

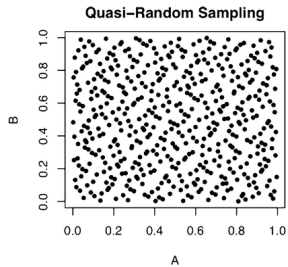
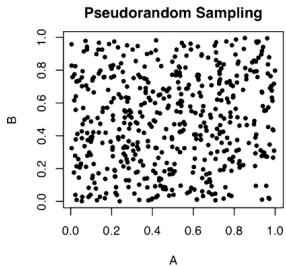
CS 595 Project: Quasi-Monte Carlo Integration using Lattice Cubature on GPU

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November 29, 2018

Introduction

- Monte Carlo to used for numerical integration of high dimensional problems.
- Traditional Monte Carlo uses pseudo random numbers (*rand,randn*).
- Branch of Monte Carlo is Quasi Monte Carlo that differs in how we chose our random inputs.
- Guaranteed Automatic Integration Library (GAIL)



Introduction

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QMC vs MC

- Why use Quasi-Monte Carlo?
- Generating lattices are not IID, some dependencies exists
- Gives us faster convergence than pseudo random IID numbers.
- Quasi Monte Carlo error almost $\mathcal{O}(\frac{1}{N})$ vs Monte Carlo is $\mathcal{O}(\frac{1}{\sqrt{N}})$.

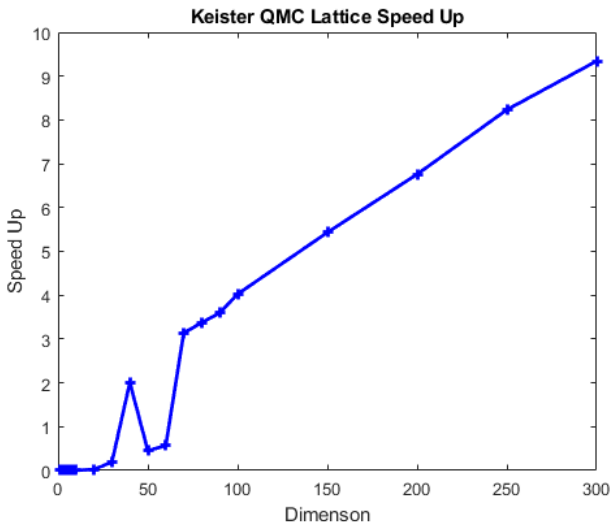
Results

- Generated lattices from 6-8 times faster at least.
- Function evaluations and averages computed on GPU.
- Using Keister example we can get speed ups to about 4-10 times faster over CPU for high dimensions.

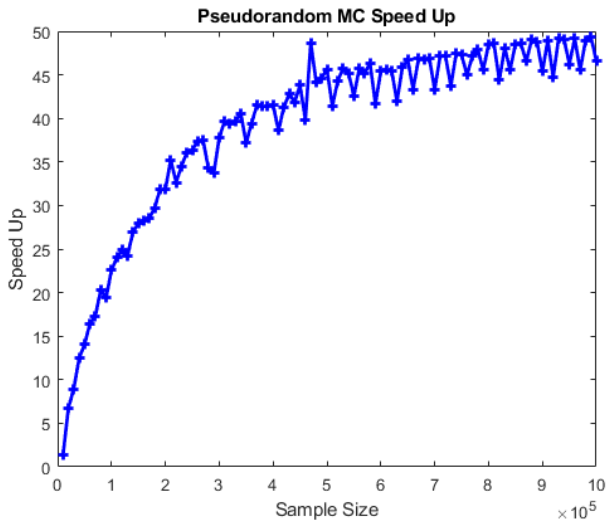
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$$\int_{\mathbb{R}^d} \cos(\|x\|) e^{-\|x\|^2} dx$$

QMC Keister Speed Up



Pseudorandom MC



Shortfalls

- We do not need FFT in parallel.
- In GAIL GPU implementation of *cublattice_g* is good, but...
- Complete integration into library is buggy.
- GPU may not always be faster than CPU depending on problem

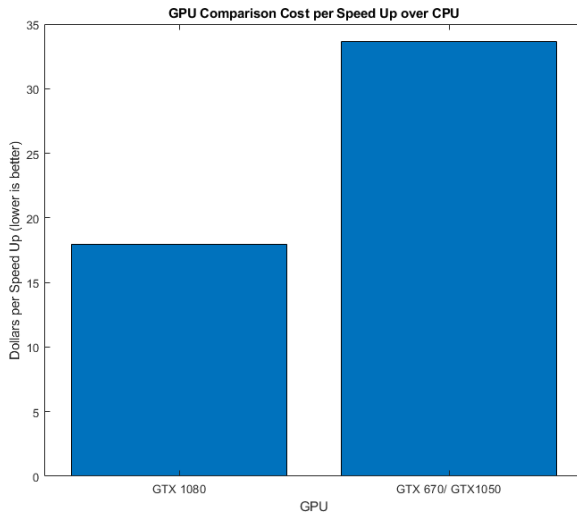
GPU Hardware Bias

- GPU computation is heavily hardware biased
- Variety of products and manufactures
- Measure dollars per performance speed up $\frac{\$}{\text{speed up}}$

MC Examples of Hardware Bias

- Easiest to see with a Pseudo-random number example
- IID best for GPU do not have to rely on dependent samples
- GTX 670 MSRP \$400 vs GTX 1080 (Desktop GPU) MSRP \$700

GPU Speed Up Cost



Further Work

- Full integration of *cublattice_g* to work with entirety of GAIL
- Extend GPU capabilities to IID Monte Carlo
- Other Quasi-Monte Carlo methods can sped up with GPU despite non IID sampling

References



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