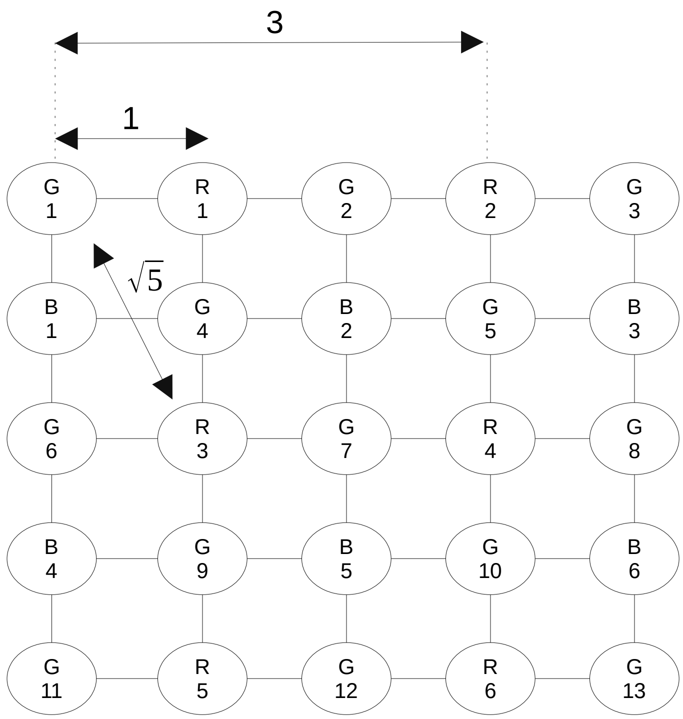


$$\frac{1}{1+\sqrt{5}+3}+\frac{\sqrt{5}}{1+\sqrt{5}+3}+\frac{3}{1+\sqrt{5}+3}=\frac{1+\sqrt{5}+3}{1+\sqrt{5}+3}=1$$

0.48107 0.35857 0.16035

$$\overline{R}=a_1\cdot\overline{R_1}+a_2\cdot\overline{R_2}+a_3\cdot\overline{R_3}$$

$$R^*=\overline{a_1}\cdot R_1+\overline{a_2}\cdot R_2+\overline{a_3}\cdot R_3$$

R * reads as R dual

$$\begin{aligned}R=&0.48107\cdot\left(r_7^1\cdot 2^7+\ldots+r_0^1\cdot 2^0\right)\\&+0.35857\cdot\left(r_7^3\cdot 2^7+\ldots+r_0^3\cdot 2^0\right)\\&+0.16035\cdot\left(r_7^2\cdot 2^7+\ldots+r_0^2\cdot 2^0\right)\end{aligned}$$

$$\overline{R}=0.48107\cdot\begin{pmatrix}r_7^1\\r_6^1\\r_5^1\\r_5^1\\r_4^1\\r_3^1\\r_2^1\\r_1^1\\r_0^1\end{pmatrix}+0.35857\cdot\begin{pmatrix}r_7^3\\r_6^3\\r_5^3\\r_5^3\\r_4^3\\r_3^3\\r_2^3\\r_1^3\\r_0^3\end{pmatrix}+0.16035\cdot\begin{pmatrix}r_7^2\\r_6^2\\r_5^2\\r_5^2\\r_4^2\\r_3^2\\r_2^2\\r_1^2\\r_0^2\end{pmatrix}$$

$$R^*=r_7^1r_6^1r_5^1r_4^1r_3^1r_2^1r_1^1r_0^1r_7^3r_6^3r_5^3r_4^3r_3^3r_2^3r_1^3r_0^3r_7^2r_6^2r_5^2r_4^2r_3^2r_2^2r_1^2r_0^2$$ call it R_{BIG}

Ex1 Px0, three surrounding red pixels

$$R_1=128_{10}=10000000_2$$
$$R_2=53_{10}=00110101_2$$
$$R_3=127_{10}=01111111_2$$

$$R_{IDEAL}=0.48\cdot 128+0.36\cdot 127+0.16\cdot 53=115.64=01110011_2$$

$$R_{BIG}=R_1\,R_3\,R_2=\underbrace{100000000111111100110101}_{\text{24bit}},$$ $R=10000100_2=132$ not good approx. ■

