AP Physics C: Mechanics - Semester 1 Notes

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Contents

1 Vectors and Motion in 2 Dimensions

Unit vector: length of 1, no units (used for direction)

- Denoted with "hat"
- $\hat{x}$, $\hat{y}$, $\hat{z}$ - usually denoted as $\hat{i}$, $\hat{j}$, $\hat{k}$

For a 2D velocity vector:

$$\vec{v} = v_x\hat{i} + v_y\hat{j}$$

1.1 Polar Notation

$$\vec{v} = \sqrt{{v_x}^2 + {v_y}^2} \text{ at } \theta^\circ$$

θ is angle relative to *positive x-axis*

1.2 Vector Addition

$$\vec{A} = A_x\hat{i} + A_y\hat{j}$$
$$\vec{B} = B_x\hat{i} + B_y\hat{j}$$
$$\vec{A} + \vec{B} = (A_x + B_x)\hat{i} + (A_y + B_y)\hat{j}$$

1.3 Scalar Multiplication

scalar · vector = vector

$$\vec{A} = A_x\hat{i} + A_y\hat{j}$$
$$2\vec{A} = 2A_x\hat{i} + 2A_y\hat{j}$$

1.4 Vector Multiplication

vector × vector

scalar/dot product: $\vec{A} \cdot \vec{B}$

- results in scalar

vector/cross product: $\vec{A} \times \vec{B}$

- results in vector

1.5 Dot Product

$$\vec{A} \cdot \vec{B} = |A||B| \cos\theta$$

- θ is the angle between the two vectors

Or,

$$\vec{A} \cdot \vec{B} = A_xB_x + A_yBy$$

1.5.1 Dot Product Properties

$$\vec{A} \cdot \vec{B} = \vec{B} \cdot \vec{A}$$
$$\vec{A} \cdot \vec{B} = 0 \text{ if } \vec{A} \text{ and } \vec{B} \text{ are orthogonal}$$

1.6 Cross Products

$$\text{If } \vec{C} = \vec{A} \times \vec{B},$$
$$|C| = |A||B| \sin\theta$$

the cross product is **orthogonal** to both original vectors (uses 3rd dimension)

1.6.1 Cross Product Properties

$$\vec{A} \times \vec{B} \neq \vec{B} \times \vec{A}$$
$$\vec{A} \times \vec{B} = -\vec{B} \times \vec{A}$$

right hand rule - cross product goes "into" or "out of" page

2 Drag Force

- resistive force by ***fluids*** on ***moving objects***
- ***opposes*** direction of motion

2.1 v^2 model

$$F_{drag} = -\frac{1}{2}C\rho Av^2$$

- C - drag coefficient
- ρ - density of fluid - greek letter "rho"
- A - cross-sectional area of object
- v - velocity of object
- negative sign just indicates direction opposite to motion

Sometimes, the coefficients $\frac{1}{2}C\rho Av^2$ are grouped into one coefficient (b or k)

- So, $F_{drag} = -bv^2$
- also called the drag coefficient
- specific to object and location
- calculated experimentally

2.2 v model

for ***small objects*** moving at ***slow speeds***,

$$F_{drag} = -bv$$

2.3 Terminal Velocity

For an object in free fall, terminal velocity occurs when $\left|\vec{F}_{drag}\right| = \left|\vec{F}_{gravity}\right|$

$$\begin{aligned} F_{net} &= F_g - F_{drag} \\ 0 &= mg - bv_t \\ v_t &= \frac{mg}{b} \text{ (v model)} \\ v_t &= \sqrt{\frac{mg}{b}} \text{ (v^2 model)} \end{aligned}$$

3 Differential Equations

Differential equations define relationship between a function and one or more *derivatives* of that function

Drag force is modeled with a differential equation:

$$\begin{aligned} F_{drag} &= bv \\ ma &= bv \\ m\frac{dv}{dt} &= bv \end{aligned}$$

3.1 Integrating Differential Equations

$$\begin{aligned} m\frac{dv}{dt} &= bv && \text{rewrite equation to see dependent and independent variables} \\ \frac{1}{v}dv &= \frac{b}{m}dt && \text{separate each variable to one side} \\ \int \frac{1}{v}dv &= \int \frac{b}{m} && \text{integrate} \end{aligned}$$

4 Circular Motion

Centripetal force (F_c or $\sum F_c$) isn't really a force - it is the **net** force along the centripetal axis (the axis going through a point on the circular path and the center of said circle).

- could be weight, thension, friction, etc.
- is always orthogonal (perpendicular) to velocity
- depends on mass and tangential velocity of object and radius of circular path

4.1 Uniform Circular Motion

Uniform circular motion occurs when an object moving in a circle with a fixed radius has a constant tangential (and therefore angular) speed

frequency (f) is measured in Hertz Hz
period (T) is measured in seconds (s)

4.2 Velocity

$$v_t = \frac{\Delta x}{T} = \frac{2\pi r}{T} = 2\pi r f$$

4.3 Acceleration

$$a_c = \frac{v^2}{r} = \frac{4\pi^2 r}{T^2}$$

Centripetal acceleration always points toward the center of the circular path

- does not change *speed* of object; only changes *direction*

4.4 Force

Centripetal force is the sum of all forces on the centripetal axis

$$\sum F = ma$$
$$\sum F_c = ma_c$$
$$\sum F_c = \frac{mv^2}{r}$$

points toward the center of the circular path

5 Rotational Motion

rigid bodies: objects that do not change shape (deform)
translational motion: motion of the *center of mass* of the object
rotational motion: rotation around a fixed point (not always the center of mass)

Circular Motion	**Rotational Motion**
circular path	spins about axis of rotation
all parts of object move with same velocity	different points on the object travel at different speeds

