Analog Electronics lab session

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Session 3 : Design of differential pairs and of an OTA

In this exercise you will learn ...

- Fundamentals about differential pairs
- How to "design on paper"
  - Differential pair with resistive load
  - OTA
- Learn to value $\frac{g_m}{I_D}$- and $\frac{g_m}{g_{ds}}$-plots !
  - with those plots one can identify design trade-offs!
- How to code in MATLAB the OTA-design

Session 3 : Design of differential pairs and of an OTA

- Solve exercise 1 a-d
  - ETA: 20 min

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Exercise 1

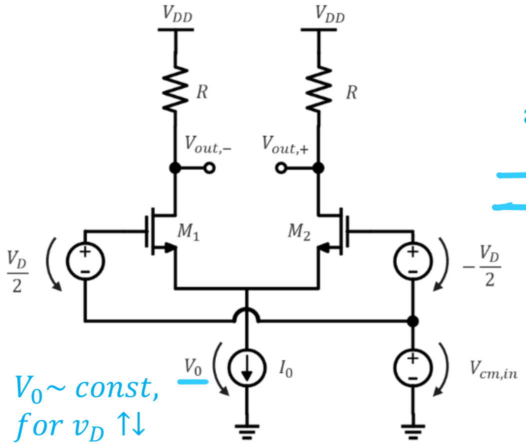
a) Give the equation of the small signal voltage gain $\frac{v_{out,-}}{v_D}$, $\frac{v_{out,+}}{v_D}$ and the differential voltage gain, when $V_D = 0$?

a) Remark: M1 = M2

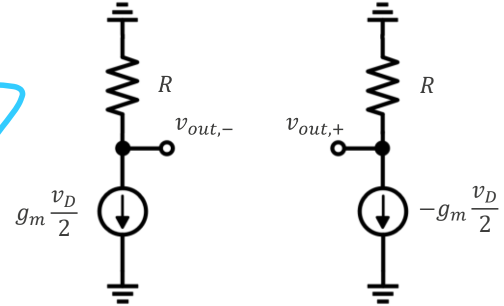
$$A_{V,-} = \frac{v_{out,-}}{v_D} = -\frac{g_m}{2} \cdot R$$

$$A_{V,+} = \frac{v_{out,+}}{v_D} = \frac{g_m}{2} \cdot R$$

$$A_{V,diff} = g_m \cdot R$$



small signal analysis



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Exercise 1

b) Draw on Fig. 2 the voltages $V_{out,+}$, $V_{out,-}$ and V_0 and on Fig. 3 ($V_{out,+} - V_{out,-}$) qualitatively. Moreover, indicate on both figures:

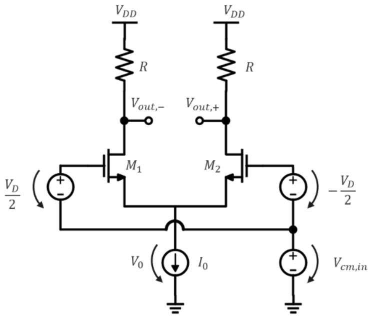
(i) V_{OV} and $\sqrt{2}V_{OV}$,

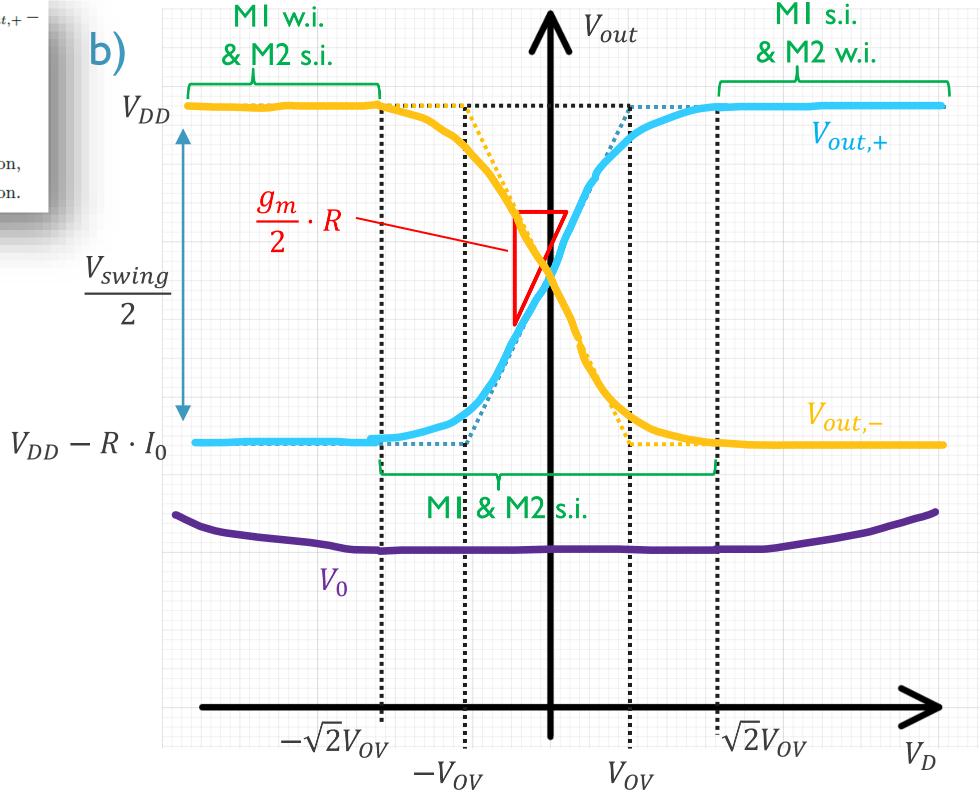
(ii) A_V,

(iii) range, where M_1 and M_2 are in strong inversion,

(iv) range, where M_1 is in weak inversion, but M_2 in strong inversion,

(v) range, where M_1 is in strong inversion, but M_2 in weak inversion.





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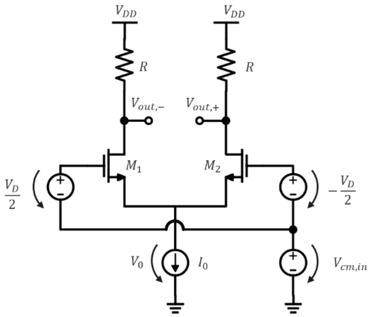
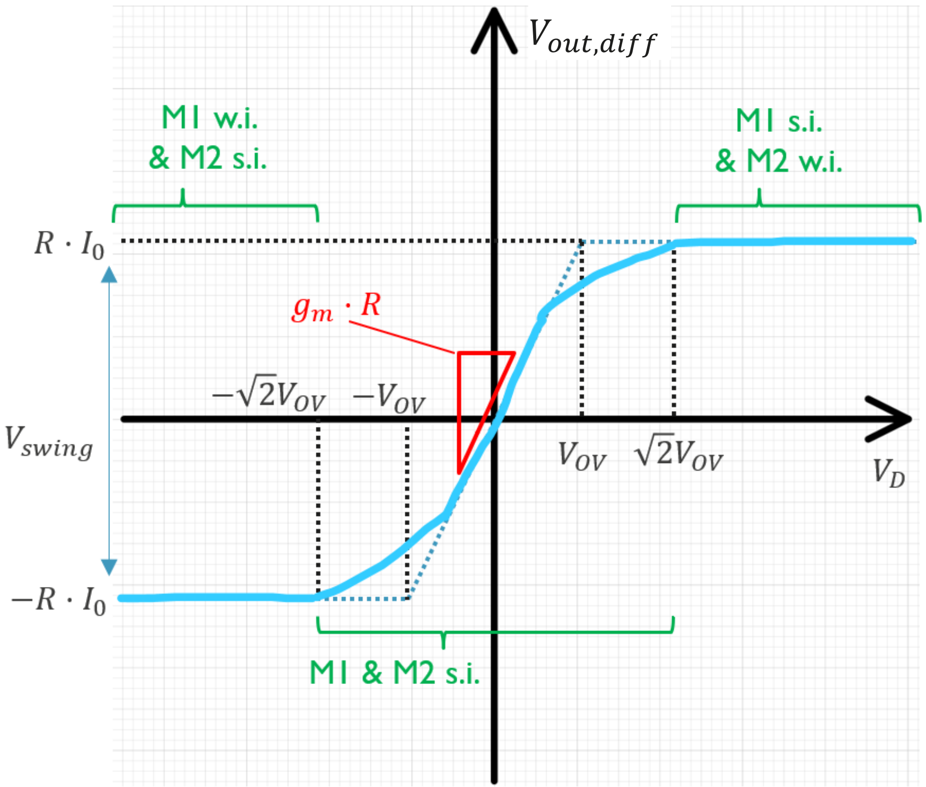


b)



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Exercise 1

c) Give the amount of current, which is flowing through M_1 and M_2, when

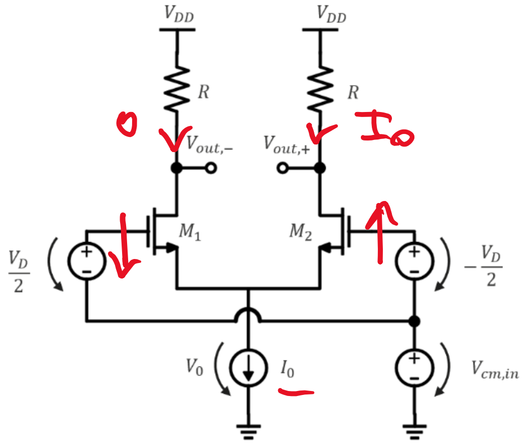
(i) $V_D = 0$,
(ii) $V_D < -\sqrt{2} \cdot V_{OV}$,
(iii) $V_D > \sqrt{2} \cdot V_{OV}$.