Ex2

Px0, three surrounding red pixels

$$R_1=128_{10}=10000000_2$$
$$R_2=53_{10}=00110101_2$$
$$R_3=127_{10}=01111111_2$$

$$R_{IDEAL}=0.48\cdot128+0.36\cdot127+0.16\cdot53$$
$$=115.64$$
$$=01110011_2$$

Let $R_{12}=\overline{(R_1 \underset{1}{>>} 2)\oplus(R_2 \underset{1}{<<} 1)}=01110100_2$,

where $\underset{1}{>>}$ and $\underset{1}{<<}$ - logical shift with '1' fill

$\oplus$ - XNOR boolean operator

- better approx. ■

Let $R=r_7r_6r_5r_4r_3r_2r_1r_0$

Define Я as: $Я=r_4r_5r_6r_7r_0r_1r_2r_3$ And define reversal operator Я as: $Я(R)=Я$

Ex3

From Ex2 $R_{12}=01110100_2$

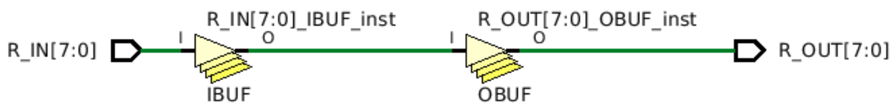
Let $Я_{12}=11100010_2$

$$\overline{Я_{12}\oplus(R_3 \underset{1}{<<} 1)}=11100010_2$$

$Я(\overline{Я_{12}\oplus(R_3 \underset{1}{<<} 1)})=01110100_2=R_{12}$ - identity operator ■

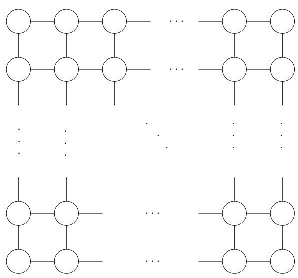
reverser_8b.vhd

```
entity reverser_8b is
  Port ( R_IN : in std_logic_vector(7 downto 0);
    R_OUT : out std_logic_vector(7 downto 0));
end reverser_8b;

architecture Dataflow of reverser_8b is

begin
   R_OUT(7) <= R_IN(4);
   R_OUT(6) <= R_IN(5);
   R_OUT(5) <= R_IN(6);
   R_OUT(4) <= R_IN(7);
   R_OUT(3) <= R_IN(0);
   R_OUT(2) <= R_IN(1);
   R_OUT(1) <= R_IN(2);
   R_OUT(0) <= R_IN(3);
end Dataflow;
```



Recall $R^{*}=r_7^1 r_6^1 r_5^1 r_4^1 r_3^1 r_2^1 r_1^1 r_0^1 r_7^3 r_6^3 r_5^3 r_4^3 r_3^3 r_2^3 r_1^3 r_0^3 r_7^2 r_6^2 r_5^2 r_4^2 r_3^2 r_2^2 r_1^2 r_0^2$ call it R_{BIG}

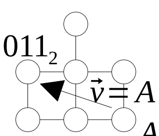
Therefore R_{BIG} is a finite space and dual space R^{*} can be viewed as a rectangle or a plane consisting of 2^{24} elements in it arranged in a lattice pattern.



Thus we can define our binary operations such as AND,OR, etc as two different things:
1) Means of traversing the grid by flipping some of the bits
2) vectors
We choose one boolean operator to measure similarity and go from there. Let it be XNOR. Other operations will have to do with adjusting weights as we arrived here from the weighted sum and the end goal is weighted average.

Ex4

$B=011_2$



$\vec{v}=A\oplus 100_2=(2,1)$

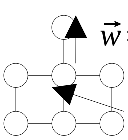
$A=000_2$

■

Ex5

$B=111_2$

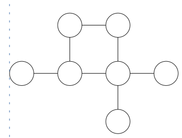
$\vec{w}=A \underset{1}{>>} 1=(1,1)=\vec{v}$



$A \underset{1}{>>} 1=100_2$

$A=000_2$

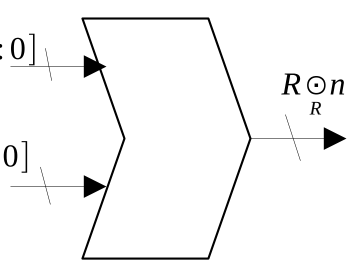
Therefore $\vec{v},\vec{w}$ are orthogonal (hint from linear algebra) and these two operations (right shift with '1' fill and XNOR) span the entire lattice. ■



Can try different geometries/origin also.
Actual notion of distance in this space may depend on the amount of such binary operations required to arrive from origin to any other point.
Remember that we are looking for a point in this grid, that surely does exist, that is the closest approximation to the weighted average.
Also remember once we contrived this space to be linear and dual by assuming 0.48 0.36 and 0.16 real numbers are independent (orthogonal) thus their products are equal to 0, all normal algebra went out the window.

