Book 10 Proposition 86

To find a second apotome.

Let the rational (straight-line) A, and GC (which is) commensurable in length with A, be laid down. Thus, GC is a rational (straight-line). And let the two square numbers DE and EF be laid down, and let their difference DF be not square [Prop. 10.28 lem. I]. And let it have been contrived that as FD (is) to DE, so the square on CG (is) to the square on GB [Prop. 10.6 corr.]. Thus, the square on CG is commensurable with the square on GB [Prop. 10.6]. And the (square) on CG (is) rational. Thus, the (square) on GB [is] also rational. Thus, BGis a rational (straight-line). And since the square on GCdoes not have to the (square) on GB the ratio which (some) square number (has) to (some) square number, CG is incommensurable in length with GB [Prop. 10.9]. And they are both rational (straight-lines). Thus, CGand GB are rational (straight-lines which are) commensurable in square only. Thus, BC is an apotome [Prop. 10.73] So, I say that it is also a second (apotome).

For let the (square) on H be that (area) by which the (square) on BG is greater than the (square) on GC [Prop. 10.13 lem.]. Therefore, since as the (square) on BG is to the (square) on GC, so the number ED (is) to the number DF, thus, also, via conversion, as the (square) on BG is to the (square) on H, so DE (is) to

EF [Prop. 5.19 corr.] . And DE and EF are each square (numbers). Thus, the (square) on BG has to the (square) on H the ratio which (some) square number (has) to (some) square number. Thus, BG is commensurable in length with H [Prop. 10.9] . And the square on BG is greater than (the square on) GC by the (square) on H. Thus, the square on BG is greater than (the square on) GC by the (square) on (some straight-line) commensurable in length with (BG). And the attachment CG is commensurable (in length) with the (prevously) laid down rational (straight-line) A. Thus, BC is a second apotome [Def. 10.12]. †

Thus, the second apotome BC has been found. (Which is) the very thing it was required to show.