Joshua Mbego 8.022 Pset 5

[Question 1] (3.53)

$$C = \frac{A}{\epsilon_0 s} \rightarrow Q = CV \rightarrow C = \frac{Q}{V}$$

$$C_2 = \frac{Q/2}{\frac{V}{4}}$$

, where $\frac{V}{4}$ is because electric field on parallel plates is constant and is only dependent on q because q is q/2 and the distance this electric field has to travel is 1/2

$$C_2 = \frac{Q4}{2V} = \frac{2Q}{V}$$

and there are two capacitors

V is $\frac{1}{4}$

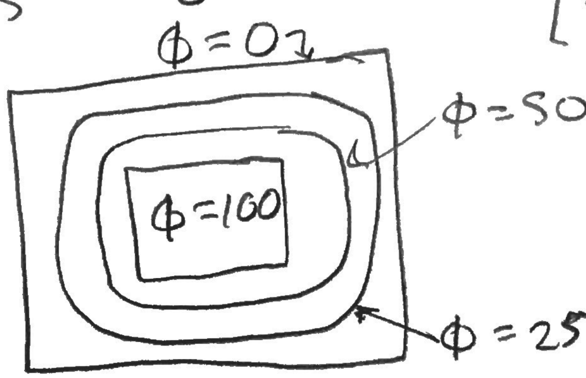
so

$$C_{new} = 2(C_2) = \frac{4Q}{V} = 4C$$

[Question 2] (3.76)

$$K = \begin{bmatrix} a & b \\ c & d \\ e & f \\ g & 0 \end{bmatrix} \quad k_0 = \begin{bmatrix} 50 & 25 \\ 50 & 25 \\ 50 & 25 \\ 25 & 0 \end{bmatrix} \quad k_1 = \begin{bmatrix} 56.25 & 25 \\ 56.25 & 25 \\ 37.5 & 25 \\ 12.5 & 0 \end{bmatrix}$$

$$k_2 = \begin{bmatrix} 59.375 & 26.5625 \\ 54.6875 & 26.5625 \\ 40.625 & 18.75 \\ 12.5 & 0 \end{bmatrix} \quad k_3 = \begin{bmatrix} 61.23 & 28.32 \\ 55.47 & 26.17 \\ 38.28 & 17.77 \\ 9.96 & 0 \end{bmatrix}$$

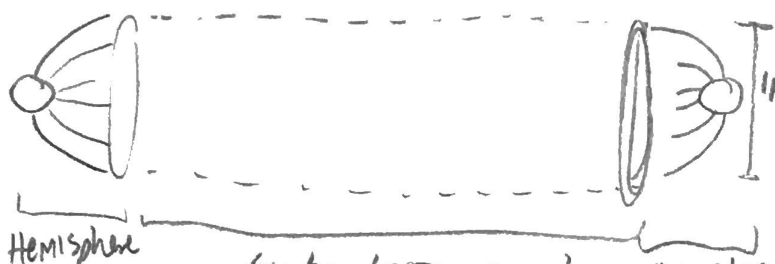


[Question 3] (4.22)

(a) $R = \rho \frac{L}{A} = \frac{(1.3 \times 10^{-8}\ \text{ohm-meter})(3{,}000 \times 10^{3}\ \text{m})}{7(\pi(.000365)^2)}$

$R = 3.0719 \times 10^{4}$ ohm

(b) we can imagine the lines of current density to look something like this



The most important part of this diagram is the tubular part and calculating its resistance

Lets say that by the time these lines look like a cylinder the diameter of the cylinder is 1km

$R = \rho \frac{L}{A} = \frac{(0.25\ \Omega m)(3 \times 10^{6} m)}{\pi (10^{3} m)^2 / 4} = 0.955\ \Omega$

We can then approximate the resistance of the hemispheres in this problem to be equal to $(2)(0.25\ \Omega m)/(0.1 m) = 5\ \Omega$

in total the water is about $6\ \Omega$ which is much smaller than the cable

[Question 4] Electric Field of Line Charge

(a)

$$\vec{E} = \frac{\lambda}{2\pi\epsilon_0 r}\hat{r}, \quad \lambda = \frac{q}{t}$$

$$q = CV$$

$$\Delta\Phi = -\int_b^a \frac{\lambda}{2\pi\epsilon_0 r}\,dr = \frac{q}{t\,2\pi\epsilon_0}\ln(b/a) = V$$

$$q = \frac{V\,2\pi\epsilon_0 t}{\ln(b/a)} \Rightarrow \vec{E} = \left(\frac{V\cancel{2\pi\epsilon_0}}{\ln(b/a)}\right)\left(\frac{1}{\cancel{2\pi}\epsilon_0 r}\right)\hat{r}$$

