Delta-Gap Model Justification

We assume the antenna is excited at a single angular frequency ω, so all fields are time-harmonic. This means the electric field can be written as:

$$\vec{E}(\vec{r},t) = \mathrm{Re}\left\{\vec{E}(\vec{r})e^{i\omega t}\right\}$$

In the frequency domain, Maxwell's equations give:

$$\vec{E}(\vec{r}) = -\nabla V(\vec{r}) - i\omega \vec{A}(\vec{r})$$

First of all, no tangential electric field can exist on the conductor surface. But a voltage perturbation (i.e. point-like voltage source) is introduced at the center of the antenna $z = 0$...

We now integrate the z-component of this expression along the wire axis (at the surface) over a small interval $[-\epsilon, \epsilon]$ centered at the feed point:

$$\int_{-\epsilon}^{+\epsilon} E_z(z)\,dz = -\int_{-\epsilon}^{+\epsilon} \frac{\partial V}{\partial z}\,dz - i\omega \int_{-\epsilon}^{+\epsilon} A_z(z)\,dz$$

We make the following assumptions:

- The vector potential $\vec{A}$ is generated by smooth, finite currents on the antenna (recall plots of current in standing wave antenna section). Thus, $A_z(z)$ is continuous near $z = 0$, and

$$\int_{-\epsilon}^{+\epsilon} A_z(z)\,dz \to 0 \quad \text{as } \epsilon \to 0$$

- The scalar potential $V(z)$ has a discontinuity across the small feed gap, representing an imposed voltage V_1. Thus,

$$\int_{-\epsilon}^{+\epsilon} \frac{\partial V}{\partial z}\,dz = V(-\epsilon) - V(+\epsilon) = -V_1$$

Therefore, we find:

$$\int_{-\epsilon}^{+\epsilon} E_z(z)\,dz = V_1$$

This motivates modeling the (idealized) electric field near the feed point as a delta function:

$$E_z(z) = -V_1\delta(z)$$

This is the essence of the **delta-gap source model**: a localized voltage drop across a small region of the antenna is idealized as a delta-function electric field, consistent with Maxwell's equations in the limit as $\epsilon \to 0$.