Signals and Systems Final Project: Controlled Acoustic Interference

Michael Bocamazo and Dennis Chen

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1 Introduction

In this project, we investigated acoustic interference with an without control. Using two oscillators and a backdriven speaker as a microphone, we first showed experimentally the principle of acoustic interference using oscilloscopes and a map of expected points of interference. Then, we connected the output of our amplified receiver to the threshold voltage (V_+) of one of the two oscillators so that the received signal sent an error signal to the frequency of the oscillator. This created control around destructive interference which drove the received signal to zero by setting the phase such that the two received signals were opposed at the receiver. Phase is the integral of frequency, so a temporary change in the frequency of the oscillator changed the phase offset, which then reduced the error. Here, we will develop an analysis of the system in terms of the transfer functions of each of the components. We will first qualitatively explain how stability is reached, then develop an approximate quantitative of the incremental change around the set point.

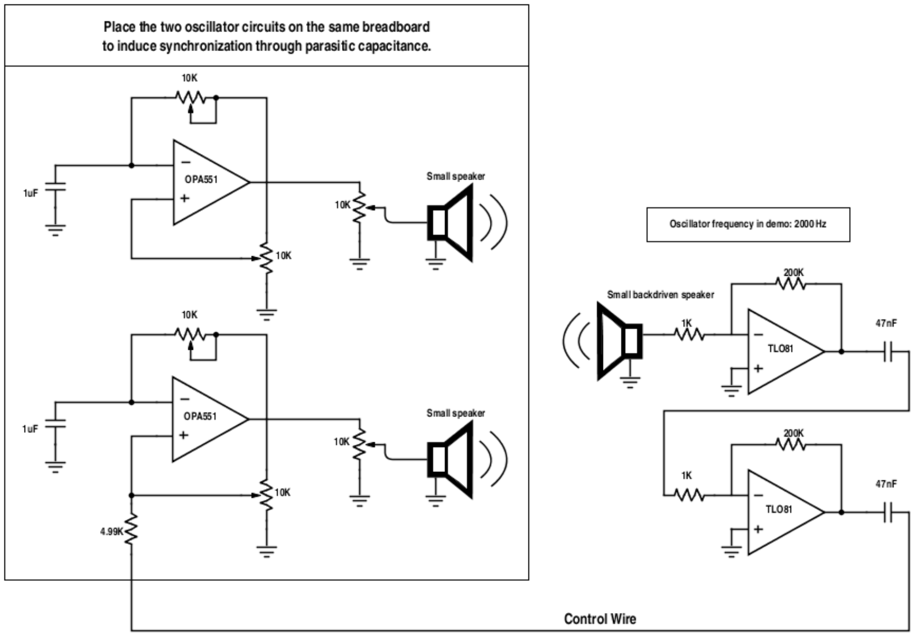


Figure 1: Schematic for controlled acoustic interference

2 Acoustic Interference

Our two oscillators were synchronized in both phase and frequency without control because of parasitic capacitance of the breadboard. This was quite useful in the construction and analysis of the circuit, because there is a trapping region of the values of the oscillators within which they will naturally synchronize. Without this effect, small disagreement in frequency would make observation of acoustic interference nearly impossible.
If the two speakers were locked in phase, the waves would add as follows:

$$s(t) = \cos(\omega t) + \cos(\omega t + \phi_d) \tag{1}$$

where $s(t)$ is the transmitted signal, ω is the frequency, and ϕ_d is the phase offset due to difference in distance from the two speakers to a point. By a trig. identity,

$$s(t) = 2\cos\left(\omega t + \frac{\phi_d}{2}\right)\cos\left(\frac{\phi_d}{2}\right) \tag{2}$$

$$\lambda = \frac{v}{\omega} \tag{3}$$

$$\phi_d = \frac{2\pi x}{\lambda} \tag{4}$$

where λ is the wavelength, and x is the difference in distance from the two sources.

$$s(x,t) = 2\cos\left(\omega t + \frac{\pi x}{\lambda}\right)\cos\left(\frac{\pi x}{\lambda}\right) \tag{5}$$
$$A = 2\cos\left(\frac{\pi x}{\lambda}\right) \tag{6}$$

where A is the amplitude of the received signal. With a time difference in the speakers, ϕ_t represents the phase offset in time,

$$s(x,t) = 2\cos\left(\omega t + \frac{\pi x}{\lambda} + \frac{\phi_t}{2}\right)\cos\left(\frac{\pi x}{\lambda} + \frac{\phi_t}{2}\right) \tag{7}$$

Destructive interference should drive ϕ_t so that the received amplitude is zero.

