Book 10 Proposition 52

Let the two numbers AC and CB be laid down such that AB does not have to either of them the ratio which (some) square number (has) to (some) square number [Prop. 10.38 lem.]. And let some rational straight-line D be laid down. And let EF be commensurable [in Thus, EF (is) a rational (straightlength with D. line). And let it have been contrived that as CA (is) to AB, so the (square) on EF (is) to the (square) on FG[Prop. 10.6 corr.]. And CA does not have to AB the ratio which (some) square number (has) to (some) square number. Thus, the (square) on EF does not have to the (square) on FG the ratio which (some) square number (has) to (some) square number either. Thus, EF and FGare rational (straight-lines which are) commensurable in square only [Prop. 10.9]. Thus, EG is a binomial (straight-line) [Prop. 10.36] So, I say that (it is) also a fifth (binomial straight-line).

For since as CA is to AB, so the (square) on EF (is) to the (square) on FG, inversely, as BA (is) to AC, so the (square) on FG (is) to the (square) on FE [Prop. 5.7 corr.]. Thus, the (square) on GF (is) greater than the (square) on FE [Prop. 5.14]. Therefore, let

(the sum of) the (squares) on EF and H be equal to the (square) on GF. Thus, via conversion, as the number AB is to BC, so the (square) on GF (is) to the (square) on H [Prop. 5.19 corr.]. And AB does not have to BCthe ratio which (some) square number (has) to (some) square number. Thus, the (square) on FG does not have to the (square) on H the ratio which (some) square number (has) to (some) square number either. Thus, FG is incommensurable in length with H [Prop. 10.9]. Hence, the square on FG is greater than (the square on) FE by the (square) on (some straight-line) incommensurable (in length) with (FG). And GF and FEare rational (straight-lines which are) commensurable in square only. And the lesser term EF is commensurable in length with the rational (straight-line previously) laid down, D.

Thus, EG is a fifth binomial (straight-line).[†] (Which is) the very thing it was required to show.