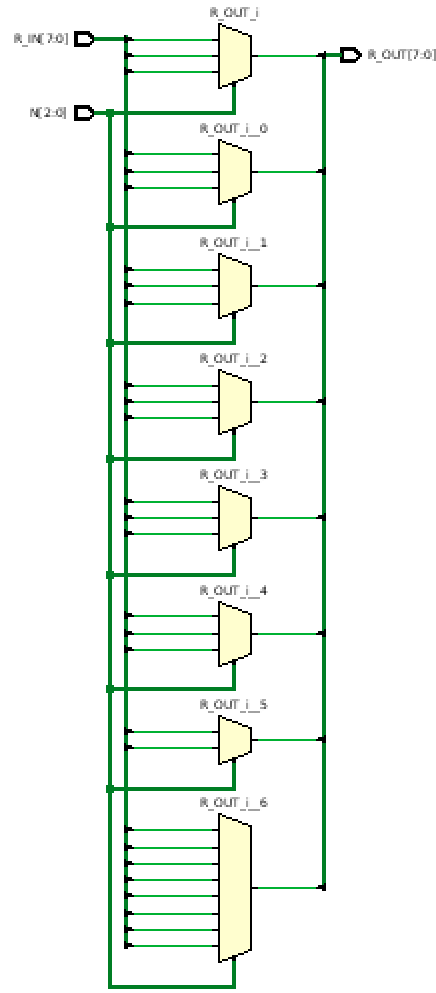
Let $R=r_7 r_6 r_5 r_4 r_3 r_2 r_1 r_0$

Define RPUSH operator as:



where n – control signal choosing which bit to push to the top position rpush_8b.vhd

```
entity rpushn_8b is
  Port ( R_IN : in std_logic_vector(7 downto 0);
    N : in std_logic_vector(2 downto 0);
    R_OUT : out std_logic_vector(7 downto 0));
end rpushn_8b;

architecture Dataflow of rpushn_8b is

begin
     with N select
        R_OUT(7) <= R_IN(0) when "000",
          R_IN(1) when "001",
          R_IN(2) when "010",
          R_IN(3) when "011",
          R_IN(4) when "100",
          R_IN(5) when "101",
          R_IN(6) when "110",
          R_IN(7) when others;
      with N select
        R_OUT(6) <= R_IN(6) when "111",
          R_IN(7) when others;
      with N select
        R_OUT(5) <= R_IN(5) when "111",
          R_IN(5) when "110",
          R_IN(6) when others;
      with N select
        R_OUT(4) <= R_IN(4) when "111",
          R_IN(4) when "101",
          R_IN(5) when others;
      with N select
        R_OUT(3) <= R_IN(3) when "111",
          R_IN(3) when "100",
          R_IN(4) when others;
      with N select
        R_OUT(2) <= R_IN(2) when "111",
          R_IN(2) when "011",
          R_IN(3) when others;
      with N select
        R_OUT(1) <= R_IN(1) when "111",
          R_IN(1) when "010",
          R_IN(2) when others;
      with N select
        R_OUT(0) <= R_IN(0) when "111",
          R_IN(0) when "001",
          R_IN(1) when others;
end Dataflow;
```