magnitude of surface charge on each plate

$$\vec{E} = \frac{V}{\ln(b/a)\,r}\hat{r}$$

(b)

$$V = R\dot{Q} + \frac{Q}{C} = R\frac{dQ}{dt} + \frac{Q}{C} \Rightarrow V - \frac{Q}{C} = R\frac{dQ}{dt}$$

$$\int_0^t \frac{dt}{R} = \int_0^{Q(t)} \frac{dQ}{V-\frac{Q}{C}} \Rightarrow \frac{t}{R} = \left[-\ln\left(V-\frac{Q}{C}\right)C\right]_0^{Q(t)}$$

$$\frac{t}{R} = -\ln\left(V-\frac{Q(t)}{C}\right)C + \ln(V)C = -C\left(-\ln(V) + \ln\left(V-\frac{Q(t)}{C}\right)\right)$$

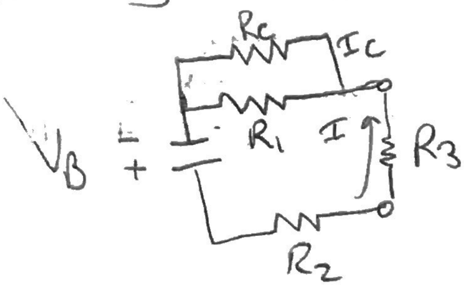
$$\frac{t}{R} = -C\left(\ln\left(1-\frac{Q(t)}{VC}\right)\right) = \frac{-t}{RC} = \ln\left(1-\frac{Q(t)}{VC}\right)$$

$$e^{-t/RC} = 1 - \frac{Q(t)}{VC} \Rightarrow e^{-t/RC} - 1 = -\frac{Q(t)}{VC}$$

$$VC - VCe^{-t/RC} = Q(t) \Rightarrow I(t) = \frac{dQ}{dt}$$

$$I(t) = \frac{-VC}{-RC}e^{-t/RC} = \frac{V}{R}e^{-t/RC}$$

[Question 5] (4.42)



$R_3 = 0\,\Omega$, $I_c = 50\,\mu A$
Scenario 1

$R_3 = 15\,\Omega$, $I_c = 25\,\mu A$
Scenario 2

Scenario 1

$I_c R_c = V_c = 0.001\,V$

$I R_2 = V - V_c = 1.499\,V$

$I = \dfrac{V - V_c}{R_2} = \dfrac{V}{R_2 + \left(\frac{R_1 R_c}{R_1 + R_c}\right)}$

$R_2 V = (V - V_c) R_2 + (V - V_c)\left(\dfrac{R_1 R_c}{R_1 + R_c}\right)$

$R_2 = \left(\dfrac{V - V_c}{V_c}\right)\left(\dfrac{R_1 R_c}{R_1 + R_c}\right)$

$= \dfrac{29980\, R_1}{R_1 + 20}$

Scenario 2

$I(R_2 + R_3) = V - V_c$, $V_c = I_c R_c = 5 \times 10^{-4}\,V$

$I = \dfrac{V - V_c}{R_2 + R_3} = \dfrac{V}{R_2 + R_3 + \left(\frac{R_1 R_c}{R_1 + R_c}\right)}$

$V(R_2 + R_3) = (V - V_c)\left(R_2 + R_3 + \dfrac{R_1 R_c}{R_1 + R_c}\right)$

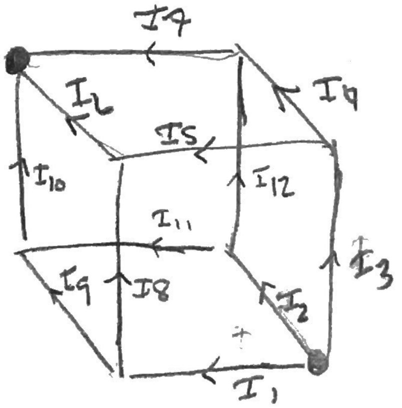
$V(R_2 + R_3) = (V - V_c)(R_2 + R_3) + (V - V_c)\left(\dfrac{R_1 R_c}{R_1 + R_c}\right)$

$R_2 = \left(\dfrac{V - V_c}{V_c}\right)\left(\dfrac{R_1 R_c}{R_1 + R_c}\right) - R = \dfrac{59980\, R_1}{R_1 + 20} - 15$

$\dfrac{59980\, R_1}{R_1 + 20} - 15 = \dfrac{29980\, R_1}{R_1 + 20}$

$30{,}000 R_1 - 15 R_1 = 300 \rightarrow \boxed{\begin{array}{l} R_1 = 0.01\,\Omega \\ R_2 = 14.98\,\Omega \end{array}}$

