Book 7 Proposition 14

If there are any multitude of numbers whatsoever, and (some) other (numbers) of equal multitude to them, (which are) also in the same ratio taken two by two, then they will also be in the same ratio via equality.

A	D —
B ──	E
C	F

Let there be any multitude of numbers whatsoever, A, B, C, and (some) other (numbers), D, E, F, of equal multitude to them, (which are) in the same ratio taken two by two, (such that) as A (is) to B, so D (is) to E, and as B (is) to C, so E (is) to F. I say that also, via equality, as A is to C, so D (is) to F.

For since as A is to B, so D (is) to E, thus, alternately, as A is to D, so B (is) to E [Prop. 7.13]. Again, since as B is to C, so E (is) to F, thus, alternately, as B is to E, so C (is) to F [Prop. 7.13]. And as B (is) to E, so A (is) to D. Thus, also, as A (is) to D, so C (is) to F. Thus, alternately, as A is to C, so D (is) to F [Prop. 7.13]. (Which is) the very thing it was required to show.