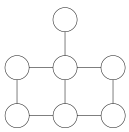


Ex6

Let $R = r_7 r_6 r_5 r_4 r_3 r_2 r_1 r_0$

$R \underset{R}{\odot} 3 = r_3 r_7 r_6 r_5 r_4 r_2 r_1 r_0$ ■

Ex7



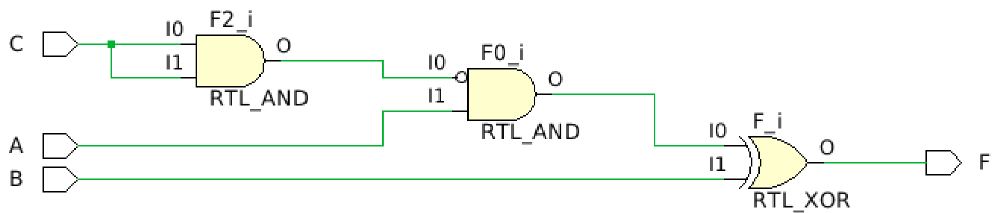
$\vec{u} = A \underset{R}{\odot} 0 = \vec{0}$

$A = 000_2$

RPUSH operator clearly doesn't do anything for the origin ■

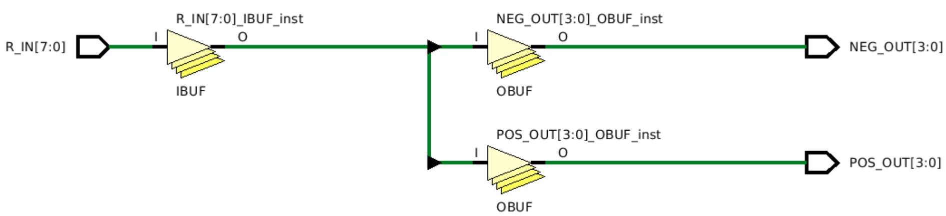
dim_1b.vhd

```
entity dim_1b is
  Port ( A : in std_logic;
    B : in std_logic;
    C : in std_logic;
    F: out std_logic);
end dim_1b;

architecture Dataflow of dim_1b is

begin
  F <= ((C NAND C) AND A) XOR B;

end Dataflow;
```



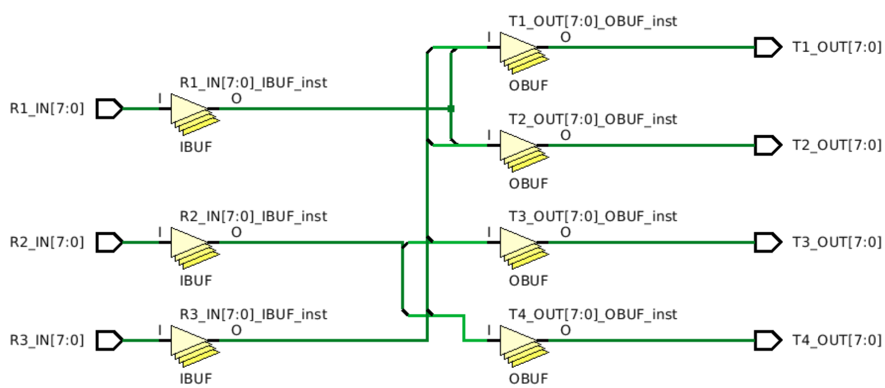
diode_8b.vhd

```vhdl
entity diode_8b is
  Port ( R_IN : in std_logic_vector(7 downto 0);
  POS_OUT : out std_logic_vector(3 downto 0);
  NEG_OUT : out std_logic_vector(3 downto 0));
end diode_8b;

architecture Dataflow of diode_8b is

begin
  POS_OUT(3) <= R_IN(7);
  POS_OUT(2) <= R_IN(6);
  POS_OUT(1) <= R_IN(1);
  POS_OUT(0) <= R_IN(0);
  NEG_OUT(3) <= R_IN(5);
  NEG_OUT(2) <= R_IN(4);
  NEG_OUT(1) <= R_IN(3);
  NEG_OUT(0) <= R_IN(2);

end Dataflow;
```