[Question 6] (4.35)



$$I_3 + I_5 - I_8 - I_1 = 0$$

$$I_3 + I_4 - I_{12} - I_2 = 0$$

$$I_2 + I_{11} - I_9 - I_1 = 0$$

$$I_4 + I_7 - I_6 - I_5 = 0$$

$$I_{12} - I_7 - I_{10} - I_{11} = 0$$

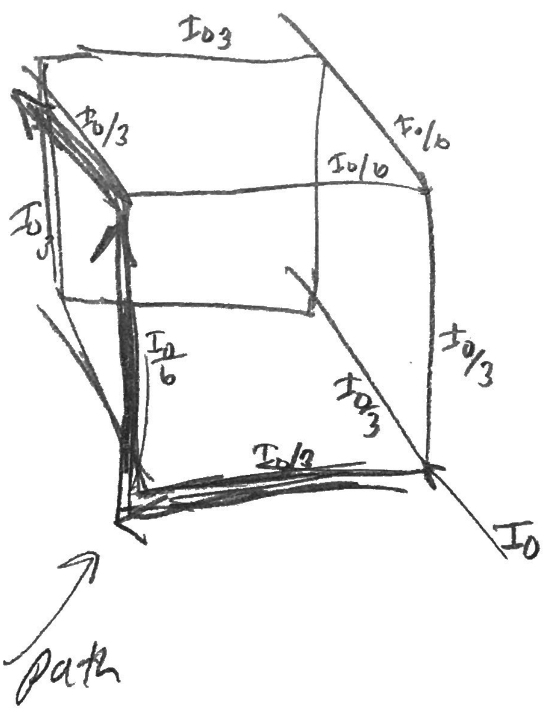
$$I_8 + I_6 - I_{10} - I_9 = 0$$

$$I_1 = I_2 = I_3$$

$$I_1 = I_8 + I_9$$

$$I_2 = I_{11} + I_{12}$$

(a)

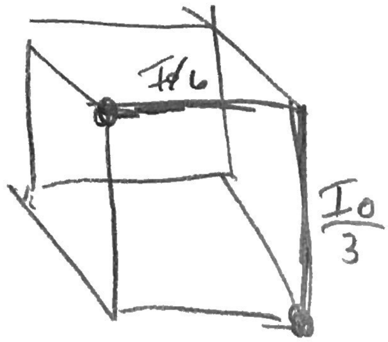


Because of symmetry no matter which direct path you take the voltage drop will be the same and this is also true because all resistors are in parrallel which means voltage drop stays constant and current splits

Along path Vdrop $= \frac{I_0}{3}(R) + \frac{I_0}{6}(R) + \frac{I_0}{3}(R) = \frac{5}{6} I_0 R$

so effective resistance is $R_{eff} = \frac{5}{6} R$

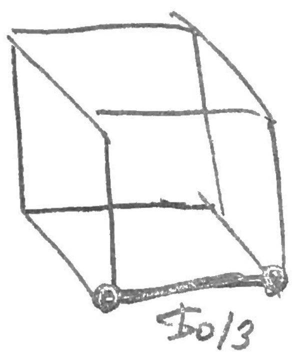
(b)



$$V_{drop} = \frac{I_0}{3}(R) + \frac{I_0}{6}(R) = \frac{1}{2} R I_0$$

$$R_{Eff} = \frac{1}{2} R$$

(c)



$$V_{drop} = \frac{I_0}{3} R \Rightarrow R_{eff} = \frac{1}{3} R$$

[Question 7]

(a) $IV = \underbrace{\frac{W}{t}\left(\frac{1}{K}\right)}_{\text{power needed to lift}} + \underbrace{I^2R}_{\text{power dissappated through internal resistance}}$

$$IVt = \frac{W}{K} + I^2Rt = \frac{mgh}{k} + I^2Rt$$

$$\frac{(IVt - I^2Rt)}{mg}K = h$$

<u>Standard D Cell</u>

$I = 10mA$
$V = 1.5V$
$t = 1.08 \times 10^6 s$
$R = 1\Omega$
$K = 0.5$
$m = 60kg$
$g = 9.8m/s^2$

$\boxed{h = 13.684m}$

<u>Alkaline D Cell</u>

$I = 10mA$
$V = 1.5V$
$t = 1.08 \times 10^6 s$
$R = 0.1\Omega$
$K = 0.5$
$m = 60kg$
$g = 9.8m/s^2$

$\boxed{h = 13.766m}$

(b) $\frac{(IV - I^2R)K}{mg} = \frac{h}{t}$ Rate m/s

we need to figure out the best current ~~to~~ pull to get a big Rate cm/s

$IV - I^2R = f(I) \Rightarrow f'(I) = 0 = V - 2IR$

$$I = \frac{V}{2R}$$

[Question 7] cont.