c) $I_{D,M1}$ and $I_{D,M2}$ given as follows

i. $V_D = 0V$: $I_{D,M1} = I_{D,M2} = 0.5 \cdot I_0$

ii. $V_D > \sqrt{2} \cdot V_{OV}$: $I_{D,M1} = I_0$ and $I_{D,M2} = 0$

iii. $V_D < -\sqrt{2} \cdot V_{OV}$: $I_{D,M1} = 0$ and $I_{D,M2} = I_0$



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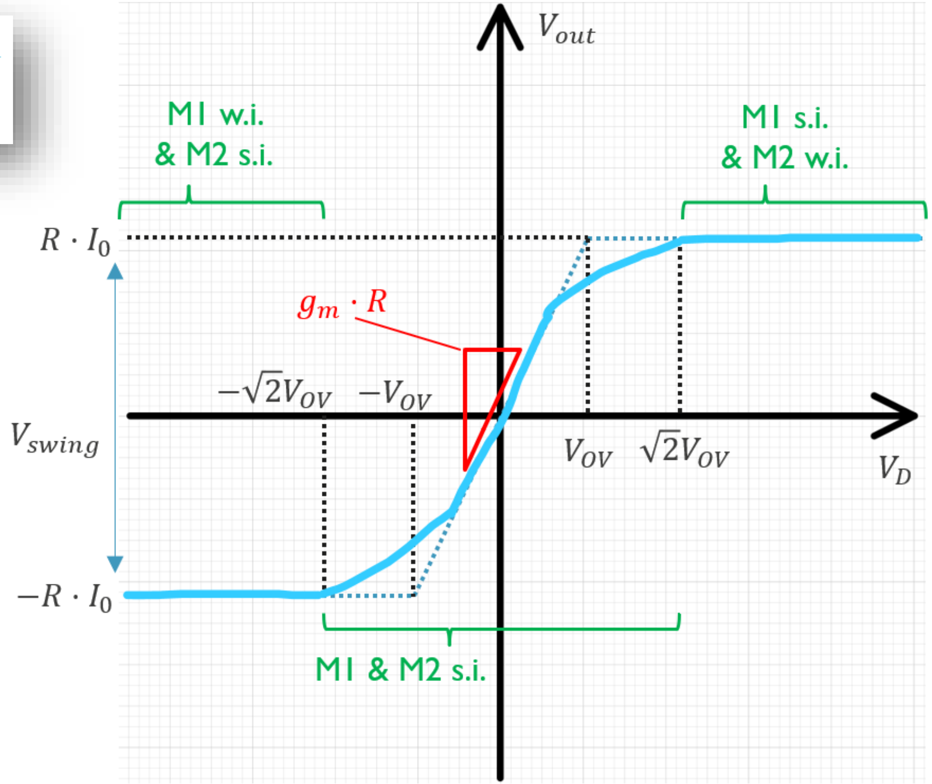
Exercise 1

d) Based on Fig. 3, derive an equation describing the relationship between V_{swing}, V_{OV} and A_V.
Remark: $V_{swing} = max(V_{out,+} - V_{out,-}) - min(V_{out,+} - V_{out,-})$

d) option 1: graphically

$$\Rightarrow A_{V,diff} = \frac{V_{swing}}{2 \cdot V_{OV}}$$



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Exercise 1

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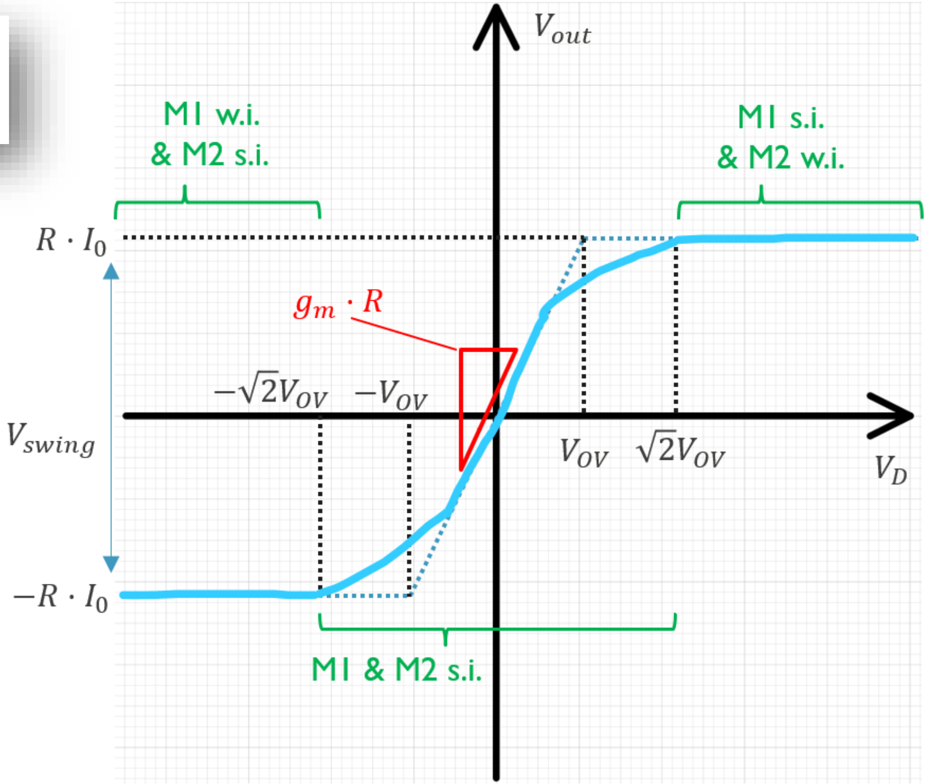
Remark: $V_{swing} = max(V_{out,+} - V_{out,-}) - min(V_{out,+} - V_{out,-})$

d) option 2: analytically

$$\frac{g_{m_{1,2}}}{I_{D_{1,2}}} = \frac{g_{m_{1,2}} \cdot R}{(I_0/2) \cdot R} = \frac{2}{V_{OV}}$$

$$\Leftrightarrow 2 \cdot A_{V,diff} = \frac{I_0}{V_{OV}} \cdot 2 \cdot R = \frac{V_{swing}}{V_{OV}}$$

$$\Leftrightarrow A_{V,diff} = \frac{V_{swing}}{2 \cdot V_{OV}}$$



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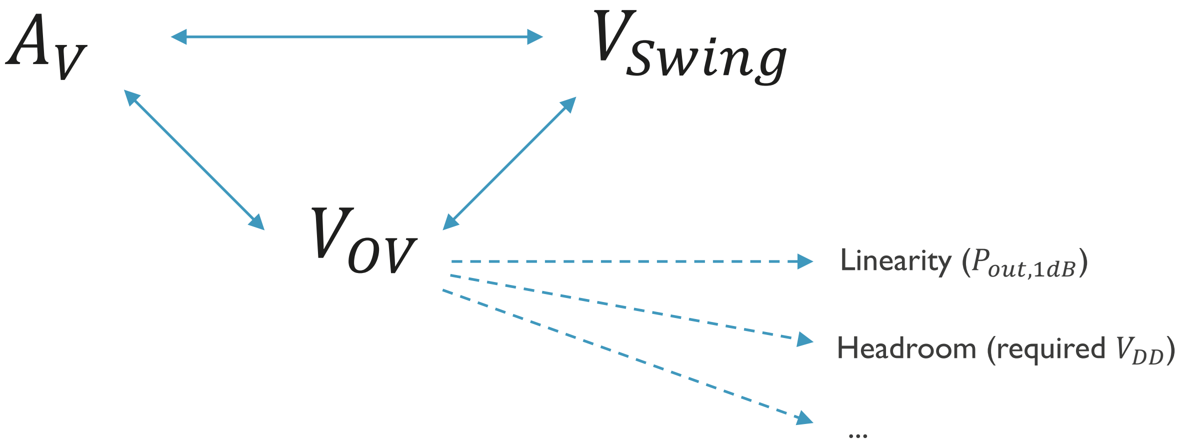
Summary

- Waveforms of an ideal differential amplifier has been presented
- (in this exercise) Trade-off triangle of differential amplifiers has been identified
  - ... in reality, one has even an octagon (see literature [1-3])



➢ Now, the differential amplifier is going to be designed

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- Solve exercise 1 e-j
    - ETA: 30 min

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Exercise 1

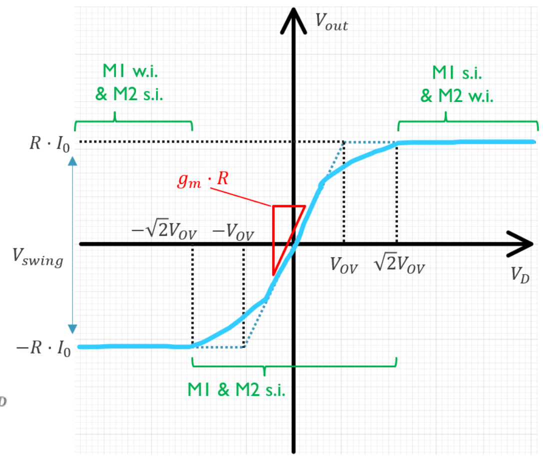
e) Give V_{swing} as a function of I_{bias}.

$V_{GS3} = V_{GS4}$

$\Leftrightarrow$ ⋮

$\Leftrightarrow \frac{I_4}{I_3} = \frac{W_4}{W_3}$

$I_D = \frac{\mu C_{ox}}{2a}\left(\frac{W}{L}\right) V_{OV}^2$

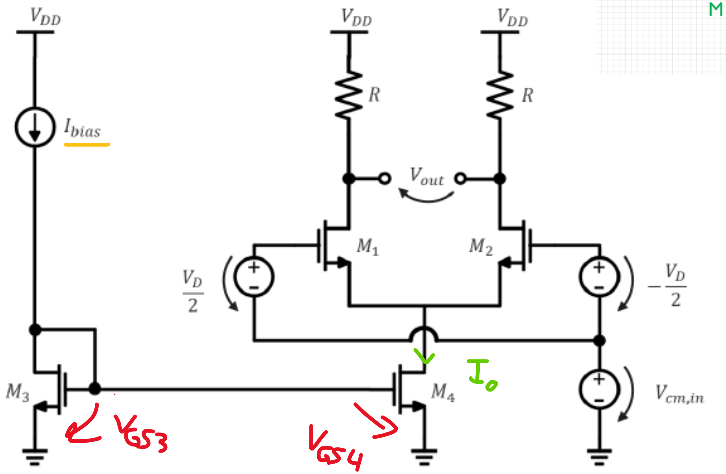


e) $V_{swing} = 2 \cdot R \cdot I_0$

current mirror equation (neglecting CLM):

$$\frac{I_0}{I_{bias}} = \frac{W_4}{W_3}$$

$$\Rightarrow V_{swing} = 2 \cdot R \cdot I_{bias} \cdot \left(\frac{W_4}{W_3}\right)$$