$$0 = \cos\left(\frac{\pi x}{\lambda} + \frac{\phi_t}{2}\right) \tag{8}$$
$$\frac{\pi x}{\lambda} + \frac{\phi_t}{2} = \frac{\pi}{2} + \pi n,\ n \in \mathbb{Z} \tag{9}$$
$$\phi_t = \frac{-2\pi x}{\lambda} + \pi + 2\pi n \tag{10}$$

By modular arithmetic, the final term on the right goes away, so

$$\phi_{t=setpoint} = \frac{-2\pi x}{\lambda} + \pi \tag{11}$$

3 Controlled Hysteretic Oscillator

In the normal hysteretic or relaxation oscillator, (see Wikipedia's derivation of the frequency) V_+, or the threshold voltage, is set to half the positive positive supply rail. By linking the output of our amplifier to the threshold voltage, we can change the frequency of the oscillator. By generalizing the threshold frequency from $\frac{V_{dd}}{2}$ to $V_{dd}x$, where x is some fraction, we can find how the frequency of oscillation depends on the threshold. By substitution starting here,

$$x = \frac{V_+}{V_{dd}} \tag{12}$$
$$xV_{dd} = V_{dd}\left(1 - \frac{3}{2}e^{\frac{-1}{RC}\frac{T}{2}}\right) \tag{13}$$
$$\left(\frac{-2}{3}x + \frac{2}{3}\right) = e^{\frac{-1}{RC}\frac{T}{2}} \tag{14}$$

Make the same steps as Wikipedia's derivation to get

$$\omega = \frac{1}{-2RC\log\left(\frac{2}{3}(1-x)\right)} \tag{15}$$

Using Wolfram Alpha, we found the derivative to be

$$\frac{d\omega}{dx} = \frac{1}{2RC(x-1)\log^2\left(\frac{-2}{3}(x-1)\right)} \tag{16}$$

Finally, note that phase is the integral of frequency, so we can link back to the previous acoustic interference section:

$$\phi_t = \int_0^t \omega dt + C \tag{17}$$

4 Controller

Now we will examine the behavior of the receiver circuit, which acts as the amplifier and controller. The analysis can be greatly simplified by recognition of subcomponents: the two op-amps act as simple multipliers (G_1, G_2) of the input voltage and minimal current is passed through the gain resistor. The gain is the ratio of the gain resistor and the resistor preceding the inverting input (referring back to the circuit diagram, the gains of both amplifiers are 200). There are two high-pass filters (a capacitor followed by a resistor), one between the amplifiers (C_1 and R_1), and another after the second amplifier (C_2 and R_2). The resistor preceding the first amplifier should be unimportant because there is negligible current that the op-amp input can pass or goes through the gain resistor. The inverting input voltage is effectively the voltage from the backdriven speaker. We can now write the transfer function as:

$$\frac{V_{out}}{V_{in}} = G_1 G_2 \left(\frac{j\omega R_1 C_1}{1 + j\omega R_1 C_1}\right)\left(\frac{j\omega R_2 C_2}{1 + j\omega R_2 C_2}\right) \tag{18}$$

5 Behavior

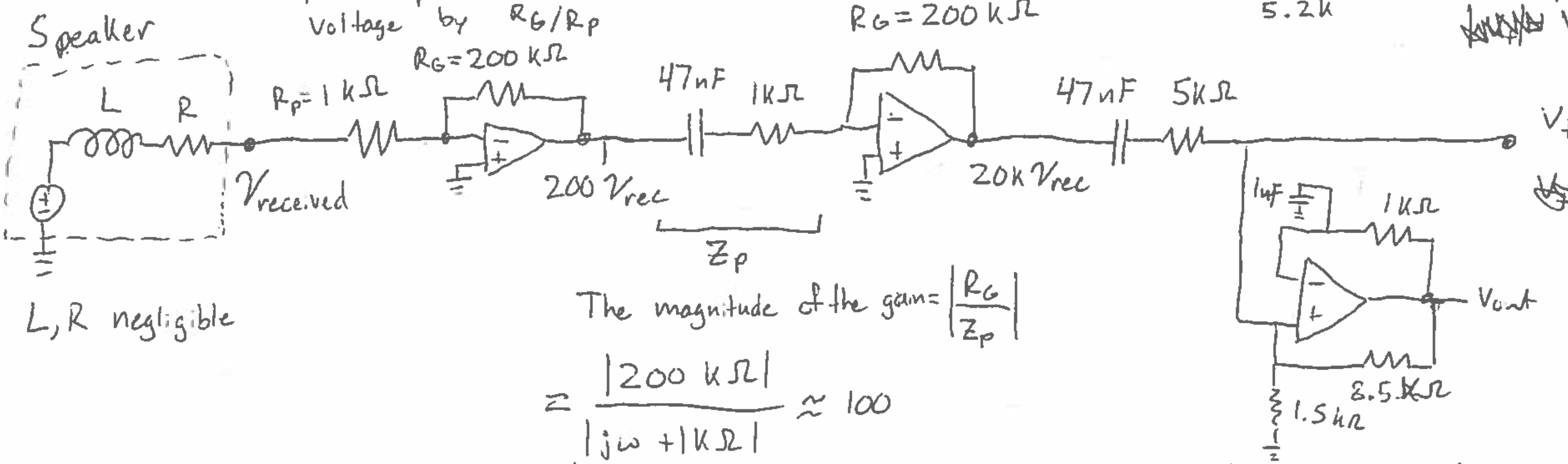
First, we will give the qualitative behavior of the system, and then we will examine the transfer functions of each component more closely to develop a small-signal linearized analysis of the change around the setpoint of the phase.

