Rationality, Risk aversion and Economic theories

Decision making under uncertainity

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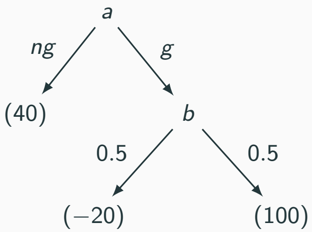
Risk and uncertainity

A gamble



Expected Value

How much is a risky decision, worth? It is simply the sum of all the possible outcomes of a gamble, multiplied by their respective probabilities.

Say you're feeling lucky one day, so you join your office betting pool on a derby and place 10 EUR on a horse, at 25/1 odds. You know that in the unlikely event of the horse winning the race, you'll be richer by 10 * 25 = 250 EUR.

What this means is that, according to the bookmaker of the betting pool, horse has a one in 25 chance of winning and a 24 in 25 chance of losing, or, to phrase it mathematically, the probability that the horse will win the race is 1/25.

So what's the expected value of your bet?

Expected Value - Illustration

Well, there are two possible outcomes - either the horse wins the race, or it doesn't. If it wins, you get 250; otherwise, you get nothing. So the expected value of the gamble is: $(250 * 1/25) + (0 * 24/25) = 10 + 0 = 10$ And 10 EUR is exactly what you would pay to participate in the gamble.

Expected Value - Illustration

A pharmaceutical company faced with the opportunity to buy a patent on a new technology for 200 million, might know that there would be a 20% chance that it would enable them to develop a life-saving drug that might earn them, say 500 million; a 40% chance that they might earn 200 million from it; and a 40% chance that it would turn out worthless.

The expected value of this patent would then be: $(500{,}000{,}000 * 0.2) + (200{,}000{,}000 * 0.4) + (0 * 0.4) = 180$ million .So of course, it would not make sense for the firm to take the risk and buy the patent.

Risk vs Uncertainity

We are going to differentiate between two ways in which the future may not be known

- Horse races
- Roulette wheels

What is the difference?

When playing a roulette wheel the probabilities are known

- Everyone agrees on the likelihood of black
- So we (the researcher) can treat this as something we can observe
- Probabilities are objective

This is a situation of risk

Risk vs Uncertainity

When betting on a horse race the probabilities are unknown

- Different people may apply different probabilities to a horse winning
- We cannot directly observe a person's beliefs
- Probabilities are subjective

This is a situation of uncertainty (or ambiguity)

Risk vs Uncertainity

Risk: Probabilities are observable

- There are 38 slots on a roulette wheel
- Someone who places a \$10 bet on number 7 has a lottery with pays out \$350 with probability 1/38 and zero otherwise

Uncertainty: Probabilities are not observable

- Say there are 3 horses in a race
- Someone who places a \$10 bet on horse - does not necessarily have a 1/3 chance of winning.

Choice under risk

So, how should you make choices under risk? Let's consider the following (very boring) fairground game.

- You flip a coin
- If it comes down heads you get \$10
- If it comes down tails you get \$0

What is the maximum amount x that you would pay in order to play this game?

Choice under risk (Expected value)

You have the following two options

- Not play the game and get \$0 for sure
- Play the game and get $-x$ with probability 50% and $10 - x$ with probability 50%

The expected amount that you would earn from playing the game is

- $0.5(-x) + 0.5(10 - x)$; This is bigger than 0 if
- $0.5(-x) + 0.5(10 - x) > 0$
- $5 > x$

Should pay at most \$5 to play the game

The St. Petersburg Paradox

Daniel Bernoulli suggested the following modification of the game

- Flip a coin
- If it comes down heads you get \$2
- If tails, flip again
- If that coin comes down heads you get \$4
- If tails, flip again
- If that comes down heads, you get \$8
- Otherwise flip again and so on ..

How much would you pay to play this game?

The St. Petersburg Paradox

The expected value is:

$$1/2 * 2 + 1/4 * 4 + 1/8 * 8 + ... = 1 + 1 + 1.... = \infty$$

So you should pay an infinite amount of money to play this game. Which is why this is St. Petersburg paradox!!

The St. Petersburg Paradox

So what is going wrong here? Consider the following example:

Say a pauper finds a magic lottery ticket, that has a 50% chance of $1 million and a 50% chance of nothing. A rich person offers to buy the ticket for $499,999 for sure. According to our expected value method, the pauper should refuse the rich person's offer!

Marginal utility

Bernoulli argued that people should be maximizing expected utility not expected value. $u(x)$ is the expected utility of an amount x.