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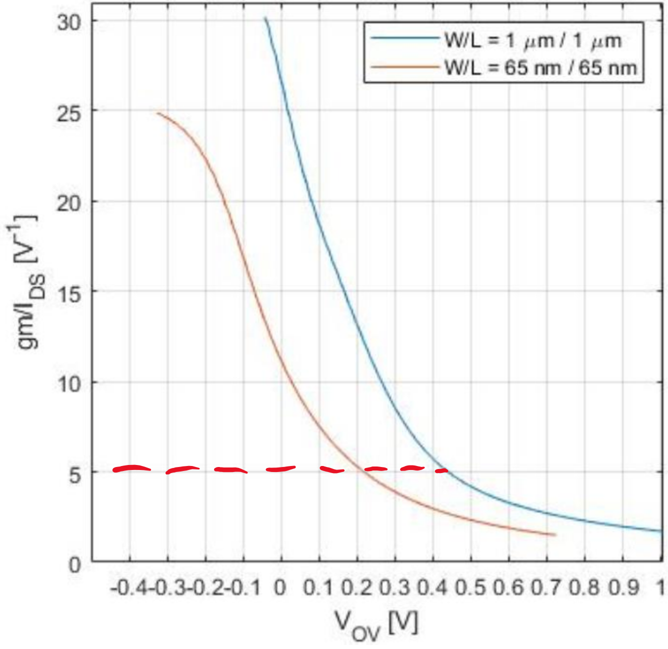
Exercise 1

f) Calculate, what $\frac{g_m}{I_D}$ is required to have exactly A_V and V_{swing}.

f) $$\frac{g_{m_{1,2}}}{I_{D_{1,2}}} = \frac{g_{m_{1,2}}}{(I_0/2)} = \frac{g_{m_{1,2}} \cdot R}{(I_0 \cdot R/2)} = \frac{A_V}{\left(\frac{V_{swing}}{2}\right)/2} = \frac{A_V}{V_{swing}/4} = 4 \cdot \frac{A_V}{V_{swing}} = 5$$



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Exercise 1

g) Calculate R and g_m such that this circuit passes exactly the V_{swing}-, P_{diss} and A_V-specification.

g) First, calculate the max. allowed current

$$P_{diss} = V_{DD} \cdot (I_0 + I_{bias})$$

$$\Leftrightarrow I_0 = \frac{P_{diss} - V_{DD} \cdot I_{bias}}{V_{DD}} \approx 1mA$$

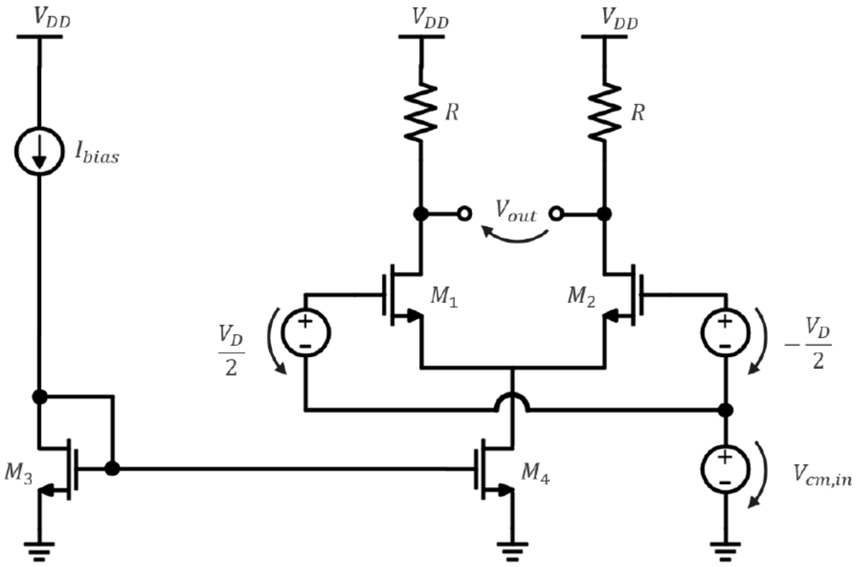
Then, calculate $g_{m_{1,2}}$

$$g_{m_{1,2}} = \left(\frac{I_0}{2}\right) \cdot 5 = 2.5mS$$

Now, calculate R

$$A_{V,diff} = g_{m_{1,2}} \cdot R$$

$$\Leftrightarrow R = \frac{A_{V,diff}}{g_{m_{1,2}}} = 400\ \Omega$$



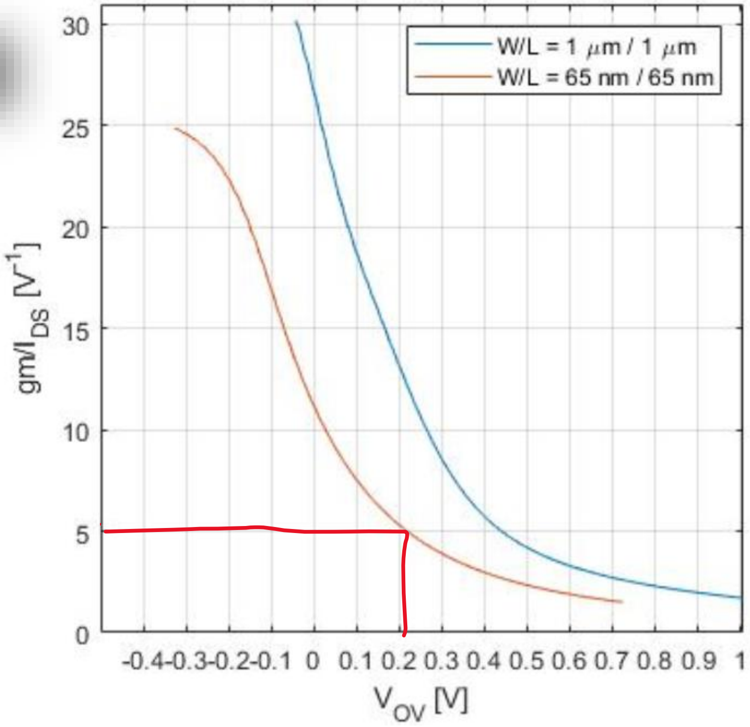
Exercise 1

h) Calculate the required width W_1 and W_2 as well as L_1 and L_2 with aid of Fig. 5 and Fig. 6.

h) Reminder: $\frac{g_{m_{1,2}}}{I_{D_{1,2}}} = 5$

Set length to minimum, i.e. $L_{1,2} = 65nm$ $\left(f_T \sim \frac{1}{L^2}\right)$

Determine V_{OV} $\rightarrow$ 0.2V



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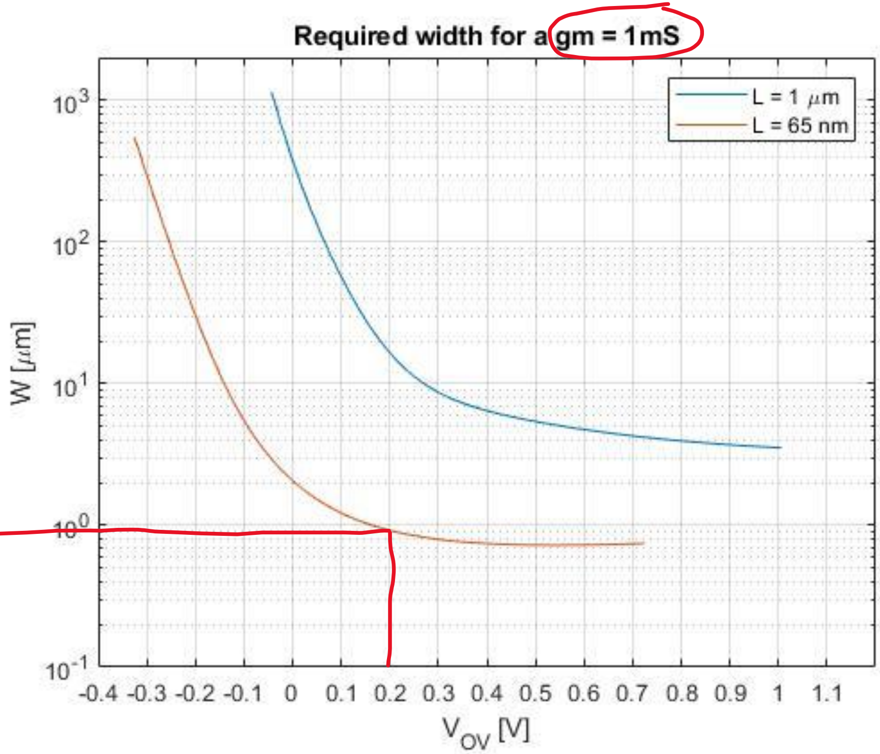
Exercise 1

h) Now determine the width $W_{1,2}$ at $V_{OV} = 0.2V$

from g): $g_m = 2.5mS = 2.5 \cdot 1mS$

$\Rightarrow 0.9\mu m \cdot 2.5 = 2.25\ \mu m$



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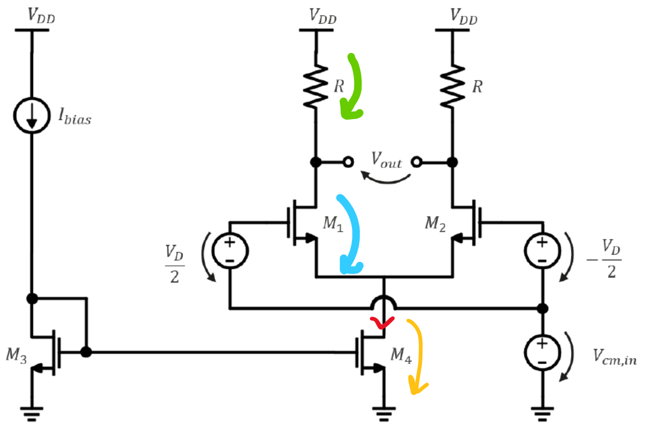
Exercise 1

$V_{Dsat} \approx V_{OV}$ (very pessimistic assumption)

i) Decide what length L_1, L_2 and V_{OV} you would choose for M_3 and M_4. Explain why? Then, calculate the width W_3 and W_4 with aid of Fig. 6.

