1 Problem Description

Imagen Healthcare is a MRI scanners manufacturer and they want to design an inventory sharing plan for their RF amplifier to minimize the shipping cost due to emergency orders among its 20 dealers. So we are hired to advice on pool configuration and contact sequence rules.

To approach this problem, we designed several pool configurations and contact sequencing rules and employed a stochastic model to evaluate total shipping costs under these different plans. We also used a LP model to further test the shipping costs under two pool configuration methods. Then, we compared the above shipping cost estimates with the shipping cost under no-sharing plan and made the final recommendations to Imagen Healthcare on the inventory sharing plan. All technical details and processes are listed in Appendix.

2 Problem Assumptions

These following assumptions are shared by the two models introduced in section 4.

1. According to the information from Imagen's supply chain planner, we assume that Imagen uses a periodic review with lead time model for its inventory control. In the beginning of the 1st week, Imagen has enough inventory for the demand of the 1st week and place an order to meet the 90% service level for the 2nd week, i.e. Imagen place an order for demand over the following week at beginning of each week to meet service rate. We calculate expected shortage and excess under this assumption.
2. Since there is no apparent seasonality in the demand data, we assume demand for each week is normally distributed. For proof please refer to Appendix 6.1.

3 Parameter Calculation

Parameters used in this report are as follows:

S = Order-up-to-level

X = Demand

μ = Demand mean for weekly demand

σ = Standard Deviation for weekly demand

$f(x)$ = PDF for weekly demand

ϕ (x) = Value of CDF evaluated at x where x is normally distributed with mean of zero and Std. Deviation of 1

For calculation process of Q and μ, σ for each dealer please refer to Appendix 6.2.

4 Stochastic and LP Models

4.1 The Stochastic Model

4.1.1 Model Description

4.1.1.1 Model Assumption

1. Each dealer will only order from a single dealer within its pool that has more than it needs (Expectation of shortage) for emergency order. If no one in its pool has enough inventory (Expectation of excess), it will order from DC.

In the original problem, there is no fixed cost associated with placing an emergency order. However in a practical setting, placing an emergency order from another dealer or DC will incur fixed costs besides regular shipping cost such as over time costs, processing costs at both ends and express shipping cost. Therefore, the more order you place, the higher fixed costs will be. Additionally, multiple emergency orders from different dealers may arrive at different times so that there may be short term shortage despite the total amount equals what is needed. We make this assumption based these two considerations.

2. If group each dealer with those closer than its average distance from the other 19 dealers, we could significantly optimize the average distance for each dealer from its pool members compared to other pool configurations.

The heuristic proof is in Appendix 6.3.

3. When grouping dealers, first consider those dealers with large average distance from others since controlling the average distance within their pools will be the key to reduce total average distance.
This can be referred from the 80:20 rule.

4. When calculating shipping cost, we assume each dealer will have A units of shortage for each period where A is expected shortage as recipient and B units of excess as donor.
We make this assumption due to the fact that demand, shortage and excess are all random variable while expectation accurately captures their average value over all periods.

4.1.1.2 Two Pooling Configurations

Distance Based Pool Configuration (DB): Since in this problem, minimizing shipping costs is the sole objective, it is natural to group dealers which are close to each other to achieve optimal pool configuration. But due to the fact that each dealer serves as potential sink and source, only picking the shortest path within all the pairs of dealer will not necessarily minimize the total shipping costs. So according to Assumption 2, we design a heuristic method to find out the sub-optimal pool configuration.

First of all we found 5 of all 20 dealers (Atlanta, Houston, Jacksonville, Kansas City and Louisville respectively) were significantly closer to DC than the rest of the dealers. For proof please refer to Appendix 6.4. Therefore, we do not group them with any other dealers but DC due to the fact that DC has unlimited stock and there is no penalty for ordering from DC.

Then we found there are 5 dealers (Los Angeles, Phoenix, Seattle, San Francisco and Salt Lake City respectively) which were very far from the rest 15 dealers and DC. Nevertheless, they are closer to each other. For proof please refer to Appendix 6.4. So it is essential to group them together since placing them with other dealers will increase average distance within that pool sharply according to Assumption 3.

From Assumption 3 again we group the rest 10 dealers according to the fact that Denver and Miami are very far from each other. After separating them into two pools, we match the rest 8 dealers by their closeness to Denver and Miami. For detail please refer to Appendix 6.4. In the end, the pools are as follows:
Atlanta, Houston, Wichita, Kansas City and Louisville do not pool with other dealers, i.e. they only order from DC; Los Angeles, Phoenix, Seattle, San Francisco and Salt Lake City form the second pool and Buffalo, Charlotte, New York, Miami and Philadelphia form the third pool and Denver, Des Moines, Wichita, Minneapolis and Chicago the fourth pool. The total pool configuration can be understood from the table below –

Pool 1	Pool 2	Pool 3	Pool 4
Atlanta	Los Angeles	Buffalo	Denver
Houston	Phoenix	Charlotte	Des Moines
Jacksonville	Seattle	New York	Wichita
Kansas City	San Francisco	Miami	Minneapolis
Louisville	Salt Lake City	Philadelphia	Chicago

