Book 8 Proposition 13

If there are any multitude whatsoever of continuously proportional numbers, and each makes some (number by) multiplying itself, then the (numbers) created from them will (also) be (continuously) proportional. And if the original (numbers) make some (more numbers by) multiplying the created (numbers) then these will also be (continuously) proportional [and this always happens with the extremes].

Let A, B, C be any multitude whatsoever of continuously proportional numbers, (such that) as A (is) to B, so B (is) to C. And let A, B, C make D, E, F (by) multiplying themselves, and let them make G, H, K (by) multiplying D, E, F. I say that D, E, F and G, H, K are continuously proportional.

A — — — — — — — — — — — — — — — — — — —	L
В	0
C ———	$\mathbf{M} \longmapsto$
D⊢──	N
_	P
E ———	Q
F	Y
G⊢──	
$H \vdash \longrightarrow$	
K	⊣

For let A make L (by) multiplying B. And let A, B make M, N, respectively, (by) multiplying L. And, again, let B make O (by) multiplying C. And let B, C make P, Q, respectively, (by) multiplying O.

So, similarly to the above, we can show that D, L, E

and G, M, N, H are continuously proportional in the ratio of A to B, and, further, (that) E, O, F and H, P, Q, K are continuously proportional in the ratio of B to C. And as A is to B, so B (is) to C. And thus D, L, E are in the same ratio as E, O, F, and, further, G, M, N, H (are in the same ratio) as H, P, Q, K. And the multitude of D, L, E is equal to the multitude of E, O, F, and that of G, M, N, H to that of H, P, Q, K. Thus, via equality, as D is to E, so E (is) to F, and as G (is) to H, so H (is) to K [Prop. 7.14]. (Which is) the very thing it was required to show.