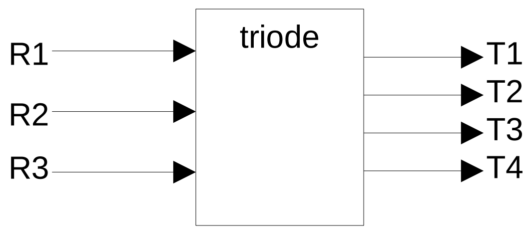


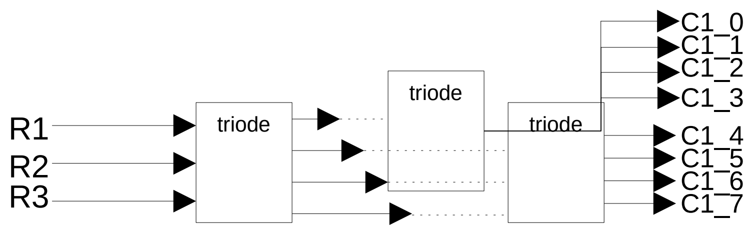
triode_8b.vhd

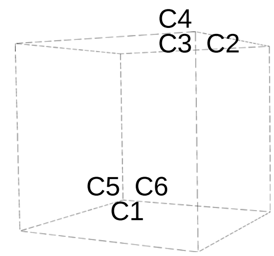
```
entity triode_8b is
  Port ( R1_IN : in std_logic_vector(7 downto 0);
  R2_IN : in std_logic_vector(7 downto 0);
  R3_IN : in std_logic_vector(7 downto 0);
  T1_OUT : out std_logic_vector(7 downto 0);
  T2_OUT : out std_logic_vector(7 downto 0);
  T3_OUT : out std_logic_vector(7 downto 0);
  T4_OUT : out std_logic_vector(7 downto 0));
end triode_8b;

architecture Dataflow of triode_8b is

begin
   T1_OUT(7) <= R1_IN(7);
   T1_OUT(6) <= R1_IN(6);
   T1_OUT(5) <= R3_IN(3);
   T1_OUT(4) <= R3_IN(4);
   T1_OUT(3) <= R3_IN(7);
   T1_OUT(2) <= R3_IN(0);
   T1_OUT(1) <= R1_IN(1);
   T1_OUT(0) <= R1_IN(0);

    T2_OUT(7) <= R1_IN(5);
    T2_OUT(6) <= R1_IN(4);
    T2_OUT(5) <= R3_IN(2);
    T2_OUT(4) <= R3_IN(5);
    T2_OUT(3) <= R3_IN(6);
    T2_OUT(2) <= R3_IN(1);
    T2_OUT(1) <= R1_IN(3);
    T2_OUT(0) <= R1_IN(2);

    T3_OUT(7) <= R2_IN(5);
    T3_OUT(6) <= R2_IN(4);
    T3_OUT(5) <= R3_IN(1);
    T3_OUT(4) <= R3_IN(6);
    T3_OUT(3) <= R3_IN(5);
    T3_OUT(2) <= R3_IN(2);
    T3_OUT(1) <= R2_IN(3);
    T3_OUT(0) <= R2_IN(2);

    T4_OUT(7) <= R2_IN(7);
    T4_OUT(6) <= R2_IN(6);
    T4_OUT(5) <= R3_IN(0);
    T4_OUT(4) <= R3_IN(7);
    T4_OUT(3) <= R3_IN(4);
    T4_OUT(2) <= R3_IN(3);
    T4_OUT(1) <= R2_IN(1);
    T4_OUT(0) <= R2_IN(0);

 end Dataflow;
```



Series triodes. 8X6 matrices. Cubes.

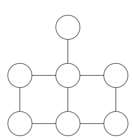






Six possible permutations of R1,R2,R3 fed into series triodes in the arrangement shown above give us a cube – unit of a three-dimensional lattice

Ex8



$A=000_2$

1-bit triode: $Triode(R_1,R_2,R_3)=\begin{matrix} r_0^3 \\ r_0^1 \\ r_0^2 \\ r_0^3 \end{matrix}$ $\quad Triode(R_1,R_3,R_2)=\begin{matrix} r_0^2 \\ r_0^1 \\ r_0^3 \\ r_0^2 \end{matrix}$

$R_1=r_0^1 \quad R_2=r_0^2 \quad R_3=r_0^3$

$$Triode(R_2,R_1,R_3)=\begin{matrix} r_0^3 \\ r_0^2 \\ r_0^1 \\ r_0^3 \end{matrix} \quad Triode(R_2,R_3,R_1)=\begin{matrix} r_0^1 \\ r_0^2 \\ r_0^3 \\ r_0^1 \end{matrix} \quad Triode(R_3,R_1,R_2)=\begin{matrix} r_0^2 \\ r_0^3 \\ r_0^1 \\ r_0^2 \end{matrix} \quad Triode(R_3,R_2,R_1)=\begin{matrix} r_0^1 \\ r_0^3 \\ r_0^2 \\ r_0^1 \end{matrix}$$

