j\omega t}\,d\omega$$

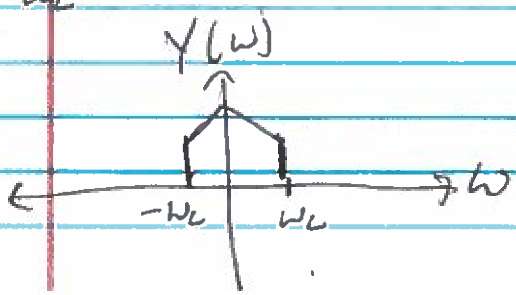
$$h(t) = \frac{1}{2\pi}\underbrace{\int_{-\omega_c}^{\omega_c}(1)e^{j\omega t}\,d\omega}_{0\text{ everywhere else}} = \frac{1}{2\pi j\cancel{\omega}t}\left[e^{j\omega_c t} - e^{-j\omega_c t}\right] \qquad \theta = \omega_c t$$

$$\frac{1}{\pi \cancel{\omega} t}\left(\frac{1}{2j}\right)\left[e^{j\theta} - e^{-j\theta}\right]$$

$$h(t) = \frac{1}{\pi t}\sin(\omega_c t)$$

b.

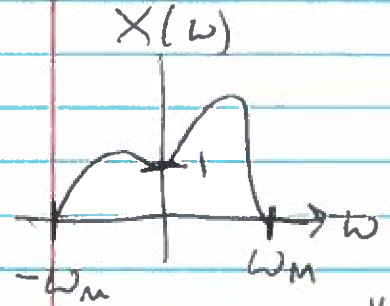




(c) Why this is an ~~LTI System~~ Low-pass filter:
~~All~~ Because ~~below~~ above ω_c, the multiplication of the filter in the frequency domain ~~eliminates~~ ~~eliminates~~ eliminates all components.
~~There~~. There is no drop-~~off~~ ~~region~~ or roll-off region, so it is considered ideal.

(d) see later pages

3. $X(\omega) = 0$ for $\omega < -\omega_M$
$\omega > \omega_M$



$y(t) = x(t)\cos(\omega_c t)$ where $\omega_c \gg \omega_M$

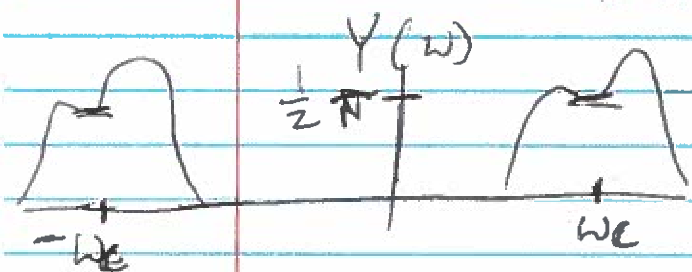
sketch $Y(\omega)$

$y(t) \rightarrow Y(\omega)$

Property: $x(t)h(t) \quad \frac{1}{2\pi} X * \underbrace{H(\omega)}$

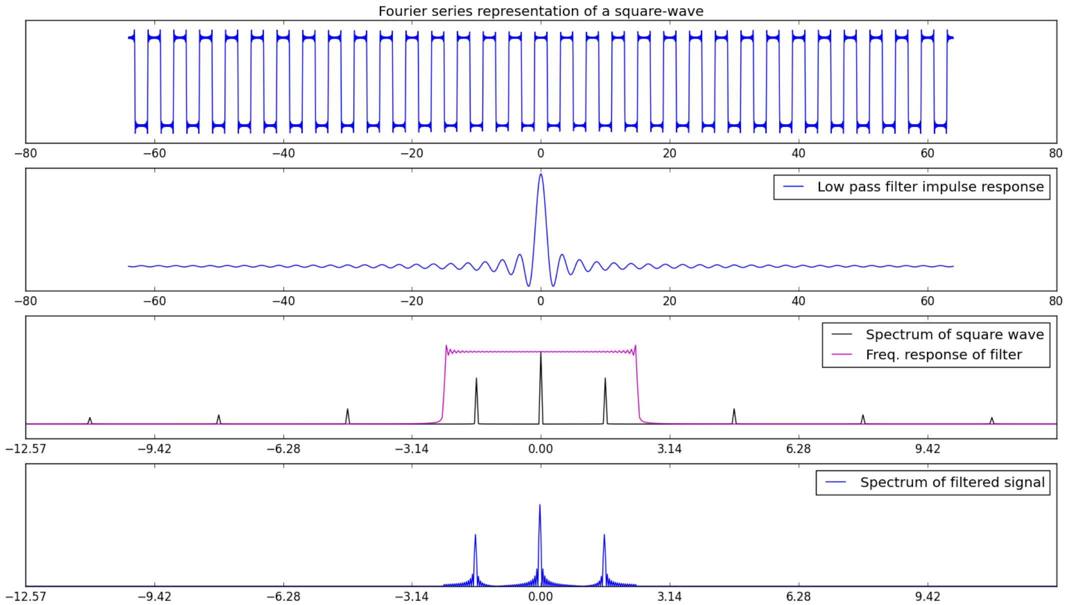
$\cos(\omega_0 t) \rightarrow [\pi\delta(\omega-\omega_0) + \pi\delta(\omega+\omega_0)]$

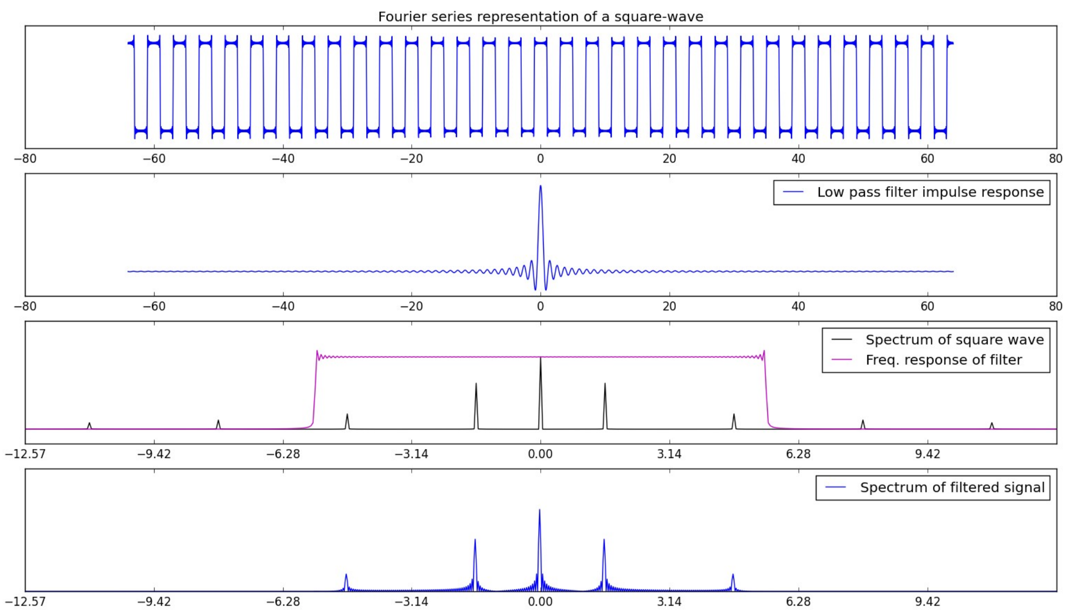
$Y(\omega) = \frac{1}{2\pi} X * [\pi\delta(\omega-\omega_c) + \pi\delta(\omega+\omega_c)]$



$\frac{1}{2\pi} X * [\pi\delta(\omega-\omega_c)] + \pi\delta($
$\frac{1}{2\pi}[\pi\delta(\omega+\omega_c)]$

$\frac{1}{2} X\delta(\omega-\omega_c) + \frac{1}{2} X\delta(\omega+\omega_c)$


Fourier series representation of a square-wave
Low pass filter impulse response
Spectrum of square wave
Freq. response of filter
Spectrum of filtered signal
−80
−60
−40
−20
0
20
40
60
80
−12.57
−9.42
−6.28
−3.14
0.00
3.14
6.28
9.42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