Minimization of the H_2 production cost

Operational Optimization - Mathematical formulation

Assumptions

(1) Cost related to H_2O consumption neglected
(2) Time step of one hour ($delta\_t=1h$)
(3) Total simulation over N_{day} days
(4) Linearization of the current voltage (polarization curves) for $j>2500 A/m^2$

Parameters

Operating pressure in the stack [bar]	P
Minimum operating temperature [°C]	T_{min}
Maximum operating temperature [°C]	T_{max}
Electrolyte KOH concentration [%wt]	Koh
Dynamic Hydrogen production degradation coefficient [€/Nm^3]	β_{dyn}
Nominal production capacity of the electrolyzer [Nm^3/h]	C_{nom}
Minimum production capacity to avoid shutdown (in [0,1])	$Coeff_{buff}$
Initial Hydrogen storage	S_{t0}
Nominal storage capacity [Nm^3]	S_{vess_nom}
Nominal battery pack capacity [kWh]	S_{batt_nom}
“Big M” method parameter to adjust the production efficiency [kWh]	$M1$
“Big M” method parameter to adjust the production capacity [Nm^3/h]	$M2$
“Big M” method parameter to adjust the production efficiency [kWh]	$M3$

Simulation parameters

Number of hours in the simulation [] (multiple of 24)	N_t
Number of days in the simulation []	N_{day}
Time step duration [h]	$delta\_t$
List of time steps	$\boldsymbol{T}=[0,N_t]$
List of days of the simulation	$D=[0,N_{day}]$

Decision variables

Hydrogen production rate [Nm^3/h]	$H2_f$
Hydrogen mass production rate [kg/h]	$H2_{mf}$
Hydrogen stored in at each time step t [Nm^3]	S_{in}
Hydrogen stored out at each time step t [Nm^3]	S_{out}
Electrolyzer turned On/Off [binary variable: 0->Off, 1->On]	y

Other variables

Cost due to degradation of the electrolyzer [€/h]	$\mathbf{K}_d$
Cost due to electricity purchases [€/h]	$\mathbf{K}_e$

Oxygen production rate [Nm^3/h]	$O2\_f$
Oxygen mass production rate [kg/h]	$O2\_mf$
Power consumption [kW]	ϕ
Actual Power consumption, accounting for cold/hot starts [kW]	ϕ_eff
Actual Power consumption, accounting for $\eta\_conso=\eta\_conso(H2\_mf)$ [kW]	ϕ_eff2
Power provided by day ahead market contract [kW]	ϕ_day
Power provided by spot on contract [kW]	ϕ_spot
Production efficiency [kWh/Nm^3]	η_conso
Hydrogen stored at time t [Nm^3]	S_t
Net amount of hydrogen coming out of the plant [kg]	$H2\_mf\_net$
Daily hydrogen contract allocation (flexible contracts) [kg/h]	$H2\_dc\_h$
Account for cold starts (binary: 0->Cold start, 1->No cold start)	$y1$

Inputs

Daily hydrogen delivery contract (flexible contracts) [kg/day]	$H2\_dc$
Hourly hydrogen delivery contract (fixed contracts) [kg/h] .	$H2\_hc$
Electricity price on the day ahead market [€/MWh]	π_da
Electricity price on the spot on market [€/MWh]	π_so

Objective function: minimization of the daily H2 production cost

$$\sum_{t\ in\ T} (\mathbf{K}\_e(t) + \mathbf{K}\_d(t))$$

Constraints:

- Cost of electricity purchase

  For $t \varepsilon \boldsymbol{T}$, $\mathbf{K}\_e(t)= (\phi\_spot(t)+\phi\_eff(t)-\phi(t))*\pi\_spot(t) + \phi\_reg(t)*\pi\_reg(t)$

- Energy balance

  For $t \varepsilon \boldsymbol{T}$, $\phi(t) = \phi\_day(t) + \phi\_spot(t)$

- Cost related to electrolyzer degradation

  For t=1, $\mathbf{K}\_d(t) = \beta_{stat}*H2\_f(t)$

  For $t \varepsilon \boldsymbol{T}$, $\mathbf{K}\_d(t) = \beta_{stat}*H2\_f(t) + \beta_{dyn}*(H2\_f(t) - H2\_f(t-1))$

- Limited production capacity (design optimisation)