=> 1-bit cube is $$Cube(R_1,R_2,R_3)=\begin{matrix} r_0^2 & r_0^3 & r_0^1 & r_0^3 & r_0^1 & r_0^2 \\ r_0^3 & r_0^2 & r_0^3 & r_0^1 & r_0^2 & r_0^1 \\ r_0^1 & r_0^1 & r_0^2 & r_0^2 & r_0^3 & r_0^3 \\ r_0^2, & r_0^3, & r_0^1, & r_0^3, & r_0^1, & r_0^2 \\ r_0^3 & r_0^2 & r_0^3 & r_0^1 & r_0^2 & r_0^1 \\ r_0^1 & r_0^1 & r_0^2 & r_0^2 & r_0^3 & r_0^3 \\ r_0^2 & r_0^3 & r_0^1 & r_0^3 & r_0^1 & r_0^2 \\ r_0^3 & r_0^2 & r_0^3 & r_0^1 & r_0^2 & r_0^1 \end{matrix}$$

■

Denoise. Piecewise linear functions.

Recall $\overline{R}=a_1\cdot\overline{R_1}+a_2\cdot\overline{R_2}+a_3\cdot\overline{R_3}$ is the value of the red component in the Bayer color filter array.

Introduce noise as a constant in our linear equation:

$\overline{R}=a_1\cdot\overline{R_1}+a_2\cdot\overline{R_2}+a_3\cdot\overline{R_3}+\overline{C}$,where C is noise

$$\overline{R}=0.48107\cdot\begin{pmatrix}r_7^1\\r_6^1\\r_5^1\\r_5^1\\r_4^1\\r_3^1\\r_2^1\\r_1^1\\r_0^1\end{pmatrix}+0.35857\cdot\begin{pmatrix}r_7^3\\r_6^3\\r_5^3\\r_5^3\\r_4^3\\r_3^3\\r_2^3\\r_1^3\\r_0^3\end{pmatrix}+0.16035\cdot\begin{pmatrix}r_7^2\\r_6^2\\r_5^2\\r_5^2\\r_4^2\\r_3^2\\r_2^2\\r_1^2\\r_0^2\end{pmatrix}+\begin{pmatrix}c_7\\c_6\\c_5\\c_5\\c_4\\c_3\\c_2\\c_1\\c_0\end{pmatrix}$$

Therefore now our Rbig looks like:

$R^*=r_7^1r_6^1r_5^1r_4^1r_3^1r_2^1r_1^1r_0^1r_7^3r_6^3r_5^3r_4^3r_3^3r_2^3r_1^3r_0^3r_7^2r_6^2r_5^2r_4^2r_3^2r_2^2r_1^2r_0^2c_7c_6c_5c_4c_3c_2c_1c_0$ call it R_{BIG}

Now, taking it back to the inherent orthogonality of flips and shifts and keeping in mind their vector nature. As this is our new basis, and it is clear that the order of operations is strict, i.e.

$$f\circ g\neq g\circ f$$

Naturally, interpret our red component vectors as:

$$i=\begin{Bmatrix}0\, induce\ shift\\1\, induce\ flip\end{Bmatrix}$$

Ex9

Px0, three surrounding red pixels

$R_1=128_{10}=10000000_2$ $R_1^*=F(S(S(S(S(S(S(S(x))))))))$

$R_2=53_{10}=00110101_2$ $R_2^*=S(S(F(F(S(F(S(F(x))))))))$

$R_3=127_{10}=01111111_2$ $R_3^*=S(F(F(F(F(F(F(F(x))))))))$

Notion of most and least significance of bits comes to help here.

Therefore our digital circuit core has to contain 2^8 = 256 dual vectors. ■

So what we really meant with R* and Rbig is:

$R^*=r_7^1r_6^1r_5^1r_4^1r_3^1r_2^1r_1^1r_0^1\wedge r_7^3r_6^3r_5^3r_4^3r_3^3r_2^3r_1^3r_0^3\wedge r_7^2r_6^2r_5^2r_4^2r_3^2r_2^2r_1^2r_0^2$ call it R_{BIG} - wedge product of linear functionals.