Moreover, marginal utility should be decreasing. The value of an additional dollar gets lower the more money you have. For example:

$$u(0) = 0$$

$$u(499,999) = 10$$

$$u(1,000,000) = 16$$

Under this scheme, the pauper should choose the rich person's over as long as

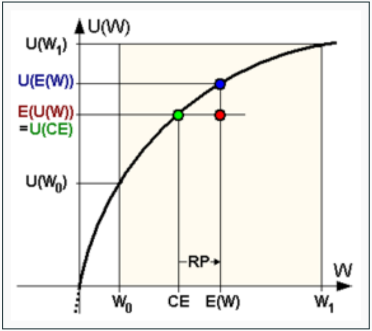
$$1/2 * u(1,000,000) + 1/2 * u(0) < u(499,999)$$

Using the numbers on the previous slide, LHS=8, RHS=10 Pauper should accept the rich persons offer

- Bernoulli suggested $u(x) = ln(x)$
- Also explains the St. Petersburg paradox
- Using this utility function, should pay about $64 to play the game

Notice that if people maximize expected utility and have decreasing marginal utility (i.e. utility is concave). They will be risk averse – will always reject a lottery in favor of receiving its expected value for sure.

if they would accept a certain payment (certainty equivalent) of less than $50 (for example, $40), rather than taking the gamble and possibly receiving nothing.



Risk Aversion

$$f(x) = \begin{cases} \dfrac{x^\lambda - 1}{\lambda} & \text{if } \lambda \neq 0 \\ \ln(x) & \text{if } \lambda = 0 \end{cases}$$

Risk Aversion

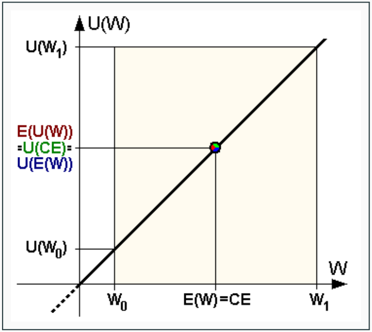
The utility function for risk aversion, known as the Arrow-Pratt utility function, is given by:

$$U(W) = \frac{W^{1-\gamma}}{1-\gamma}$$

where W is the wealth and γ is the coefficient of absolute risk aversion.

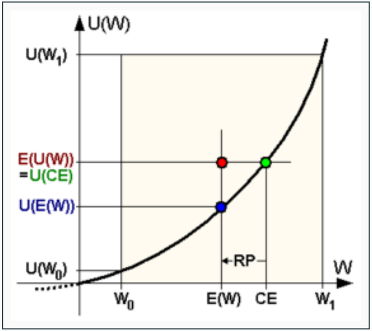
Risk neutral

if they are indifferent between the bet and a certain payment.



Risk loving

if they would accept the bet even when the guaranteed payment is more than $50 (for example, $60)



$$U(c) = \begin{cases} (1 - e^{-ac})/a & \text{if } a \neq 0 \\ c & \text{if } a = 0 \end{cases}$$

a>0 for risk aversion, a=0 for risk-neutrality, or a<0 for risk-seeking.

$U(W) = W - k \cdot \sigma^2$

Where:

1. As wealth (W) increases, utility U(W) increases linearly.
2. $-k \cdot \sigma^2$ represents the penalty due to risk

Risk Modelling

$$U(W) = W - k \cdot \sigma^2$$

1. k represents the individual's risk aversion coefficient.
2. σ^2 represents the variance of wealth, which reflects the level of risk.

Risk Modelling

Different individuals might have different levels of risk aversion, so the value of k could vary. For instance, risk-averse individuals would have higher values of k, indicating a stronger aversion to risk, while risk-seeking individuals would have lower values of k or even negative values, indicating a preference for risk.

This is just one simple form of a utility function for risk, and there are many more sophisticated models depending on the context and assumptions of risk behavior.

Condition	**Definition**	**Implication**
Risk Aversion	Reject fair gamble	$U''(0) < 0$
Risk Neutral	Indifferent to fair gamble	$U''(0) = 0$
Risk Seeking	Select a fair gamble	$U''(0) > 0$

Arrow-Pratt measure of absolute risk aversion

$A(W) = -\frac{U''(W)}{U'(W)}$

- Increasing absolute risk aversion - As wealth increases hold lesser monetary value in risky assets - A'(w) $>$0
- Constant absolute risk aversion - As wealth increases hold same monetary value in risky assets - A'(w) $=$ 0
- Decreasing absolute risk aversion - As wealth increases hold more monetary value in risky assets - A'(w) $<$0

Economic Theories

Several other economic theries

- Prospect theory
- Protection motivation theory
- Theory of planned behaviour
- Regret theory
- Dual theory
- ....

