Determination of the Charge-to-Mass Ratio of the Electron (e/m)

Properties of electrons can be studied in order to determine the e/m ratio of electrons. The e/m ratio is important because it absolutely predicts a particles direction of travel in a vacuum when subject to magnetic or electric fields. In this lab a deflection tube is used to measure the effects of an electric field on a beam of accelerated electrons from a cathode ray tube. Helmholtz coils are used in conjunction with the cathode ray tube to measure the effects of a magnetic field on a beam of accelerated electrons. Measurements of the deflection distance of the beam at various electric and magnetic field strengths can be used to determine the e/m value for an electron.

Experiment 6.3.1: Magnetic Deflection

Objective: To discover the deflecting force a magnetic field causes on an electron accelerated perpendicularly through its field and to use these data to find the e/m ration of an electron.

Procedure: See manual

Data:

Positive Deflections

V_a (V)	x (m)	y (m)	I_B (A)	B (T)	R (m)	e/m (C/kg)	σ
2500	.10	.01	0.0731	3.092×10^{-4}	0.505	2.051×10^{11}	$\pm 3.857 \times 10^{10}$
3000	.10	.01	0.0803	3.397×10^{-4}	0.505	1.019×10^{11}	$\pm 3.857 \times 10^{10}$
3500	.10	.01	0.0885	3.744×10^{-4}	0.505	9.791×10^{10}	$\pm 3.857 \times 10^{10}$

Negative Deflections

V_a (V)	x (m)	y (m)	I_B (A)	B (T)	R (m)	e/m (C/kg)	σ
2500	.10	-.01	0.0878	3.714×10^{-4}	0.505	1.412×10^{11}	$\pm 3.857 \times 10^{10}$
3000	.10	-.01	0.0989	4.183×10^{-4}	0.505	1.345×10^{11}	$\pm 3.857 \times 10^{10}$
3500	.10	-.01	0.1049	4.433×10^{-4}	0.505	1.397×10^{11}	$\pm 3.857 \times 10^{10}$

Analysis: The magnetic field produced from the helmholtz coils creates a force that causes the originally straight electron beam to be deflected, either positively or negatively, depending on the direction of the current. The measurements recorded are then used to calculate the e/m of an electron. The above calculations were completed using the following equations:

$B = 4.23 \times 10^{-3} I_B$ $\qquad R = (x^2 + y^2)/(2y)$ $\qquad e/m = (2V_a)/(BR)^2$

$\sigma = [1/(N-1) \sum_{i=1}^{N} (\overline{e/m} - e/m_i)^2]^{1/2}$, N = 6 $\qquad \bar{\sigma} = \sigma/\sqrt{N}$

$\overline{e/m} = 1.369 \times 10^{11}$ C/kg $\qquad \bar{\sigma} = 1.574 \times 10^{10}$

The results of the individual calculations for the e/m are fairly far from the accepted value of e/m = 1.7589×10^{11} C/kg. However, the average value of 1.369 C/kg is closer, less than 2.5 standard deviations from the accepted value.

Choosing a point with a large x-value value will increase the accuracy of the measurements because it will yield larger y-values, which will decrease the relative error. However, a source of error in the data may be because the beam was aimed to intersect (0.10, 0.01) instead of trying to keep I_B as close to 0.150 A. This technique may have caused increased error in the measurements, such as that found in the positive deflections where the experimental e/m is calculated to be less than the accepted value for e/m. Due to the force of the earth's magnetic field higher values than the accepted e/m would be expected.

Charge-to-mass ratio calculation accuracy increases when balancing both the positive and negative values which serves to negate the effects of the earth's magnetic field. Additionally, the electron beam may not have been perfectly at zero y and finding the average value would eliminate this error in measurement.

The objective of this experiment was accomplished – the data revealed that the electron beam will move in a circular path in accordance with the magnetic field force equations (F = iL x B which causes an inward acceleration, leading to a circular path). Also, the e/m ratio for the electron was successfully calculated using the data collected. The experiment was important because a method for calculating e/m was learned.

