Anatomy of Dynamical Systems

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Dynamical System

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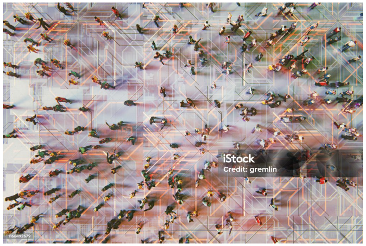

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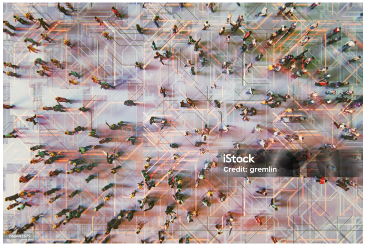

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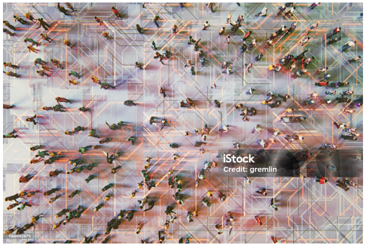

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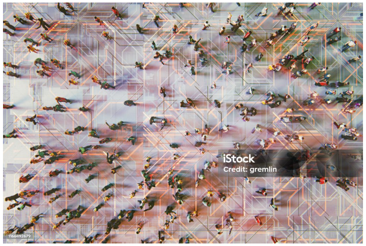

Dynamical System

Consider the following autonomous ordinary differential equation (ODE):

$$\frac{d\mathbf{x}}{dt} = \mathbf{F}(\mathbf{x}) \tag{1}$$

where $\mathbf{F}(\mathbf{x})$ is vector valued function of $\mathbf{x}$.

Dynamical System

Consider the following non-autonomous ordinary differential equation (ODE):

$$\frac{d\mathbf{x}}{dt} = \mathbf{F}(t, \mathbf{x}) \tag{2}$$

where $\mathbf{F}(t, \mathbf{x})$ is a vector valued function of both t and $\mathbf{x}$.

Dynamical System

Consider the following non-autonomous ordinary differential equation (ODE) with control $u(t)$ and parameters p:

$$\frac{d\mathbf{x}}{dt} = \mathbf{F}(t, \mathbf{x}, \mathbf{u}; \mathbf{p}) \tag{3}$$

where $\mathbf{F}(t, \mathbf{x}, u, p)$ is a function of time t, state $\mathbf{x}$, control $\mathbf{u}$, and parameters $\mathbf{p}$.

Navier-Stokes equation

The Navier-Stokes equation for an incompressible fluid in Cartesian coordinates is given by:

$$\rho\left(\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u}\cdot\nabla)\mathbf{u}\right) = -\nabla p + \nu\nabla^2\mathbf{u} + \mathbf{f} \tag{4}$$

with the continuity equation:

$$\nabla\cdot\mathbf{u} = 0 \tag{5}$$

where **u** represents the velocity vector field, t is time, p is pressure, ρ is density, ν is the kinematic viscosity, and **f** denotes external forces.

Schrödinger wave equation

The time-dependent Schrödinger wave equation is given by:

$$i\hbar\frac{\partial\Psi}{\partial t} = \hat{H}\Psi \tag{6}$$

where i is the imaginary unit, $\hbar$ is the reduced Planck constant, Ψ is the wave function, t is time, and $\hat{H}$ is the Hamiltonian operator.

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6. Noise, Stochastic
7. Uncertainty

Uses of models

1. Predict using ‘f’ (ensemble)

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4. Understanding mechanisms (interpretable)
5. Generalize

Why Model?

Everybody models - all of you forecast the future in some way, predicting the way a friend reacts, planning your future[1]

[1] Epstein, "Why Model?"

Why Model?

1. Explain (very distinct from predict)
2. Guide data collection
3. Illuminate core dynamics
4. Suggest dynamical analogies
5. Discover new questions
6. Promote a scientific habit of mind
7. Bound outcomes to plausible ranges
8. Illuminate core uncertainties.
9. Offer crisis options in near-real time
10. Demonstrate tradeoffs / suggest efficiencies
11. Train practitioners
12. Discipline the policy dialogue
13. Educate the general public
14. Reveal the apparently simple (complex) to be complex (simple)

Why Model?

Explain (very distinct from predict)

Plate tectonics describe the mechanism of earthquakes, but doesn't allow us to predict.
generative explanation, where large scale regularities such as wealth distributions, spatial settlement patterns, or epidemic dynamics—emerge in populations of heterogeneous software individuals (agents) interacting locally under plausible behavioral rules.

Why Model?

Guide Data Collection

Inductivism:: 'Science proceeds from observation, and then models are constructed to 'account for' the data.' First collects lots of data and then runs regressions on it or machine learning.
A model is simply a theory - the model may predict a particular dynamic that you would need to measure to corroborate. This happens a lot in Physics - Maxwell's equations, Higg's Boson, etc.

Why Model?

Illuminate Core Dynamics: All the Best Models are Wrong

A lot of ABMs do this, this is really about furthering scientific understanding. Building the simplest theory you can do describe the processes and dynamics of a system without it necessarily being predictive.
The Lotka-Volterra ecosystem model, Hooke's Law, or the Kermack-McKendrick epidemic equations. They continue to form the conceptual foundations of their respective fields

Why Model?

Suggest Analogies

It is a startling and wonderful fact that a huge variety of seemingly unrelated processes have formally identical models (i.e., they can all be seen as interpretations of the same underlying formalism).
Do revolutions, or religions, or the adoption of innovations unfold like epidemics? Is it useful to think of these processes as formal analogues? If so, then a powerful pre-existing theory can be brought to bear on the unexplored field, perhaps leading to rapid advance. This type of thinking/science is endemic in Complexity Science.

Why Model?

To me, however, the most important contribution of the modeling enterprise—as distinct from any particular model, or modeling technique—is that it enforces a scientific habit of mind, which I would characterize as one of militant ignorance—an iron commitment to "I don't know."

— [Joshua M. Epstein[a]]

[a] Epstein, "Why Model?"