If there is a received signal at the speaker, the high pass filters remove the DC bias, the amplifiers increase the voltage, and then an error signal is brought into the non-inverting or threshold input, V_+. The threshold input determines the voltage to which the capacitor must charge for the oscillator to flip the output, and therefore determines the frequency with a locally linear relationship. A small change in V_+ produces a small change in the frequency of the oscillator, which is then integrated and changes the time-based phase offset, ϕ_t. This loops back and changes the difference between the received signal and so the received voltage. A positive error signal then increases V_+, which decreases ω, which decreases ϕ_d. In the final two pages, we will show a more complete analysis of the entire system with a highly approximate attempt at linearization (the transmission cannot be fully linearized, it must be a sinusoid).

Approximation of Transfer Function

From the received signal to ~~speaker~~ ~~frequency of oscillator~~ threshold voltage

The amplifier multiplies the voltage by R_G/R_P



L, R negligible

The magnitude of the gain $= \left|\frac{R_G}{Z_P}\right|$

$$= \frac{|200\,k\Omega|}{\underbrace{|j\omega + 1k\Omega|}_{2.2\,k}} \approx 100$$

$\omega = 2000$ Hz for the transmitted signal

Gain of final segment $=$

$$\frac{|1.3\,k\Omega|}{\underbrace{|j2k + 5k\Omega|}_{5.2k}} \approx 0.25$$

(time varying)

so $\tilde{V}_+ = (5\,k)\,V_{rec}$

$R_{equiv} = (1.5k \| 8.5k) = 1.3\,k\Omega$

To achieve an ω of ≈ 2000 Hz, we can use this approximation – the oscillating resistor was $\approx 1k\Omega$, and the threshold pot had $\approx 8.5\,k\Omega$ to $1.5\,k\Omega$ split.

The time varying part of V_+, $\tilde{V}_+$, is $= Q\,V_{rec}$

where $Q = 5 \times 10^3$

Approximation of linearized transfer function from the threshold voltage to the oscillator frequency.

From the Section "Controlled Hysteretic Oscillator",

$$\frac{df}{dx} = \frac{1}{2RC\,(x-1)\log^2\left(-\frac{2}{3}(x-1)\right)} \quad \text{where } x = \frac{V_+}{V_{dd}}$$

From the previous analysis, $x \approx 0.125$

Which means that around the set point with little divergence,

$$\frac{df}{dx} \approx -2\times10^6 \quad \text{by plugging in } x = 0.125$$

Then,

$$\Delta f \approx (-2\times10^6)\frac{\Delta V_+}{V_{dd}} = (-1.6\times10^4)\,\Delta V_+$$

Call this ratio A, $A = -1.6\times10^4$

So far, $\Delta f \approx QA\,\tilde{V}_{rec} = (-8\times10^3)\,\tilde{V}_{rec}$.

Next, ~~ϕ~~ phase is the integral of frequency, so

$$\tilde{\phi}_t = QA\int_0^t \tilde{V}_{rec}\,dt + C$$

This transfer function is $1/s$. (integration).

From before, $\phi_{t\,(\text{setpoint})} = -\frac{2\pi x}{\lambda} + \pi$, which is where $A = 2\cos\left(\frac{\pi x}{\lambda} + \frac{\phi_t}{2}\right) = 0$.

The argument of the cosine θ at equilibrium is $\pi/2$ where we can use a small-angle approximation, that $\cos(x) \approx -(x-\pi/2)$ for x near $\pi/2$.

So $A = -\left(\frac{2\pi x}{\lambda} + \underbrace{\tilde{\phi}_t}\right) + \frac{\pi}{2}$, and $\frac{dA}{d\phi} = -1$. Call this factor B.

The time-varying component of the signal cannot be linearized with changing ϕ, because it must be sinusoidal.

The amplitude of the signal is then attenuated in the air, which scales B.

So:

$$|\dot{V}_{rec}| = B\dot{\phi} = B\tilde{f} = BA\tilde{V}_+ = BAQ\,V_{rec}$$

Block Diagram of System: