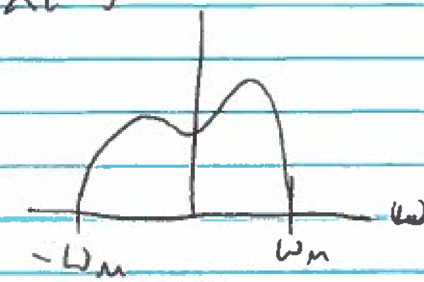
2015/03/24 | Michael Bocamazo

PS08

$x(t)$

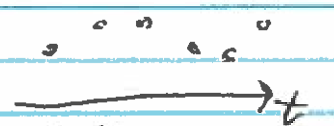
$X(\omega)$



$p(t) = \sum_{k=-\infty}^{\infty} \delta(t - kT_s)$

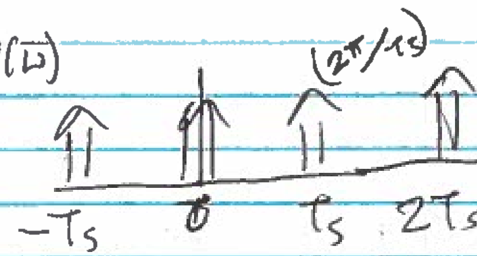
$x_p(t) = x(t)\,p(t)$

(a)

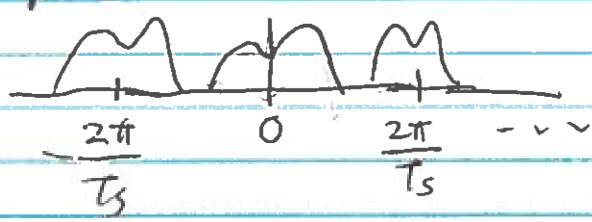


$x_p(t)$

(b) $P(\omega)$



(c) $X_p(\omega)$



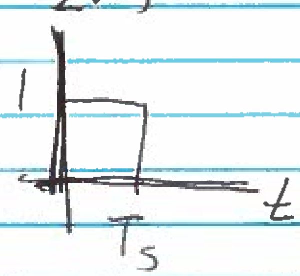
$\frac{\omega_M}{2} > \frac{2\pi}{T_s}$

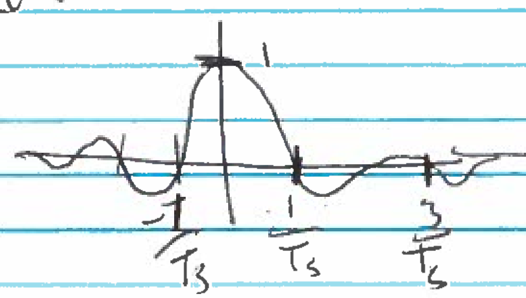
(d) ~~$\omega_M > 2T_s$~~ so that the frequency components cannot overlap

(e) Sampling over an infinite signal ~~at~~ ~~$\omega_M > 2T_s$~~ would find it, or using a bandpass with frequency $-\omega_m$ to ω_m, which in the time domain is a ~~sinc~~ sinc function.

(f) $z(t)$

which is $Z(\omega)$ in f

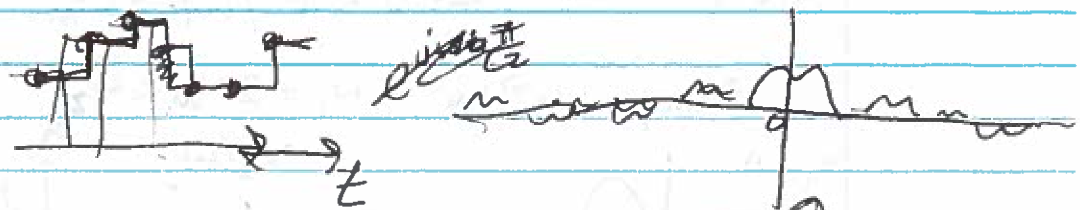




$x_z(t)$ - Zero order hold reconstruction of $x_p(t)$

(g) $x_z(t) = x_p * z(t)$

$x_z(t)$

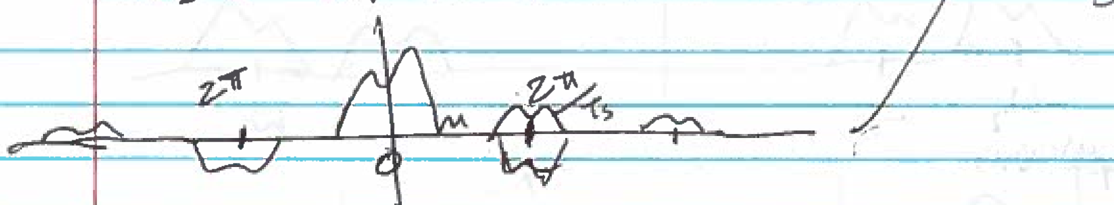


(h) $X_z(\omega)$

multiply each point by $e^{i\phi}$ to center signal, where $\phi = \pi$ to move $z(t)$ to center

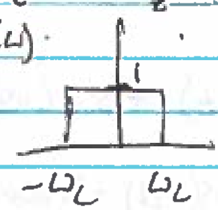
$e^{i\pi}$

$X_z(\omega) = X_p(\omega) Z(\omega)$

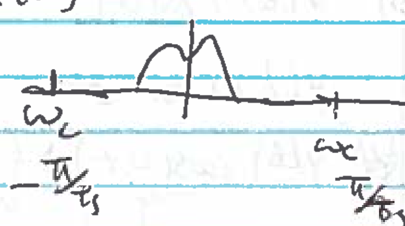


(i) $\bar{X}(\omega) = X_z(\omega) H(\omega)$ and $\hat{X}(\omega) = X_p(\omega) H(\omega)$

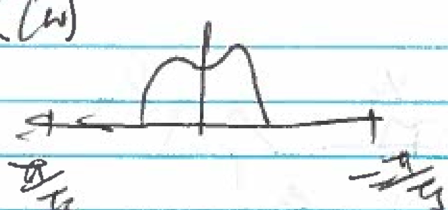
$H(\omega)$ $\omega_c = \frac{\pi}{T_s}$



$\bar{X}(\omega)$ (somewhat attenuated as $|\omega| \uparrow$)



$\hat{X}(\omega)$



(no attenuation)

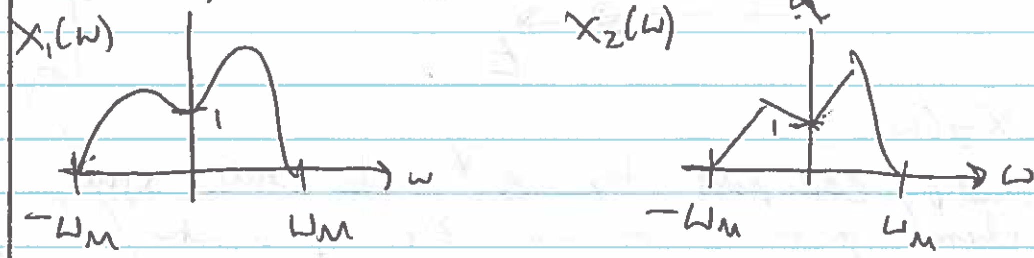
j. They differ by the attenuation caused by the multiplication of the sinc.

k. The ratio should be sinc, or is 0/0 because they should both be zero at π/T_s as that is the cutoff

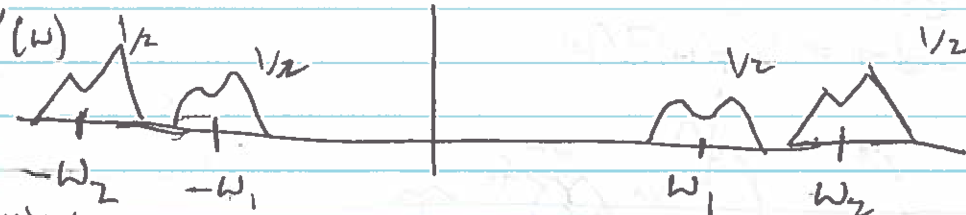
2. $y(t) = x_1(t)\cos(\omega_1 t) + x_2(t)\cos(\omega_2 t)$

$X_1(\omega)=0$ and $X_2(\omega)=0$ if $|\omega| > \omega_M$

$\omega_1 \gg \omega_M$, $\omega_2 \gg \omega_M$, $\omega_1 + 2\omega_M < \omega_2$



(a) $Y(\omega)$

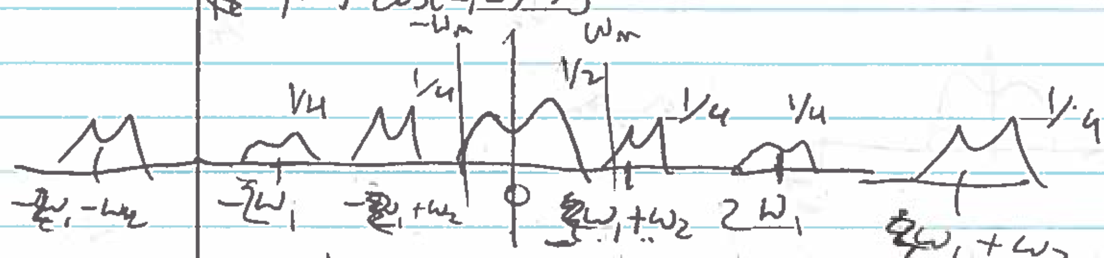


(b)