  $C\_nom<C\_nom\_max$

  $C\_nom>C\_nom\_min$

- Limited storage capacity (design optimisation)

$$S\_nom < S\_nom\_max$$

$$S\_nom > S\_nom\_min$$

- Detection of whether the electrolyzer is on or off

For $t \varepsilon \boldsymbol{T}$, H2_f(t) = y(t)*H2_f(t)

- Detection of cold start (electrolyzer off for the previous 3 time steps)

For $t \varepsilon \boldsymbol{T} \backslash \{0,1,2,3\}$, $\sum_{j=i-4}^{i-1}$ H2_f(j) >= C_nom_min*y1(i)

- Limited production and account for cold starts (3 min delay) and hot starts (15 sec delay)

For $t \varepsilon \boldsymbol{T}$, H2_f(t)<C_nom

For $t \varepsilon \boldsymbol{T} \backslash \{0,1,2,3\}$, H2_f(t)<=57/60*C_nom+M2*y1(t)

For $t \varepsilon \boldsymbol{T} \backslash \{0,1\}$, H2_f(t)<=(1-15/3600)*C_nom+M2*y(t-1)

For $t \varepsilon \boldsymbol{T}$, H2_f(t)>Coeff_b.C_nom

- Power input, accounting for the loss in efficiency when there is a cold start

For $t \varepsilon \boldsymbol{T}$, ϕ(t)<=ϕ_eff(t)

For $t \varepsilon \boldsymbol{T} \backslash \{0,1,2,3\}$, 2*$\phi$(t)-M1*y1(t)<=$\phi$_eff(t)

For $t \varepsilon \boldsymbol{T} \backslash \{0,1,2,3,4\}$, 5/4*$\phi$(t)-M1*y1(t-1)<=$\phi$_eff(t)

- Hydrogen net output at each time step

For $t \varepsilon \boldsymbol{T}$, H2_mf_net(t)=H2_mf(t)+S_out(t)-S_in(t)

- Flexible Hydrogen delivery contracts ensured

For $d \varepsilon \mathbf{D}$, H2_dc(d)= $\sum_{t=24*d}^{24*(d+1)}$ H2_dc_h(t)

- Hydrogen balance between net output of the plant and delivery

For $t \varepsilon \boldsymbol{T}$, H2_mf_net(t)=H2_hc(t)+H2_dc_h(t)

- Constraints on the operating Temperature of the electrolyzer

  For $t \in \boldsymbol{T}$, $T(t)>T\_min$

  For $t \in \boldsymbol{T}$, $T(t)<T\_max$

- Hydrogen production knowing the inlet power (accounts for initial cold start)

  $\phi(0)=2*\eta\_prod(t)*H2\_mf(0)$

  $\phi(1)=5/4\eta\_prod(t)*H2\_mf(1)$

  $\phi(2)=4/3*\eta\_prod(t)*H2\_mf(2)$

  For $t \in \boldsymbol{T}\backslash\{0,1,2\}$, $\phi(t)=\eta\_prod(t)*H2\_mf(t)$

  For $t \in \boldsymbol{T}$, $H2\_f(t)=Coeff\_mff*H2\_mf(t)$

- Hydrogen storage balance

  For $t \in \boldsymbol{T}$, $S\_t(t+1)=S\_t(t)+S\_in(t)-S\_out(t)$

- Maximum storage capacity

  For $t \in \boldsymbol{T}$, $S\_t(t)<S\_nom$

- Positivity constraints

  For $t \in \boldsymbol{T}$, $S\_t(t)>= 0$

  For $t \in \boldsymbol{T}$, $S\_in\_t(t)>=0$

  For $t \in \boldsymbol{T}$, $S\_out\_t>=0$

Indicators (post-simulation)

- O2 production related to H2 production ($H_2O \rightarrow H_2+½*O_2$)