Experiment 6.3.2: Electrostatic Deflection

Objective: To observe the deflection of an electron beam when accelerated through an electric field and to report the experimental slope of the electric field using these measurements.

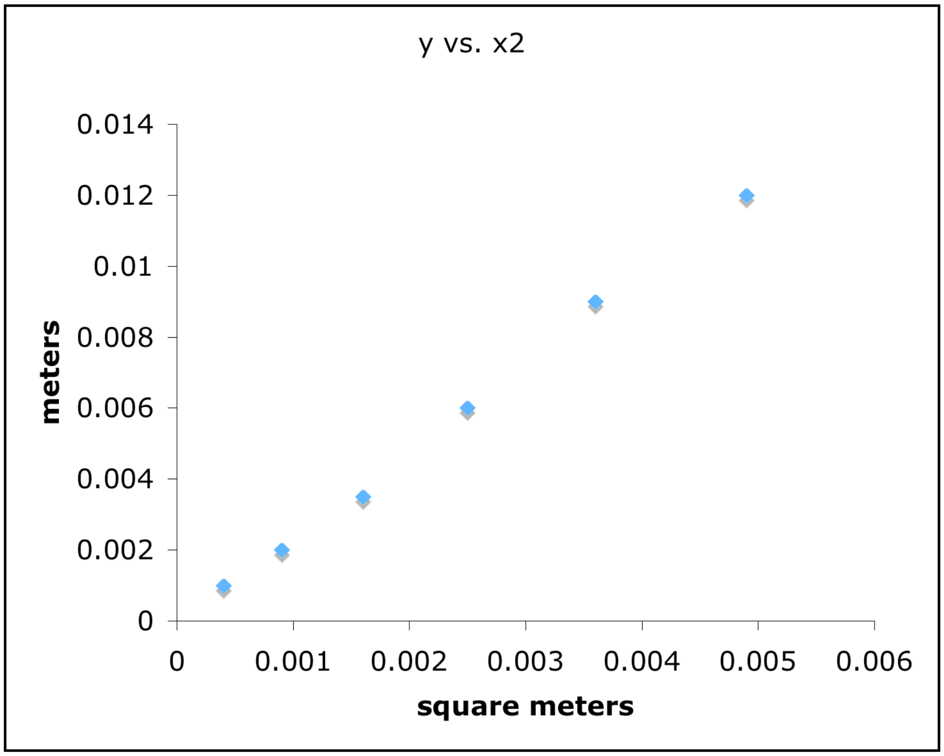
Procedure: See manual, set voltage at V_a = 2500 V, V_d = 3000 V, d = 0.052 m

Data:

Deflections

x (m)	.02	.03	.04	.05	.06	.07
y (m)	.001	.002	.0035	.006	.009	.012
x^2 (m^2)	.0004	.0009	.0016	.0025	.0036	.0049

Analysis:



The electrostatic field causes the beam of electrons accelerated perpendicularly to be deflected along a parabolic path. This is different from the magnetic field which causes electrons to travel in a circular pattern because the magnetic force causes an inward acceleration. In the above graph x^2 is used instead of x because of the results found when the following equations are solved for y. The explanation of this parabolic path and the expected slope of this graph can be derived from:

$$y = ayt^2/2 = 1/2[e/m*E](x^2/v_e^2)$$

The calculated experimental slope ($\Delta y/\Delta x^2$) of our graph is 2.44.

The electric field strength can be calculated two ways. Easily by using the equation:

$$E = V_d/d = 5.769 \times 10^4 \text{ N/C}$$

A more precise calculation can be derived from combining the following two equations and replacing v_e^2 in the first by solving for it in the second.

$$y/x2 = 1/2[e/m*E](x^2/v_e^2)$$

$$1/2m_eV_e^2 = q_eV_a$$

Now the e/m ratio can be eliminated from the equation and E can be solved for using the slope.

$$E = 4V_a y/x^2 = 2.44 \times 10^4 \text{ N/C}$$

This second value for the electric field is more accurate because the experimental electric field is not constant across the capacitor plates. The size of the plates is of the same order of magnitude as the distance between them; therefore, the electric field near the ends of the plates varies from that of the center. The equation V_d/d assumes a constant electric field, while the other equations allow the experimental slope to be used in order to more accurately represent the non-constant electric field strength.