5.1 Variables

- R - distance from point on object to axis of rotation
- θ - angular displacement - *measured in radians*
- S - Arc Length

$$S = R\theta$$

5.2 Angular Velocity

rate of change of θ

$$\omega = \frac{\Delta\theta}{t} = 2\pi f$$

- ω - angular velocity - *measured in radians/second*

all points on a rigid body have the same angular velocity

ω does ***not*** point in the direction of rotation

- use the right hand rule to determine the direction of *omega* - always orthogonal to the rotational plane
  - usually described as "into page" or "out of page"
  - sometimes described as "clockwise" or "counterclockwise"

5.3 Angular Acceleration

rate of change of *omega*

$$\alpha = \frac{\Delta\omega}{t}$$

- α - angular acceleration - *measured in radians/second²*

α does ***not*** point in the direction of rotation - same as angular velocity (ω)

- use right hand rule just like angular velocity to determine direction of α

5.4 Kinematic Equations

Translational Kinematics	**Rotational Kinematics**
$\bar{v} = \frac{\Delta x}{t}$	$\bar{\omega} = \frac{\Delta\theta}{t}$
$\bar{v} = \frac{v_1 + v_2}{2}$	$\bar{\omega} = \frac{\omega_1 + \omega_2}{2}$
$\bar{a} = \frac{v - v_0}{t}$	$\bar{\alpha} = \frac{\omega - \omega_0}{t}$
$\Delta x = v_0 t + \frac{1}{2}at^2$	$\Delta\theta = \omega_0 t + \frac{1}{2}\alpha t^2$
${v_f}^2 = {v_0}^2 + 2a\Delta x$	${\omega_f}^2 = {\omega_0}^2 + 2\alpha\Delta\theta$

6 Center of Mass (Discrete Objects)

- The point at which all **mass** of an object is though to be concentrated
- also thought of as "average location of mass"
- can be determined experimentally or mathematically
- the center of mass of *all objects* moves like a point particle *even if the object is rotating*
- could be inside or outside the object
- the geometric center of an object is *not* always its center of mass - only when the object has a uniform density
- unless specified, objects have a uniform density

6.1 Position of the Center of Mass

$$x_c m = \frac{1}{M}\sum m_i x_i$$

- $x_c m$ = position of the center of mass of the systeme
- M - total mass of the systeme
- m_i - mass of the i^{th} particle
- v_i - psoition of the i^{th} particle

A continuous object can be broken down into symmetric discrete pieces to find its center of mass

6.2 Velocity of the Center of Mass

$$v_c m = \frac{1}{M} \sum m_i v_i$$

- v_i - velocity of the i^{th} particle

6.3 Acceleration of the Center of Mass

Newton's third law ($F = ma$) actually refers to the center of mass of a system or object:

$$M a_{cm} = \sum F_{external}$$

7 Center of Mass (Continuous Objects)

Not all continuous objects have a uniform mass density

- Mass density usually varies withlocation

$$x_{cm} = \frac{1}{M} \int x \ dm$$

- x - position of particle
- dm - infinitely small mass

position is given as a function of mass
integrate along the length of the object

7.1 Position of Center of Mass

1. Define mass density function:
$$\lambda = \frac{dm}{dx}$$
2. Separate variables:
$$dm = \lambda(x) dx$$
3. Integrate along length of object to find total mass of object:
4. Use integral form of center of mass formula (see above) to solve for the position of the center of mass
    - use total mass calculated in step 4

8 Rotational Intertia

Rotational inertia (a.k.a **moment of inertia**): ability of an object to resist changes in its rotational motion

symbol: I
scalar quantity
units: $\text{kg} \cdot \text{m}^2$

8.1 Discrete Objects

$$I = \sum m_i R_i^2$$

- m_i - mass of object
- R - distance between axis of rotation and the object

The same object can have different rotational inertias depending on the location of the axis of rotation

8.2 Continuous Objects

$$I = \int R^2 \ dm$$

Object and Axis of Rotation	**Rotational Inertia**
Thin rod, about center	$\frac{1}{12}ML^2$
Thin rod, about end	$\frac{1}{3}ML^2$
Cylinder or disk, about center	$\frac{1}{2}MR^2$
Cylindrical hoop, about edge	MR^2

8.3 Parallel Axis Theorem

If the moment of inertia about the center of mass (I_{CoM}) is known, the moment of inertia about any parallel axis of rotation can be found using the formula:

$$I_{parallel} = I_{CoM} + Md^2$$

- M - mass of the objects
- d distance between the axes

8.4 Systems of Objects

For a system of discrete and/or continous objects:

$$I_{system} = I_1 + I_2 + I_3 + \cdots + I_n$$

9 Torque

Torque is the ability of a force to make an object rotate - a ”twisting force”

symbol: τ (”tau”)
units: $N \cdot m$
vector quantity

Magnitude depends on

- size of the force
- direction

- location at which the force is applied

torque is given as the cross product $R \times F$:

$$\tau = R \times F = RF \sin \theta$$

- R - distance from the axis of rotation to the applied force
- F - applied force
- θ - angle between R and F (when drawn tip-to-tail)

9.1 Direction of Torque

Use the right hand rule to find the direction of torque:

- point index finger in direction from axis of rotation to applied force
- point middle finger in direction of applied forces
- point thumb out - this is the direction of the torque

9.2 Torque in Newton's Second Law

$$\begin{aligned}
\tau_{net} &= R \times F \\
&= RF_{net} \\
&= Rma && \text{Newton's 2nd Law, } F = ma \\
&= Rm\alpha R && a = \alpha R \\
&= I\alpha && \text{rotational inertia, } I = mR^2
\end{aligned}$$