to ensure saturation for all M_x:

$$V_{OV,3,4} + V_{OV,1,2} + \frac{V_{swing}}{2} \leq V_{DD}$$



i) $L_{3,4} = 1\mu m$ to have negligible CLM (good current source)

Then, decide for a suitable value for $V_{OV_{3,4}} \rightarrow 0.3\ V$

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Exercise 1

i) Calculate the width $W_{3,4}$

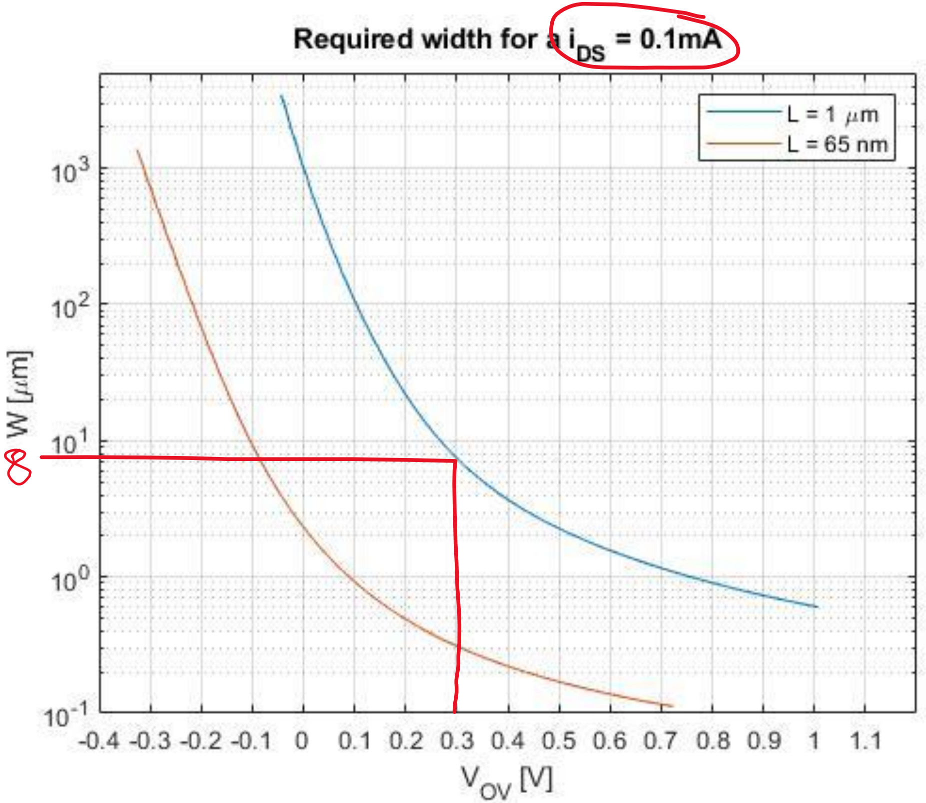
from g):

$$I_{DS,4} = I_0 = 1mA = 10 \cdot 0.1mA$$

$$\Rightarrow 8\ \mu m \cdot 10 = 80\mu m$$

Then, with current mirror equation determine W_3

$$\frac{I_0}{I_{bias}} = \frac{W_4}{W_3} \Leftrightarrow W_3 = \frac{W_4}{(I_0/I_{bias})} = \frac{W_4}{10} = 8\mu m$$



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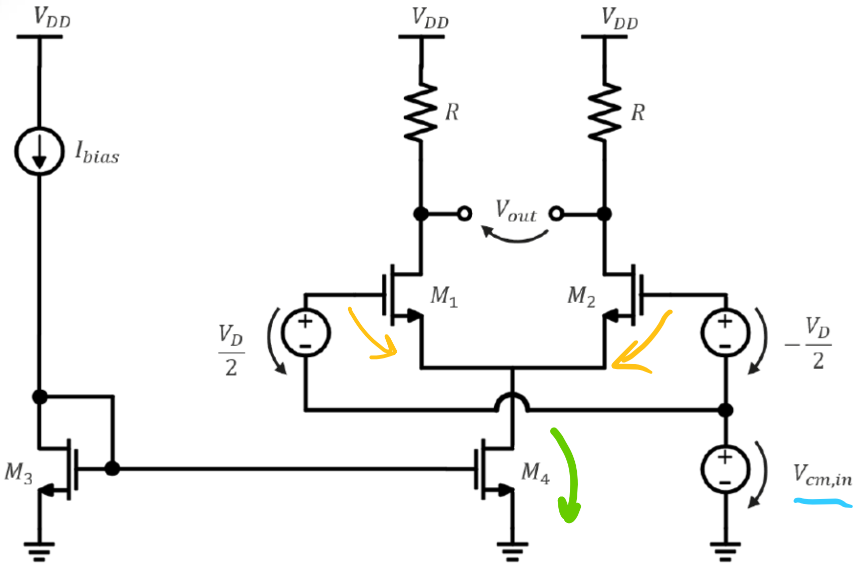
Exercise 1

j) Calculate the minimum allowed input common mode voltage $V_{cm,min,in}$. What happens, if one sets $V_{cm,in} < V_{cm,min,in}$?

j) $V_{cm,in,min}$ calculated as follows

$$
\begin{aligned}
V_{cm,in,min} &= V_{Dsat4} + V_{GS_{1,2}} \\
&= V_{Dsat4} + V_{OV_{1,2}} + V_{th,n} \\
&= 0.3V + 0.2V + 0.4V = 0.9V
\end{aligned}
$$



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Exercise 1

j) What happens, if $V_{cm} < V_{cm,in,min}$?

→ drives M4 into triode ....

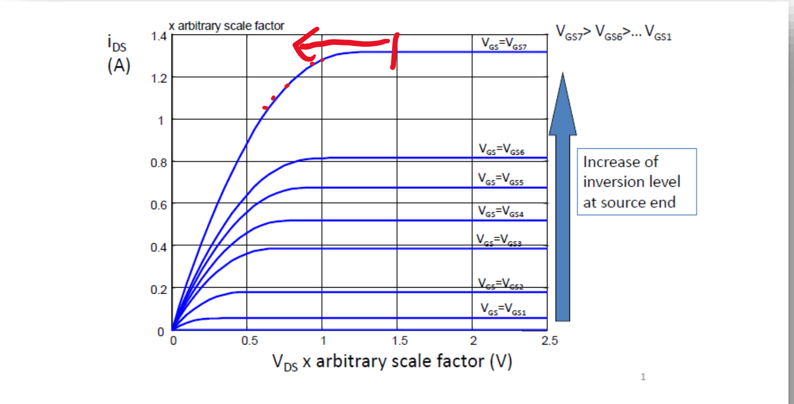
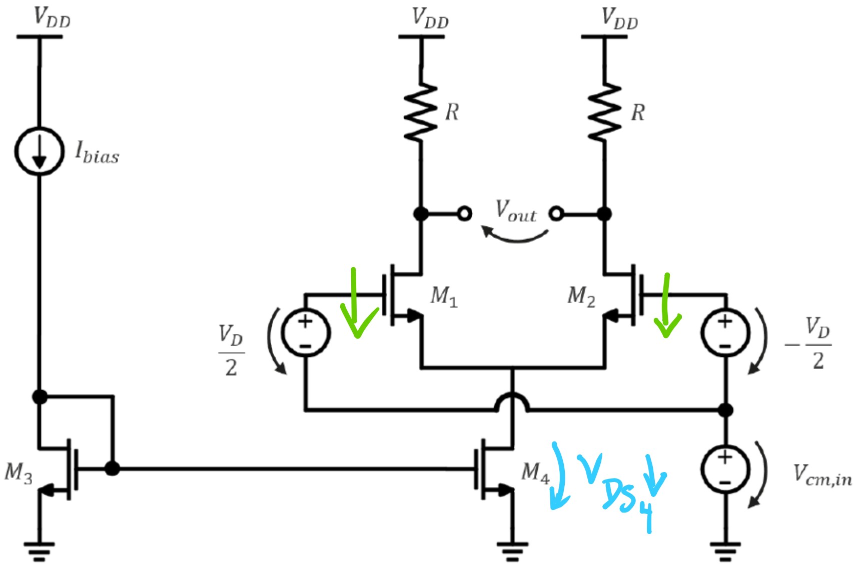


Figure 1.17: Drain current I_{DS} as a function of V_{DS} for different V_{GS} values. The flat behavior, starting at the onset of saturation, is an idealized representation, that will be refined below.



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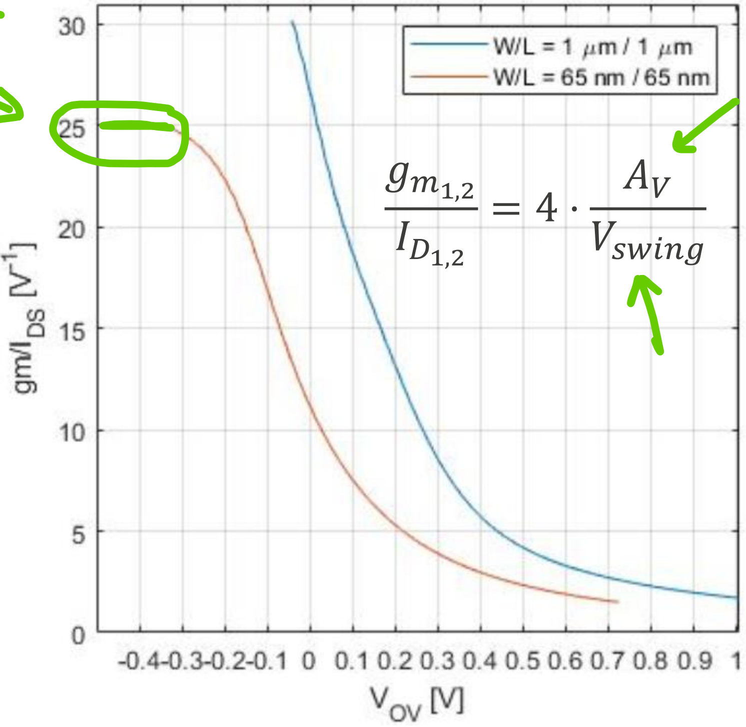
Summary

sidenote: $\frac{g_m}{I_D} = \frac{1}{n V_T}$

$n = 1{,}5$



- Value the $\frac{g_m}{I_D}$-plot !
  - With $\frac{g_m}{I_D}$ one can identify the entire trade-off in one plot !