  For $t \in \boldsymbol{T}$, $O2\_mf(t) = ½*M(O2)/M(H2)*H2\_mf(t)$

- Hydrogen production efficiency

  For $t \in \boldsymbol{T}$, $\eta\_conso(t)=\phi(t)/H2\_f(t)$

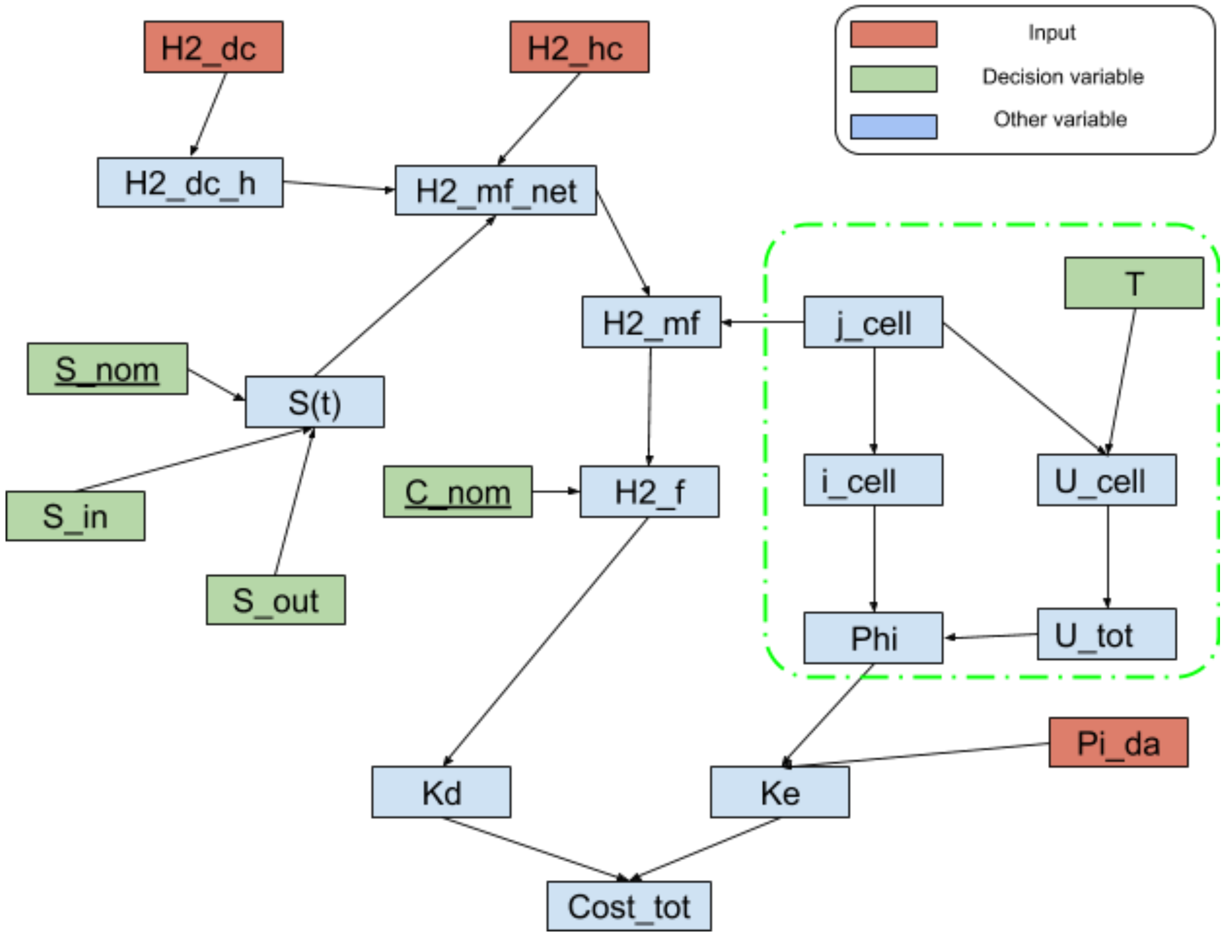
- Portion of the net Hydrogen outflow of the plant going for flexible contracts

  For $t \in \boldsymbol{T}$, $\alpha(t)=H2\_dc\_h(t)/(H2\_dc\_d(t)+H2\_hc(t))$

Parameters values

M(H2) = 32 g/mol
M(O2) = 2.016 g/mol
Coeff_mff =1/0.0899
Coeff_f=1/(2*96 485)*44.61*3600*0.0899

Schematic of the steps to evaluate
the production costs of hydrogen


Input
Decision variable
Other variable
H2_dc
H2_hc
H2_dc_h
H2_mf_net
H2_mf
j_cell
T
S_nom
S(t)
S_in
S_out
C_nom
H2_f
i_cell
U_cell
Phi
U_tot
Pi_da
Kd
Ke
Cost_tot