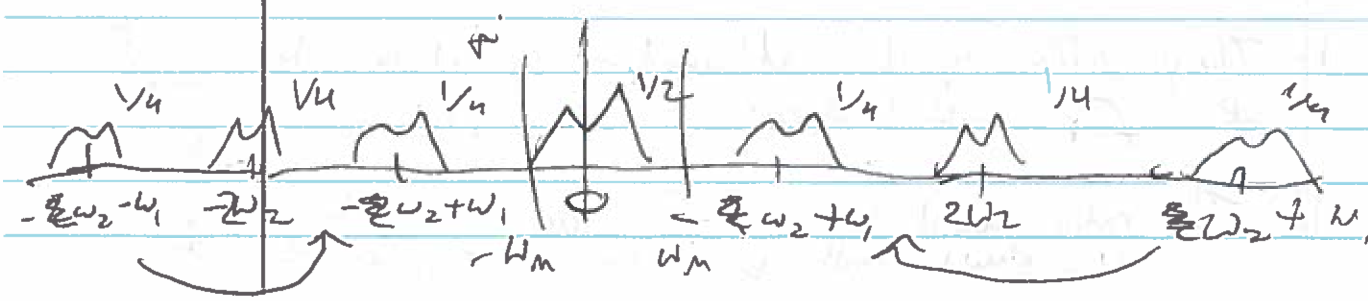
(b) $y(t)\cos(\omega_1 t) = x_1(t)\cos^2(\omega_1 t) + x_2\cos(\omega_1 t)\cos(\omega_2 t)$

$y(t)\cos(\omega_2 t) = x_2(t)\cos(\omega_1 t)\cos(\omega_2 t) + x_2\cos^2(\omega_2 t)$

$y(t)\cos(\omega_1 t) \to f$

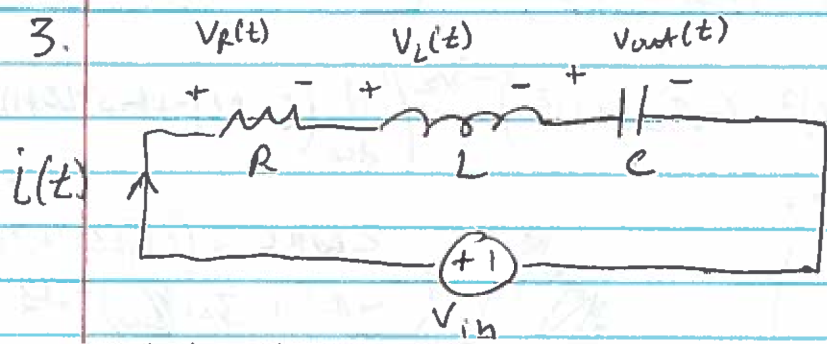


$y(t)\cos(\omega_2 t)$



c. Multiply $y(t)$ by $\cos(\omega_1 t)$, then use a band pass filter of $\pm\omega_M$, and multiply by 2 to account for the scale factor caused by the cosine. This gives the original $x_1(t)$. Similarly, multiply $y(t)$ by $\cos(\omega_2 t)$ and repeat the process to recover $x_2(t)$.
(See previous part for sketches w/ ω_M)

3.



$$i(t) = C\frac{d}{dt}V_{out}(t)$$

$$V_L(t) = L\frac{d}{dt}i(t)$$

$$i(t) = \frac{V_R}{R}$$

$$V_{in} = V_R + V_L + V_{out}$$

$$V_{in} = i(t)R + L\frac{d}{dt}i(t) + V_{out}$$

$$V_{in} = RC\frac{d}{dt}V_{out}(t) + L\frac{d}{dt}\left(C\frac{d}{dt}V_{out}(t)\right) + V_{out}$$

$$V_{in} = RC\frac{d}{dt}V_{out}(t) + LC\frac{d^2}{dt^2}V_{out}(t) + V_{out}$$

$$V_{in}(\omega) = j\omega RC\,V_{out}(\omega) + -\omega^2 LC\,V_{out}(\omega) + V_{out}(\omega)$$

$$H(\omega) = \frac{1}{j\omega RC - \omega^2 LC + 1} = \frac{1}{j\omega RC - \omega^2 LC + 1}$$