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- Solve exercise 2 a-e
  - ETA: 25 min

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Exercise 2

a) Give the small signal voltage gain equation of this circuit including the capacitances and indicate

(i) dominant pole,

(ii) non-dominant pole,

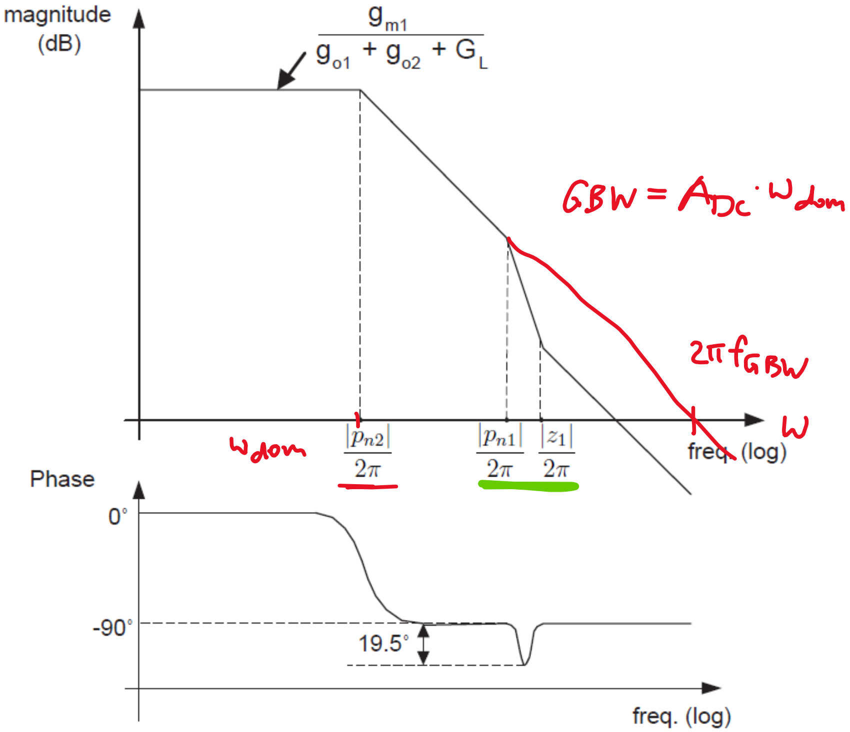
(iii) zero.

b) Assume the pole-zero-doublet to be far away from the dominant pole. Draw the bode- and phase plot and indicate location of the dominant pole and GBW.

a)

$$A_v(s) = \frac{v_{out}(s)}{v_{in}(s)} = \frac{g_{m,1,2}}{g_{02} + g_{06}} \cdot \frac{1 + \overbrace{s\frac{C_{d1,5}}{2gm_5}}^{\text{zero}}}{1 + \underbrace{s\frac{C_{d1,5}}{gm_5}}_{\text{non-dominant pole}}} \cdot \frac{1}{1 + \underbrace{s\frac{C_{d2,6}}{g_{02}+g_{06}}}_{\text{dominant pole}}}$$

b) see drawing



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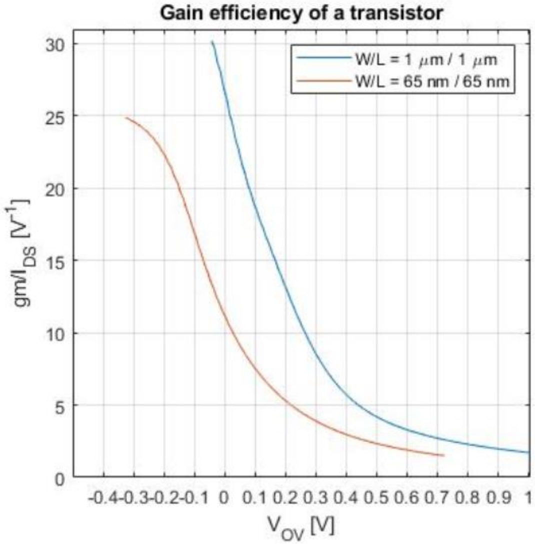
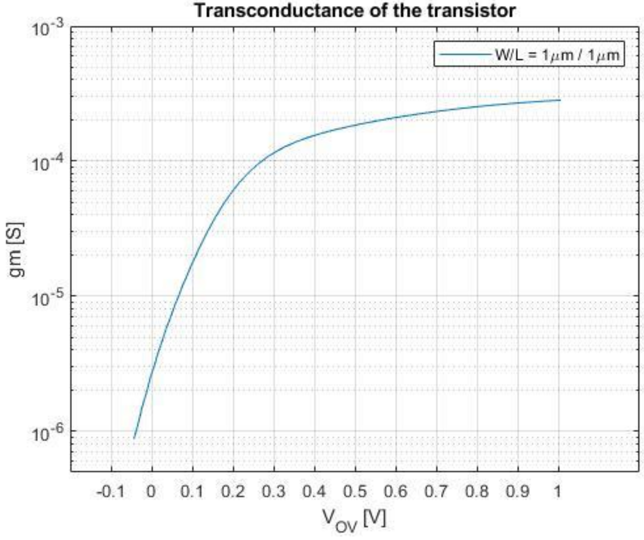
Exercise 2

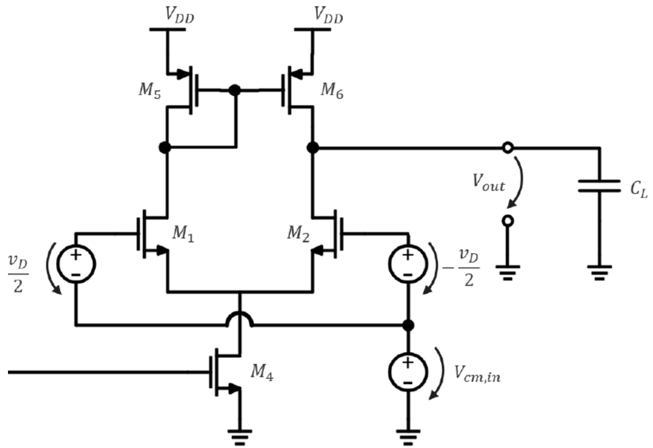
c) What happens to the pole-zero-doublet, if one increases/decreases the V_{OV} of M_5 and M_6 assuming that the current flowing through M4 is fixed as well as the lengths are fixed?

c) $p_{non-dom} \sim \frac{g_{m5}}{C_{gs_{5,6}}}$

$V_{OV,5} \uparrow \Rightarrow g_{m5} ? \Rightarrow p_{nom-dom} ?$







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Exercise 2

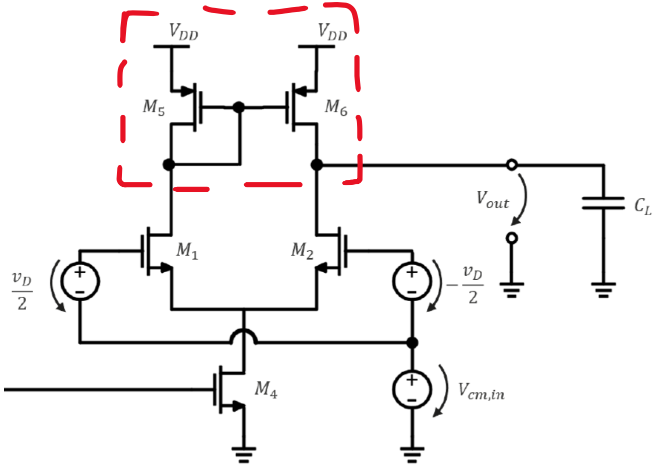
c) $$p_{non-dom} \sim \frac{g_{m5}}{C_{gs_{5,6}}} \sim \frac{2 \cdot I_{DS,5}}{V_{OV_5} \cdot W_5} \sim V_{OV_5}$$

$$C_{gs_{5,6}} \sim C_{ox} \cdot W \cdot L$$

$$W_5 \sim \frac{2I_{DS,5}}{V_{OV,5}^2}$$

$$I_{DS} = \mu C'_{ox} \cdot \frac{W}{2L} \cdot V_{OV}^2$$

$V_{OV,5} \uparrow \Rightarrow p_{nom-dom} \uparrow$



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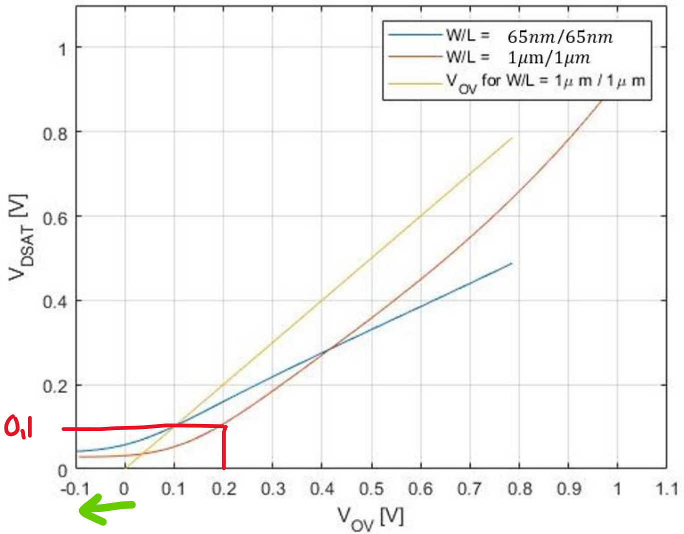
Exercise 2

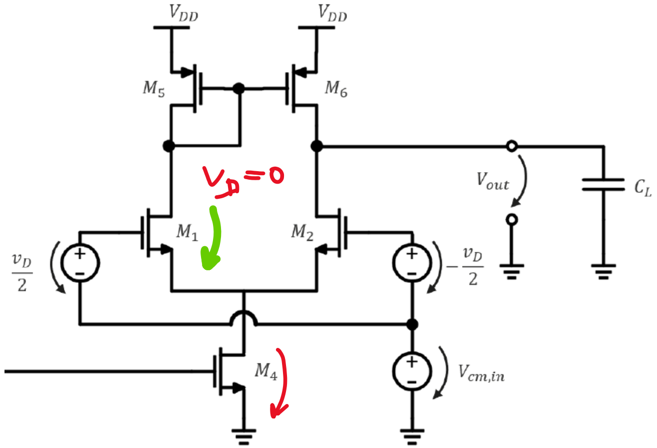
d) Assume $V_{OV,3,4} = 0.2V$, $V_{Dsat,5,6} = -0.25V$ and $V_{OV} < -0.1V$ for the remaining devices. With aid of Fig. 9, calculate the maximum/minimum possible output common mode voltage $V_{cm,max,out}$ and $V_{cm,min,out}$, where all transistors are still in saturation.