Pool Configuration Considering Both Distance and Expectation of Excess (DEE):
To take into account the fact that when one dealer A place emergency order from another dealer B, it increases B's risk of having stock-out, we consider another pool Configuration based on both distance and expectation of excess. The basic idea is to group dealers with large excess with dealers with relatively small ones to achieve risk pooling. For detail please refer to Appendix 6.5.
In the end, the pools are as follows:

Group 1	Group 2	Group 3	Group 4	Group 5	Group 6
Buffalo	L.A	Louisville	Miami	Des Moines	Denver
New York	Phoenix	Charlotte	Jacksonville	Chicago	Salt Lake city
Philadelphia	San Francisco	Memphis		Kansas City	Wichita
	Seattle	Atlanta		Minneapolis	
		Houston			

4.1.1.3 Two Contact Sequencing Rules

Closest First Rule (CFR): Under this rule, when one dealer is having shortage, it will turn to the closet pool member first to check if it has enough excess inventory. If it does not, that dealer will look into the next closest member until it finds that the next dealer is farther than DC or it exhausts all members. Then it will order directly from DC. For example please refer to Appendix 6.6.

Highest Excess First Rule (HEF): Under this rule, when one dealer is having shortage, it will turn to the pool member with the highest excess inventory first to check if it has enough to cover needs. If it does not, that dealer will look into the member with the next highest excess inventory until it finds that the next dealer is farther than DC or it exhausts all members. Then it will order directly from DC. For example please refer to Appendix 6.6.

4.1.2 Shipping Costs for Four Alternative Plans

According to Assumption 4, we calculate all annual shipping costs under four plans. For individual cost and calculation example please refer to Appendix 6.2.

4.1.2.1 DB & CFR Shipping Cost

The total annual shipping cost for all 20 dealers is 59174.25. (mile * unit)

4.1.2.2 DB & HEF Shipping Cost

The total annual shipping cost for all 20 dealers is 67834.97.

4.1.2.3 DEE & CFR Shipping Cost

The total annual shipping cost for all 20 dealers is 89234.65.

4.1.2.4 DEE & HEF Shipping Cost

The total annual shipping cost for all 20 dealers is 93824.67.

4.1.2.5 No-sharing Plan Shipping Cost

The total annual shipping cost for all 20 dealers is 184111.72.

4.1.3 Conclusions and Recommendations

From results in 4.1.2, we clearly see that DB & CFR plan is much more superior to the other 3 alternatives. This result can be seen intuitively from the problem setting. From the statistical analysis table in Appendix 6.2, we can see that the expected shortage in each week is much lower than expected excess which is a result of 90% percent service rate. Therefore, the benefits gained from ordering from member with highest excess is insignificant compared to the benefits ordering from closest member in terms of minimizing total shipping costs. For proof please refer to Appendix 6.2. The same logic applies when considering whether to pool dealers solely based on average distance. If the risk of shortage is greater for each dealer, there may be stronger incentives for the company to order from dealers with higher excess.

4.2 The Transportation Problem Model

Based on our extensive research on various transshipment problems, we noticed that most technical papers have used Linear programming for their cost optimization model in this kind of environment. In addition, this problem can also be easily formulated into a linear programming model.

A sample objective function and its corresponding constraints have been included in the appendix for instructor's reference. In order to solve this linear programming model for our different configuration pools (including no-sharing plan policy) we used a trial version of a software called "LINGO". LINGO is a comprehensive tool designed to make building and solving linear, nonlinear and integer optimization models faster, easier and more efficient. A sample program code written for this model is also included in the appendix.

Pool Configuration (1):

We have pooled the dealers based upon their distance and that at least *one* (but *not all*) of the dealers in each pool should have a large excess (large inventory). This is because if we group the dealers such that every dealer is short of supply, then they won't be able to fulfill the other dealers' demands. Likewise, if every dealer has significant large excess, they don't need to order from each other and this excess amount is wasted. So first we formed the groups based on distance and then made sure that each group has at least one dealer with a large excess.

For grouping the dealers according to the distance, we looked into the distance matrix and noticed that there are some cities that are closer to each other than to the other cities. For example dealers in cities like Buffalo, Philadelphia and New York are closer to each other than to Houston, Atlanta, etc.

Pool Configuration (2):

This pool consists of two sub-pool configurations. In one sub-pool we have grouped all the dealers together in one big pool in such a way that larger dealers (larger excess) will be supplying to smaller ones (less excess). In the other sub-pool, we've grouped all the large excess dealers that were suppliers in the above pool as sinks and MDC as the only source or provider. Meaning that they will only be ordering from Memphis.

Comparison of costs obtained from above pool config. With No shared plan

5 Result analysis and Recommendations

From the numerical results above, we know that under the demands pattern of Imagen, it will be better to divide the dealers solely based on their distances with each other. This is because the expectation of shortage is relatively small so that ordering from another dealer will not increase its risk of stock out significantly.

The same logic applies when deciding the contact sequencing rules. Therefore, our recommended pool configuration and contact sequence rules are as follows:

Pool Configuration:

Pool 1	Pool 2	Pool 3	Pool 4
Atlanta	Los Angeles	Buffalo	Denver
Houston	Phoenix	Charlotte	Des Moines
Jacksonville	Seattle	New York	Wichita
Kansas City	San Francisco	Miami	Minneapolis
Louisville	Salt Lake City	Philadelphia	Chicago

Contact Sequence Rule:

CFR.

The projected cost saving under this inventory sharing plan will be almost 70% of the current total shipping cost associated with emergency orders.