c. $$|H(\omega)| = \frac{1}{\sqrt{(\omega RC)^2 - \omega^4 L^2C^2 + 1}}$$

$\cancel{(\omega RC)^2 = \omega^2 LC}$

$\cancel{\omega^2R^2C^2 = \omega^4L^2C^2}$

$\cancel{\omega^2 = \frac{R^2}{L^2}}$ $\cancel{\omega = \frac{R}{L}}$

or $\omega = 0$

d. Find ω to maximize $H(\omega)$ as a fcn of R, L, C

$$\frac{d}{d\omega}(1-\omega^2LC)^2$$

$$0 = 2\omega RC + 2(1-\omega^2LC)(-2\omega$$

~~ ~~ imag - real $0 = 2\omega RC - 4\omega LC + 2\omega^3 L^2C^2$

$\omega = 0$

$0 = 2RC - 4LC + 2\omega^2 L^2C^2$

$$\omega = \pm\sqrt{\frac{-2RC+4LC}{2L^2C^2}}$$

$$\boxed{\omega = \pm\sqrt{\frac{1}{LC} - \frac{R}{4L^2C}}}$$

C orthogonal parts

$$\sqrt{(\omega RC)^2 + (-\omega^2LC+1)^2}$$

$$\frac{d}{d\omega}\left(\left(\omega RC\right)^2 + (-\omega^2LC+1)^2\right)^{-1/2}$$

$$-\frac{1}{2}\left((\omega RC)^2 + (-\omega^2LC+1)^2\right)^{-3/2}\left(\frac{d}{d\omega}\left((\omega RC)^2+(-\omega^2LC+1)^2\right)\right)$$

$$(\omega RC)^2 = -\omega^4L^2C^2 + 2\omega^2LC^2 + 1$$

$2\omega RC + (\omega^4(LC)^2 + 2\omega^2LC + 1)$

$(2\omega RC + 3\omega^3(LC)^2 + 4\omega LC$

$\boxed{\omega = 0}$

$0 = 2RC + 3\omega^2L^2C^2 + 4LC$

generates additional roots ~~but $\omega = 0$ is sufficient~~

$$\left[\cancel{\omega = \pm\sqrt{\frac{-2RC-4LC}{3L^2C^2}}}\right]$$

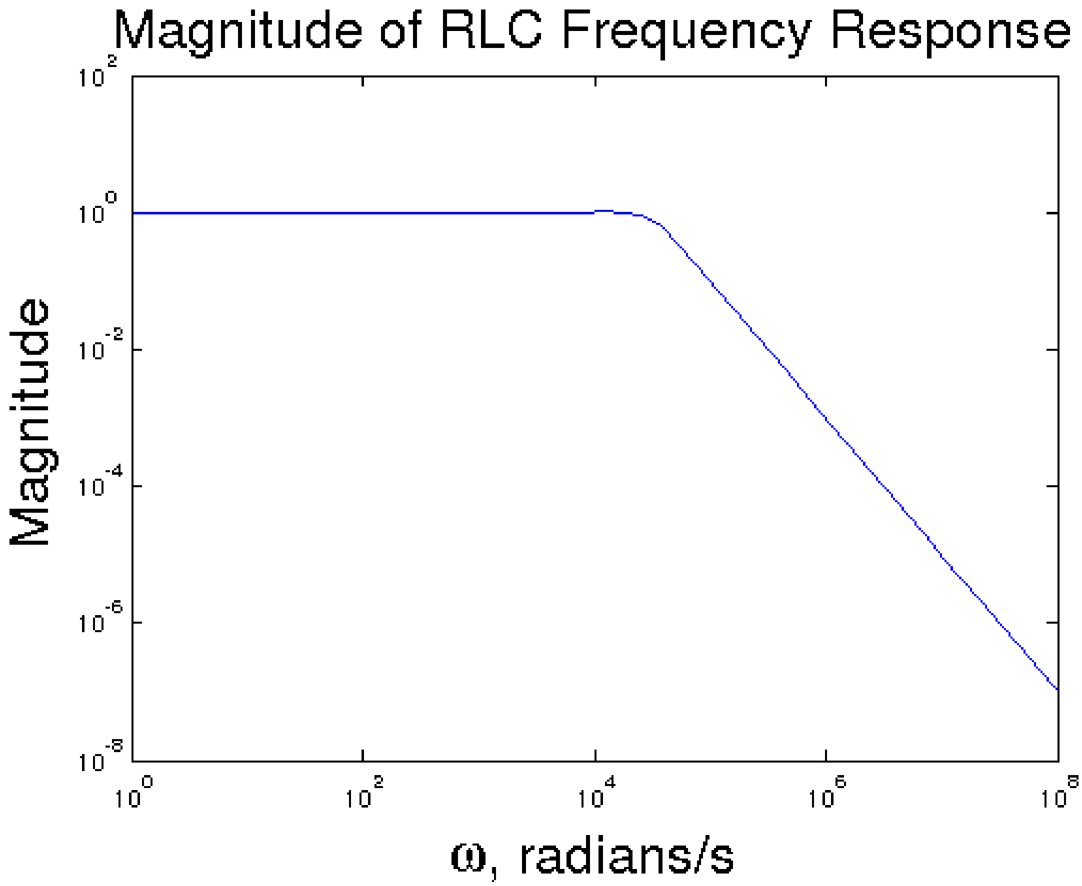
$$\omega = \pm\sqrt{\frac{1}{LC} - \frac{R}{4L^2C}}$$

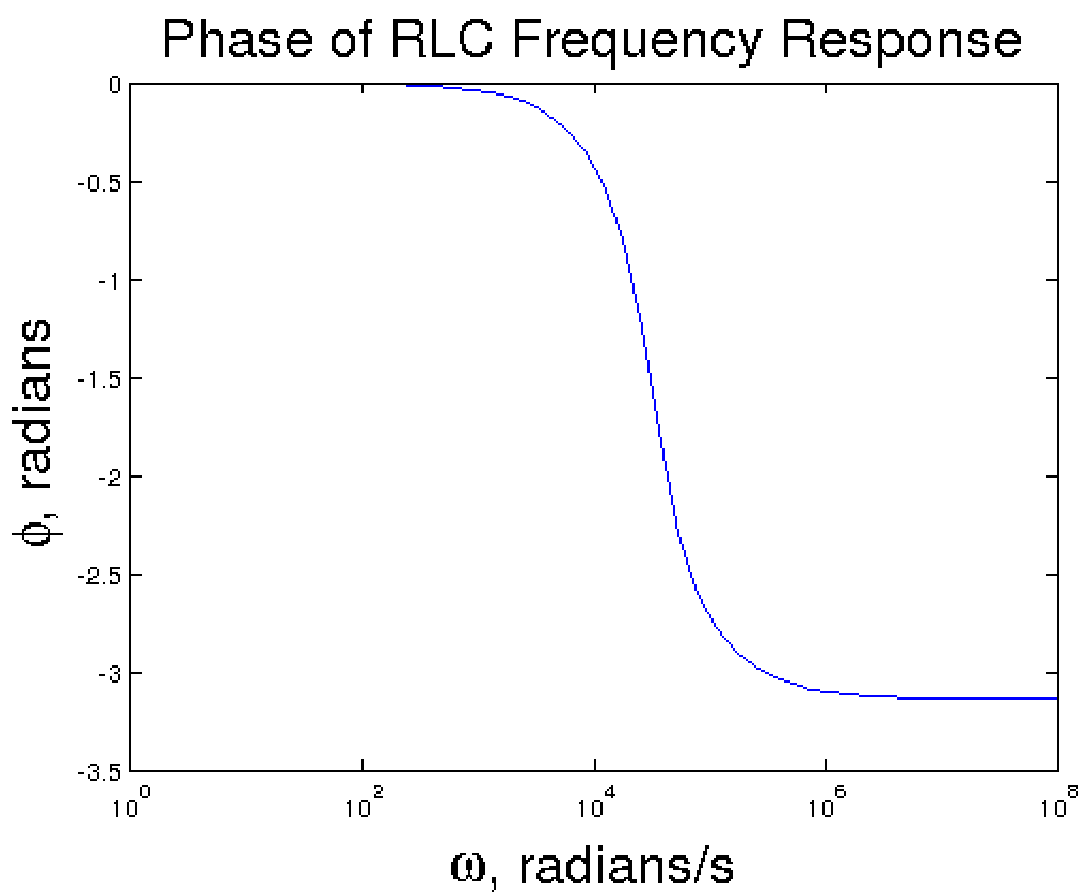
$$(RC)^2\left(\frac{1}{RC} - \frac{R}{4L^2C}\right) + \left(\left(\frac{1}{LC} - \frac{R}{4L^2C}\right)LC + 1\right)^2$$

$$\left(1 - \frac{RC}{4} + 1\right)^2$$

minimizes → $\frac{(RC)^2}{LC} - \frac{R^3C}{4L^2} + \left(\frac{RC}{16}\right)^2$

For the first set of values, $C = 10{-}7F$, $L = 10{-}2H$, $R = 400\Omega$.





For the second set of values, C = 10−7F, L = 10−2H, R = 50Ω.