d) Calculate $V_{out,cm,min}$ (for $V_{OV,4} = 0.2V$ and remaining $V_{OV} < 0$V)

$$V_{out,cm,min} = V_{Dsat,4} + V_{Dsat,1,2}$$

$$= 0.1\ V + 0.05V = 0.15\ V$$





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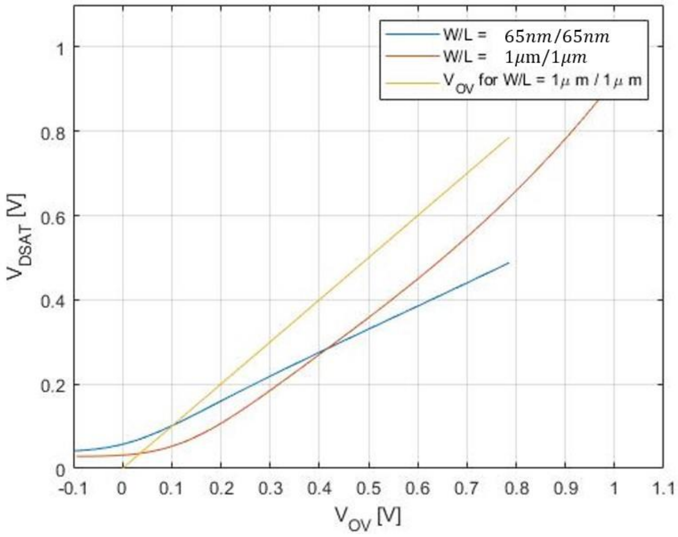
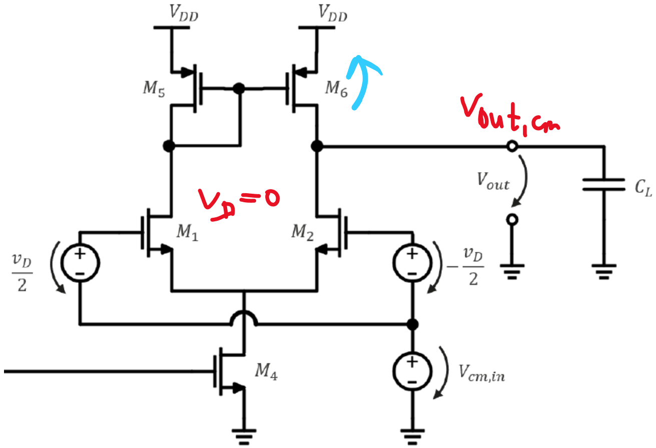
Exercise 2

d) Calculate $V_{out,cm,max}$ (for $V_{OV,4} = 0.2V$ and $V_{Dsat,5,6} = -0.25\text{V}$)

$$V_{out,cm,max} = V_{DD} + V_{Dsat,5,6}$$

$$= 1.1\ V - 0.25V = 0.85\ V$$





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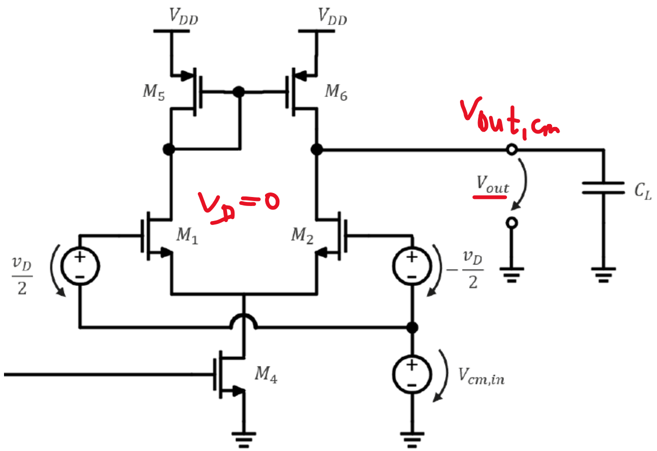
Exercise 2

e) The designer decides to set the output common mode voltage for $v_D = 0$ to the midpoint between $V_{cm,max,out}$ and $V_{cm,min,out}$. Explain how and calculate for that the required V_{OV}.
Hint: Examine the PMOS pair for $v_D = 0$.

e) $V_{out,cm}$ calculated as follows

$$V_{out,cm} = \frac{V_{out,cm,max} + V_{out,cm,min}}{2} = 0.5V$$

How to setup $V_{out,cm}$ in OTA's ?



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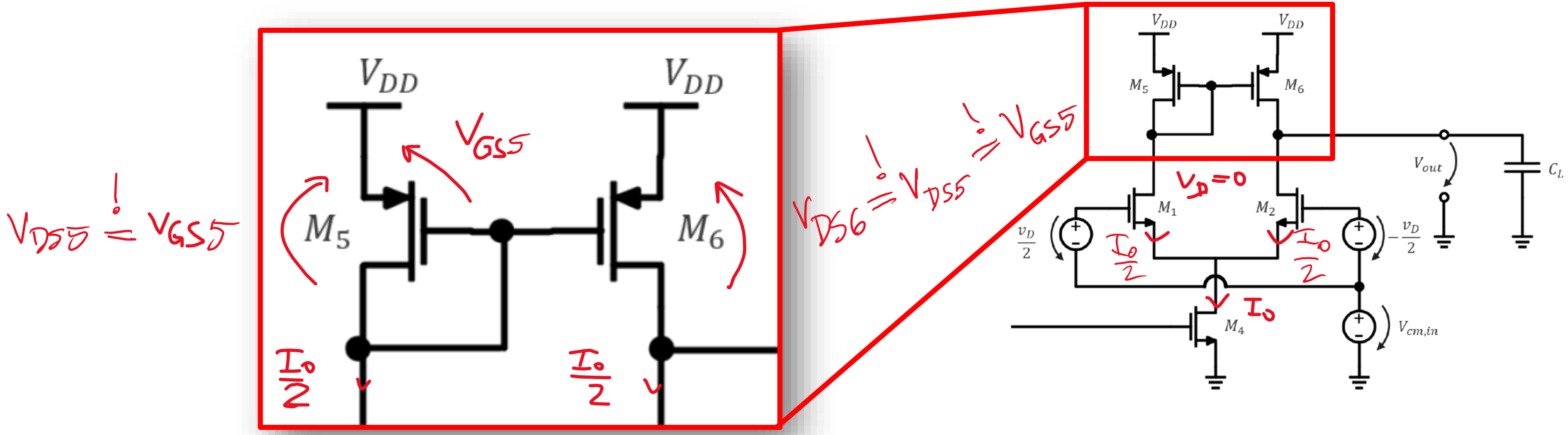
Exercise 2

e) $$V_{out,cm} = \frac{V_{out,cm,max} + V_{out,cm,min}}{2} = 0.5V$$

How to setup $V_{out,cm}$ in OTA's ?



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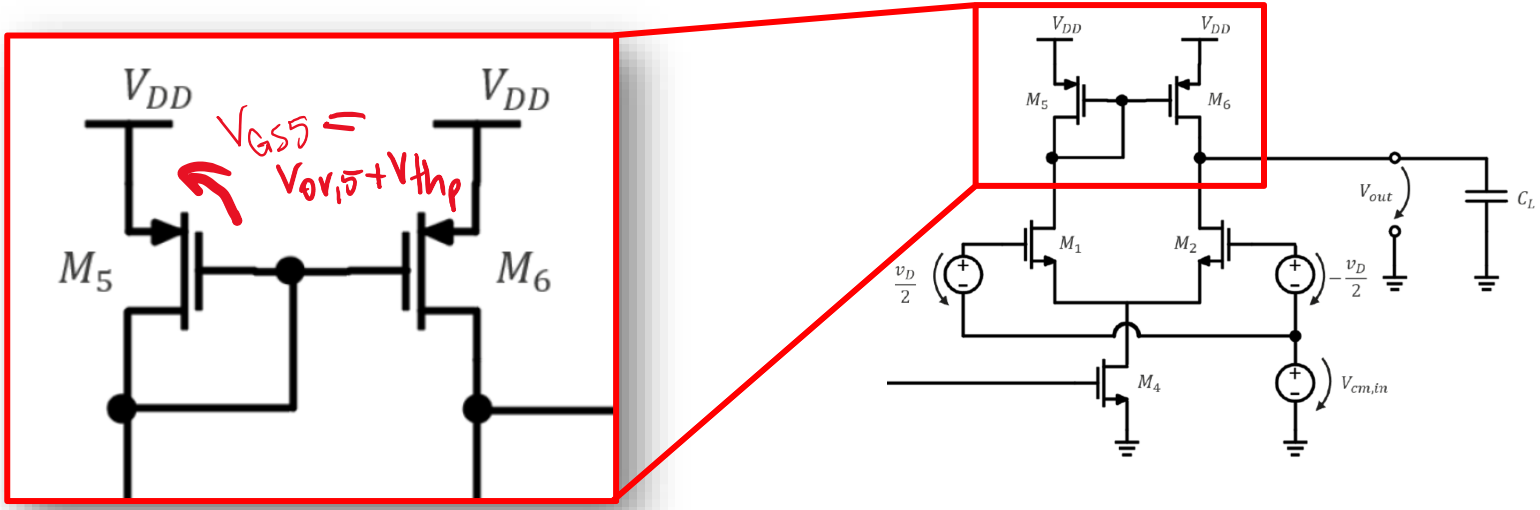
Exercise 2

e) Consequently, $|V_{OV,5}| = V_{DD} - V_{out,cm} - |V_{th,p}| = 0.38V$

$V_{GS5} = V_{ov,5} + V_{thp}$



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- Solve exercise 2 f-i
  - ETA: 20 min

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Exercise 2

f) Find and explain a suitable simplification of the equation from "a)" such that Fig. 8 becomes useful for designing the OTA. Then, calculate the required V_{OV} and $V_{in,cm,min}$ for M_1 and M_2 to achieve $A_V > 8$.