The experiment's objective was accomplished. The experiment was important because the slope of the electric field was calculated which can now be used in the last experiment to more accurately find the e/m value. The experimental slope yields more accurate results than using the equation V_d/d when capacitor plates are not infinite.

<u>Experiment 6.3.3: Determination of the Ratio e/m by Balanced Deflection</u>

Objective: To balance the forces from both an electric and magnetic field on a beam of accelerated electrons and use the experimental deflection values to find the e/m ration of an electron.

Procedure: See manual, vary V_a from 2000 V – 4500 V

Data:

Deflections

V_a (V)	2000	2500	3000	3500	4000	4500
V_d (V)	3000	3000	3000	3000	3000	3000
I_B (A)	0.311	0.284	0.264	0.245	0.229	0.220
e/m (C/kg)	4.8×10^{11}	4.61×10^{11}	4.48×10^{11}	4.43×10^{11}	4.43×10^{11}	4.27×10^{11}
e/m* (C/kg)	8.6×10^{10}	8.25×10^{10}	7.96×10^{10}	7.92×10^{10}	7.93×10^{10}	7.64×10^{10}

Analysis: The above calculations for I_B, e/m, and e/m* were completed using the following equations:

$e/m_{uncorrected} = 1/(2V_a)\,(V_d/d)^2 1/(KI_B)^2$
$k = 4.23 \times 10^{-3}$

$e/m^*_{corrected} = 1/(2V_a)\,(E)^2 1/(KI_B)^2$,
$E_{using\ slope} = 4V_a y/x^2$

$$\sigma = [1/(N-1) \sum_{i=1}^{N} (\overline{e/m} - e/m_i)^2]^{1/2},\ N = 6 \qquad \bar{\sigma} = \sigma/\sqrt{N}$$

$\overline{e/m} = 4.498 \times 10^{11}$ C/kg $\qquad \bar{\sigma} = 7.423 \times 10^9$
$\overline{e/m^*} = 8.06 \times 10^{10}$ C/kg $\qquad \bar{\sigma} = 1.355 \times 10^9$

Charge-to-mass ratio values calculated using E calculated from the slope instead of V_d/d serve to increase accuracy by accounting for the non-infinite plates of the capacitor. The mean values for the e/m in both cases are very far from the accepted value of 1.759×10^{11} C/kg. Subtracting the average values shows that the value corrected by using slope is closer to the accepted value. The corrected value does have a smaller standard deviation which may have resulted from increased precision with this method. However, both e/m's are very far away from the accepted value.

There are many compounding sources of error in this experiment. Again, there is the problem that the electric field is not constant across the plates because they are not infinite. This causes systematic error in the experiment such that the trajectory of the electron beam is bowed in the middle with the B field unable to completely cancel out the E field. Also, the effects of the earth's magnetic field have been disregarded and may have caused inaccurate measurements.

The goal of the experiment was poorly accomplished with the measured e/m being very far from the accepted value. This experiment was valuable because it facilitated the investigation of various sources of systematic error.

Question 6.5.3

Theoretically, there would be no change in the trajectory of an electron doubled in mass and charge in a magnetic field because it is dependent on the *ratio* of charge to mass which would be unchanged.

Conclusion:

The charge-to-mass ratio of an electron was calculated using two separate methods. First, the deflection data of an electron beam accelerated between two capacitor plates were used to find the e/m. The second method involved balancing the forces on an electron beam from an electric field and a magnetic field. More accurate results were found using the first method which may be because the second method had more sources of error due to error from both the E field (capacitor plates) and B field (earth's own magnetic field). Additionally, it was learned that the slope method for calculating E yielded more accurate results than using the equation V_d/d when finding the e/m of an electron.