Standard D Cell

$I = \frac{1.5V}{2\Omega} = 0.75$ Amps

$\frac{h}{t} = 4.783 \times 10^{-4}$ m/s

$\boxed{= 4.783 \times 10^{-2} \text{ cm/s}}$

Alkaline D Cell

$I = \frac{1.5V}{0.2\Omega} = 7.5$ Amp

$\frac{h}{t} = 4.783 \times 10^{-3}$ m/s

$\boxed{= 4.783 \times 10^{-1} \text{ cm/s}}$

(c) Total Energy in Battery $= IVt - I^2Rt$

Power output $= P = IV - I^2R$

$\frac{\text{Total Energy}}{\text{Power}} = \text{time}$

Standard D cell

Total Energy $= (0.01 \text{ Amps})(1.5V)(1.08 \times 10^6 s) - (0.01 \text{ Amps})^2(1\Omega)(1.08 \times 10^6 s)$
$= 16092$ J

Power $= (0.75 \text{ Amps})(1.5V) - (0.75 \text{ Amps})^2(1\Omega) = 0.5625 \frac{J}{s}$

$\boxed{\text{Time}} = 16092 J / 0.5625 J/s = 28608 s$

$\boxed{\text{Max Height}} = (28608 s)(4.783 \times 10^{-2} \text{ cm/s})$
$= 1368.32$ cm

Alkaline D cell

Total Energy $= 16189.2$ J

Power $= (7.5 \text{ Amps})(1.5V) - (7.5 \text{ Amps})^2(0.1\Omega) = 5.625$ J/s

$\boxed{\text{Time}} = 16189.2 J / 5.625 J/s = 2878.08 s$

$\boxed{\text{Max Height}} = (2878.08 s)(4.783 \times 10^{-1} \text{ cm/s}) = 1376.58$ cm

[Question 8]

(a) $R = \frac{\rho L}{A} \rightarrow R = \int_a^b \rho \frac{dr}{2\pi r} = \frac{\rho}{2\pi} \ln(b/a)$

(b) $Q = CV \qquad V = \Delta\Phi_{ab} = -\int \vec{E} \cdot d\vec{r} \rightarrow \vec{E} = \vec{E}_{\text{line charge}}$

$$\vec{E} = \frac{\lambda}{2\pi \epsilon_2 r} \hat{r}, \quad \lambda = \frac{q}{L}, \quad \epsilon_2 = \epsilon_0 \epsilon$$

$$\Delta\Phi_{ab} = -\int_b^a \frac{\lambda}{2\pi \epsilon_2 r} dr = \frac{\lambda}{2\pi \epsilon_2} \ln(b/a) = \frac{q}{L 2\pi \epsilon_2} \ln(b/a)$$

$$\frac{\cancel{q} L 2\pi \epsilon_2}{\cancel{q} \ln(b/a)} = C = \frac{2\pi \epsilon_2 L}{\ln(b/a)}$$

(c) $\vec{J} = \left(\frac{1}{\rho}\right) \vec{E} = \left(\frac{1}{\rho}\right)\left(\frac{q}{2\pi L \epsilon_2 r}\right)\hat{r} \rightarrow q = CV_0$

$$\vec{J} = \left(\frac{1}{\rho}\right)\left(V_0 \frac{\cancel{2\pi \epsilon_0} \epsilon \cancel{L}}{\ln(b/a)}\right)\left(\frac{1}{\cancel{2\pi L \epsilon_0} r}\right) = \frac{V_0 \epsilon}{\rho \ln(b/a) r}$$

This makes sense that it is has a $\frac{1}{r}$ factor because the surface area dA that this $\vec{J}(r)$ is going through is $dA = 2\pi r \, dr \rightarrow I = \int \vec{J} \cdot dA = \int (D)\left(\frac{1}{\cancel{r}}\right) 2\pi \cancel{r} \, dr$

(d) $Q_0 = CV_0 = \dfrac{2\pi \epsilon_0 \epsilon L}{\ln(b/a)}$

(e) $0 = R\dot{Q} + \dfrac{Q}{C} = R\dfrac{dQ}{dt} + \dfrac{Q}{C}$

$\therefore \dfrac{Q}{C} = -R\dfrac{dQ}{dt} \rightarrow \displaystyle\int_0^t \frac{-dt'}{RC} = \int_{CV_0}^{Q(t)} \frac{dQ'}{Q'} \rightarrow \frac{-t}{RC} = \left[\ln(Q)\right]_{CV_0}^{Q(t)}$

$\dfrac{-t}{RC} = \ln(Q(t)) - \ln(CV_0) = \ln\left(\dfrac{Q(t)}{CV_0}\right)$

$CV_0 e^{-\frac{t}{RC}} = Q(t)$