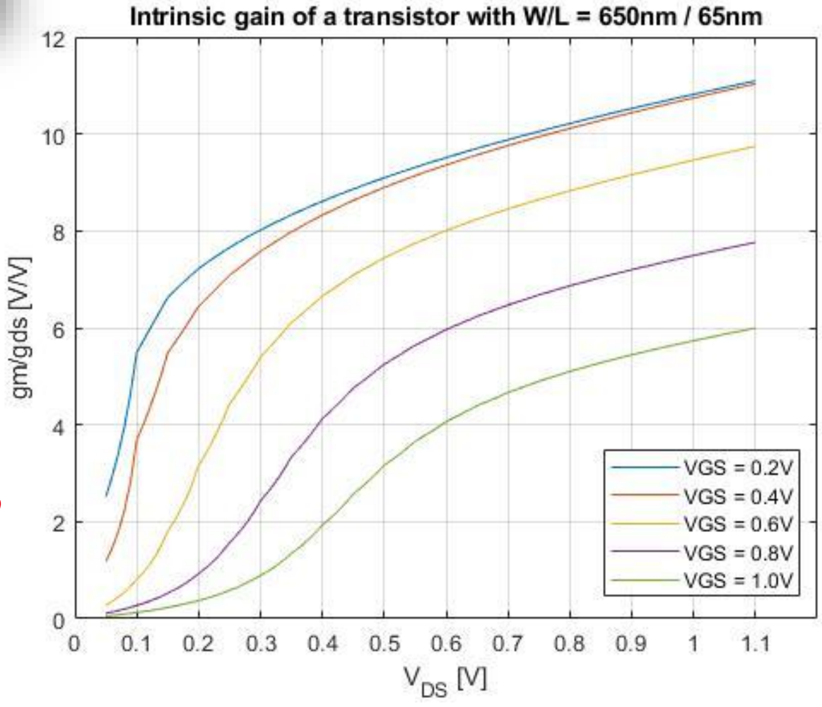
f) How to simplify $A_V(s = 0)$?

$$A_v(s) = \frac{v_{out}(s)}{v_{in}(s)} = \frac{g_{m,1,2}}{g_{02} + g_{06}} \cdot \frac{\overbrace{1 + s\cancel{\frac{C_{d1,5}}{2g_{m5}}}}^{\text{zero}}}{\underbrace{1 + s\cancel{\frac{C_{d1,5}}{g_{m5}}}}_{\text{non-dominant pole}}} \cdot \frac{1}{\underbrace{1 + s\cancel{\frac{C_{d2,6}}{g_{02} + g_{06}}}}_{\text{dominant pole}}}$$

⇒ make $L_{5,6} = 1\mu m$ for negligible CLM

⇒ $g_{02} \gg g_{06}$

$$A_V(s = 0) \approx \frac{g_{m,1,2}}{g_{02}}$$

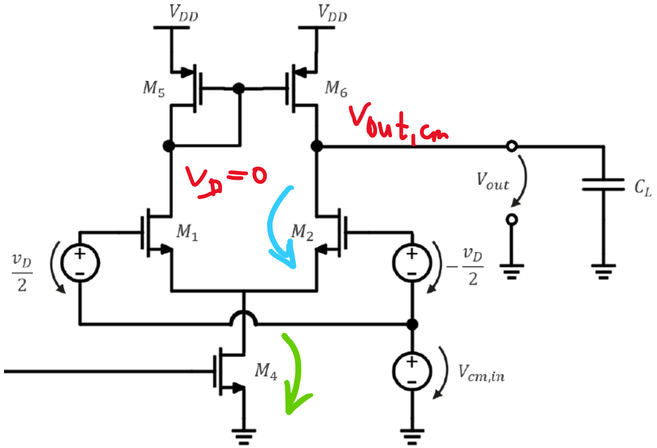


Session 3 : Design of differential pairs and of an OTA

Exercise 2

f) $V_{DS,2}$ calculated as follows

$$V_{DS,2} = V_{out,cm} - V_{Dsat,4} = 0.5 \ - 0.1V = 0.4V$$



Session 3 : Design of differential pairs and of an OTA

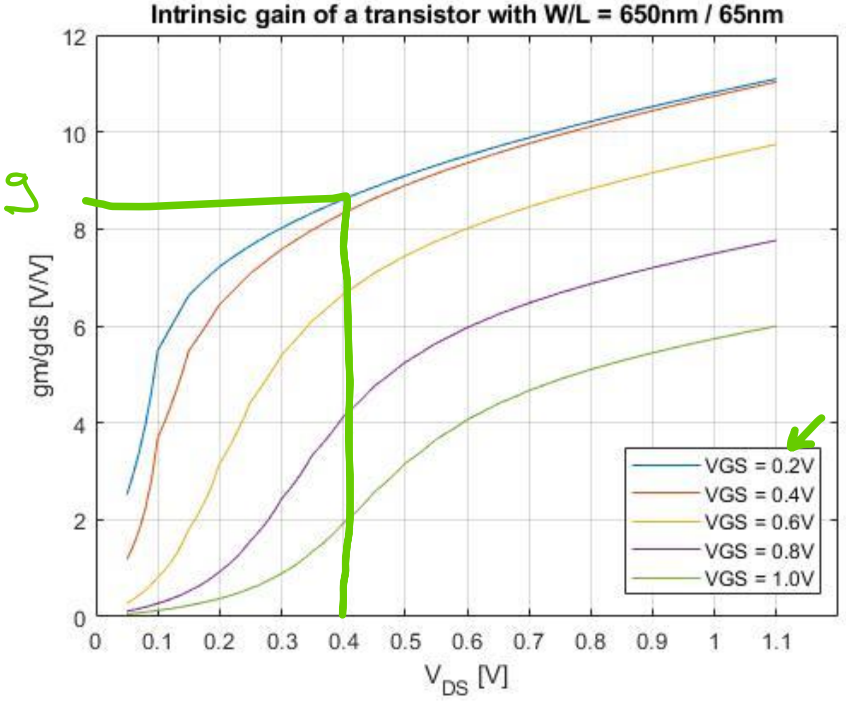
Exercise 2

f) Search $V_{OV,1,2}$ for $A_V(s = 0) > 8$

$\rightarrow V_{GS,1,2} = 0.2V \rightarrow A_V \sim 9$

$\rightarrow V_{OV,1,2} = V_{GS,1,2} - V_{th,n} = -0.2V$

$\rightarrow V_{in,cm} = V_{GS,1,2} + V_{Dsat,4} = 0.3V$



Session 3 : Design of differential pairs and of an OTA

Exercise 2

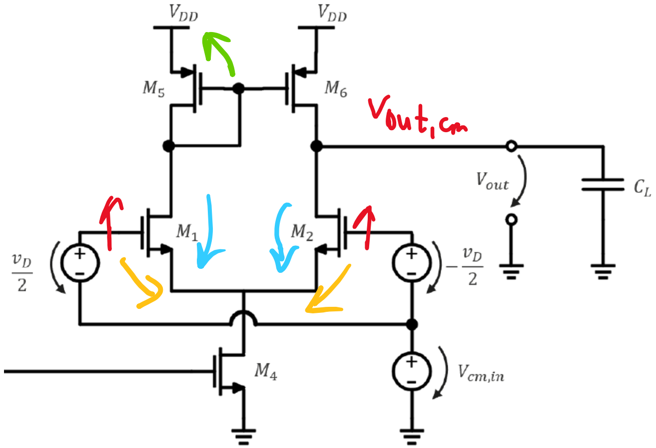
g) Calculate $V_{in,cm,max}$.

g) $V_{in,cm,max}$ calculated as follows:

$$V_{in,cm,max} = \underbrace{V_{DD} + V_{GS,5}} - V_{DSat1,2} + V_{GS,1,2}$$

$$= V_{out,cm} - V_{Dsat,1,2} + V_{GS,1,2}$$

$$= 0.5V - 0.05V + 0.2V = 0.65V$$



Session 3 : Design of differential pairs and of an OTA

Exercise 2

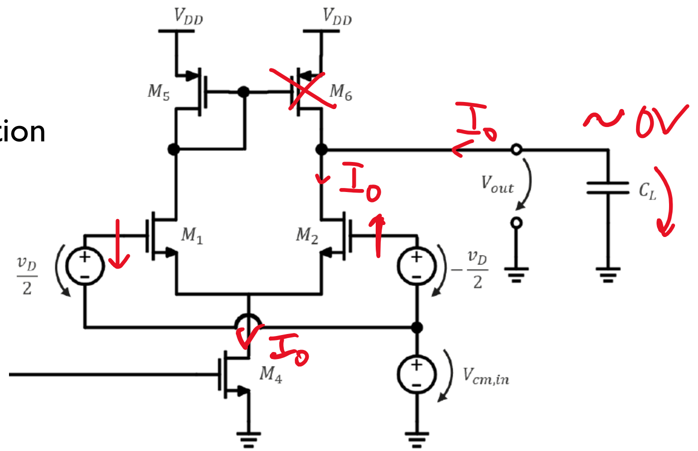
h) Assume a current of I_0 is flowing through M4. Given that all transistors are in saturation, determine what current I_{CL} flows through C_L, if

(i) $V_D = 0$,

(ii) $V_D < -X$: M_1 is in weak inversion and M_2 in strong inversion,

(iii) $V_D > X$: M_1 is in strong inversion and M_2 in weak inversion.

Moreover, determine X.



h) Following current I_{CL} flows through C_L given that all transistors are in saturation

(i) $V_D = 0V \quad : I_{CL} = 0A$

(ii) $V_D < -X : I_{CL} = -I_0$

(iii) $V_D > X \quad : I_{CL} = I_0$

What is X? $\sqrt{2} \cdot V_{OV}$?

$\rightarrow X = \sqrt{2} \cdot V_{Dsat,1,2} \sim 71\ mV$ (for $V_{OV,1,2} = -0.2V$)

Session 3 : Design of differential pairs and of an OTA

Exercise 2

i) Calculate the SR, the required I_0 and $g_{m,1,2}$ necessary to pass exactly the f_{GBW}-requirement with aid of Fig. 5. After that, calculate $g_{m,5,6}$.

