Book 5 Proposition 12

If there are any number of magnitudes whatsoever (which are) proportional then as one of the leading (magnitudes is) to one of the following, so will all of the leading (magnitudes) be to all of the following.

A	C —	• E
$B \longmapsto$	$D \longmapsto$	F -
G -		L
H		$M \vdash \!$
K	—	N

Let there be any number of magnitudes whatsoever, A, B, C, D, E, F, (which are) proportional, (so that) as A (is) to B, so C (is) to D, and E to F. I say that as A is to B, so A, C, E (are) to B, D, F.

For let the equal multiples G, H, K have been taken of A, C, E (respectively), and the other random equal multiples L, M, N of B, D, F (respectively).

And since as A is to B, so C (is) to D, and E to F, and the equal multiples G, H, K have been taken of A, C, E (respectively), and the other random equal multiples L, M, N of B, D, F (respectively), thus if G exceeds L then H also exceeds M, and K (exceeds) N, and if (G is) equal (to L then H is also) equal (to M, and K to N), and if (G is) less (than L then H is also) less (than M, and K than N) [Def. 5.5]. And, hence, if G exceeds L then G, H, K also exceed L, M, N, and if (G is) equal (to L then G, H, K are also) equal (to L, M, N) and if (G is) less (than L then G, H, K are also) less (than

L, M, N). And G and G, H, K are equal multiples of A and A, C, E (respectively), inasmuch as if there are any number of magnitudes whatsoever (which are) equal multiples, respectively, of some (other) magnitudes, of equal number (to them), then as many times as one of the (first) magnitudes is (divisible) by one (of the second), so many times will all (of the first magnitudes) also (be divisible) by all (of the second) [Prop. 5.1]. So, for the same (reasons), L and L, M, N are also equal multiples of B and B, D, F (respectively). Thus, as A is to B, so A, C, E (are) to B, D, F (respectively).

Thus, if there are any number of magnitudes whatsoever (which are) proportional then as one of the leading (magnitudes is) to one of the following, so will all of the leading (magnitudes) be to all of the following. (Which

is) the very thing it was required to show.