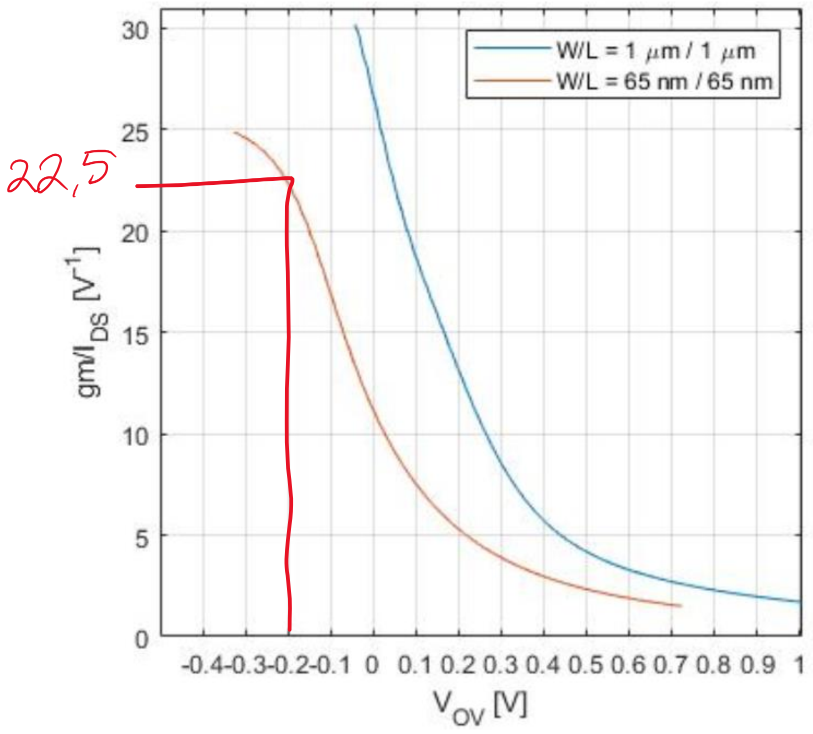
i) Determine SR (Slew Rate) for given f_{GBW}-spec and $V_{OV,1,2}$

$$\frac{g_{m_{1,2}}}{I_{D_{1,2}}} = \frac{2\pi f_{GBW} \cdot C_L}{(I_0/2)} = 2 \cdot \frac{2\pi f_{GBW}}{SR}$$

$$SR = \frac{I_0}{C_L}$$

$$\rightarrow \frac{g_{m_{1,2}}}{I_{D_{1,2}}} = 22.5 \text{ for } V_{OV_{1,2}} = -0.2V$$

$$\rightarrow SR = 2 \cdot \frac{2\pi f_{GBW}}{22.5} = 400mV/ns$$



Session 3 : Design of differential pairs and of an OTA

Exercise 2

i) Calculate I_0 for M4

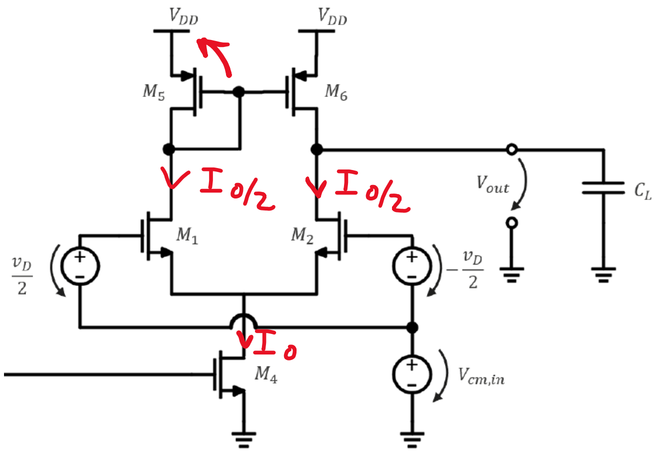
$$I_0 = SR \cdot C_L = 400mV/ns \cdot 500fF = 200\mu A$$

Determine $g_{m,1,2}$

$$g_{m_{1,2}} = 22.5 \cdot \left(\frac{I_0}{2}\right) = 2.25mS$$

Determine $g_{m,5,6}$

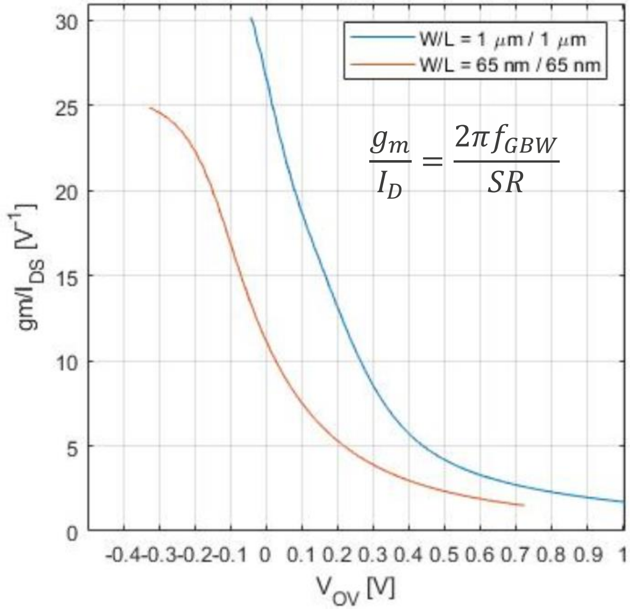
$$g_{m,5,6} = \frac{2 \cdot (I_0/2)}{V_{OV,5,6}} \approx 526\mu S$$

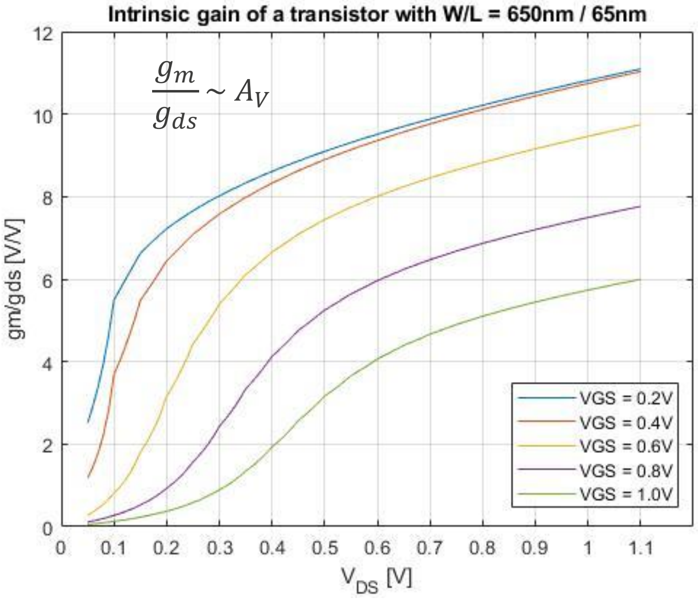


Session 3 : Design of differential pairs and of an OTA

Summary (1)

1. Value the $\frac{g_m}{I_D}$- and $\frac{g_m}{g_{ds}}$- plots!
   - One can predict the feasibility of the given specifications for an OTA in one view





Session 3 : Design of differential pairs and of an OTA

Summary (2)

2. Design flow introduced
3. Learned how to "design on paper" with
   - W-plot for fixed g_m or i_{ds}
   - $\frac{g_m}{I_D}$- and $\frac{g_m}{g_{ds}}$-plots
   - V_{dsat}-plot
   - ➢ Those plots will help you to save time when designing for the final lab session!

(1) Derive equations depending on the Specifications

(2) Decide which transistor should have a short or long length L (65nm or 1um) ?

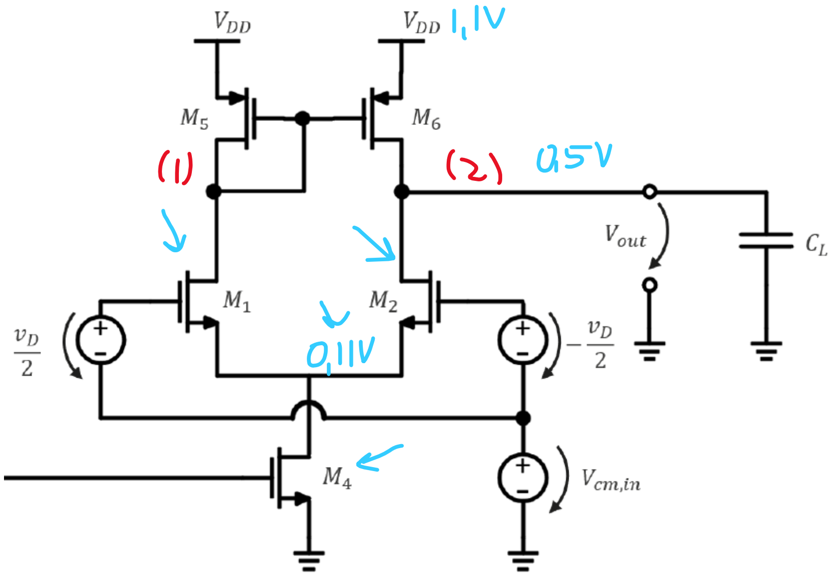
(3) Determine g_m (or I_{DS}) and V_{OV} (w.i. or s.i.?) of each transistor

(4) Determine width W of each transistor

Session 3 : Design of differential pairs and of an OTA

MATLAB session

- Exercise 3
  1. Type in the code accordingly to this presented OTA exercise
  2. Solve following scientific questions?
     1. How can one move the pole-zero-doublet?
        - 2 options
     2. How can I increase the f_{GBW}?
  - ETA: 30 min



Hints:

1. Use the following command to copy transistors e.g. M1 and M2
   1. M2 = cirElementCopy(M1, M2);
2. Before coding, draw all voltage condition on the OTA-circuit
   - i.e. vgs, vgd, vds, ids (or gm) of each transistor !
   - This will help you to speed-up your coding
3. Code it smart !
   1. code it such that you can play around with the pole-zero doublet !

Session 3 : Design of differential pairs and of an OTA

... some helpful references

[1]: “Analog Design Essentials”, Sansen, Willy M. C, 2006

[2]: “Design of Analog CMOS Integrated Circuits”, Razavi, Behzad, 2016

[3]: “Analysis and Design of Analog Integrated Circuits”, P. Gray, P. Hurst, S. Lewis and R. Meyer, John Wiley and Sons, 2009

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