Prospect Theory

Prospect theory, proposed by Kahneman and Tversky in 1979, is a descriptive model for decision-making under uncertainty. It suggests that people do not evaluate outcomes in absolute terms, but rather in terms of changes from a reference point. The utility function for prospect theory typically takes the following form:

$$U(x) = \pi(p_i)v(x_i) + \pi(p_j)v(x_j)$$

where x is the outcome, $U(x)$ is the overall utility, $v(x)$ is the value of the outcome , and π is a weighting function.

Prospect Theory

The value function in prospect theory is typically represented as:

$$V(x) = \begin{cases} x^{\alpha} & \text{if } x \geq 0 \\ -\lambda(-x)^{\beta} & \text{if } x < 0 \end{cases}$$

where:

$V(x)$ is the value function

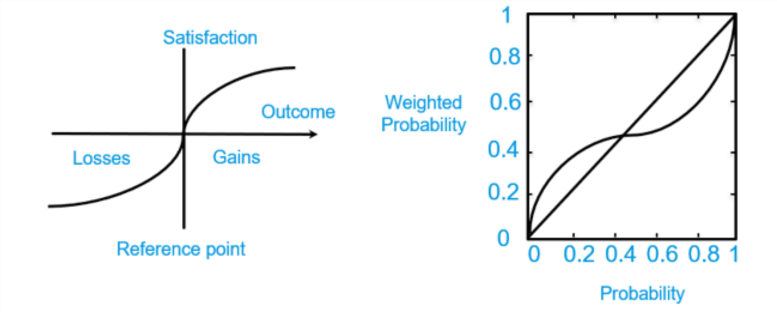
x is the outcome

α is the positive gain sensitivity parameter

β is the loss aversion parameter

λ is the loss aversion parameter

Prospect Theory



Prospect Theory

The weighting function in prospect theory is typically represented as:

$$w(p) = \frac{p^\gamma}{(p^\gamma + (1-p)^\gamma)^{1/\gamma}}$$

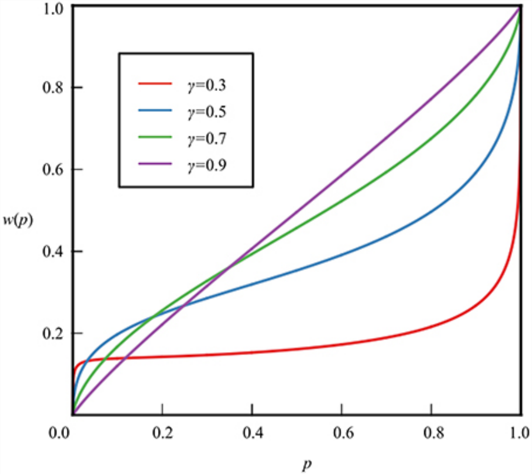
where:

$w(p)$ is the weighting function

p is the probability of an outcome

γ is the probability weighting parameter

Prospect Theory



Rationality

Perfect rationality

The perfect rationality of homo economicus imagines a hypothetical agent who has complete information about the options available for choice, perfect foresight of the consequences from choosing those options, and the wherewithal to solve an optimization problem (typically of considerable complexity) that identifies an option which maximizes the agent's personal utility. The meaning of ‘economic man’ has evolved from John Stuart Mill's description of a hypothetical, self-interested individual who seeks to maximize its personal utility.

The Emergence of Procedural Rationality

How well an organism performs in terms of accuracy (section 8.2) with its limited cognitive resources in order to investigate models with comparable levels of accuracy within those resource bounds. Effectively managing the trade-off between the costs and quality of a decision involves another type of rationality, which Simon later called procedural rationality

Satisficing

Satisficing is the strategy of considering the options available to you for choice until you find one that meets or exceeds a predefined threshold—your aspiration level—for a minimally acceptable outcome.

Satisficing

Aumann advanced arguments for bounded rationality, which we paraphrase here (1997).

- Even in very simple decision problems, most economic agents are Satisficiers and not (deliberate) maximizers. People do not scan the choice set and consciously pick a maximal element from it.
- Even if economic agents aspired to pick a maximal element from a choice set, performing such maximizations are typically difficult and most people are unable to do so in practice. Biases and Heurestics
- Experiments indicate that people fail to satisfy the basic assumptions of rational decision theory.
- Some conclusions of rational analysis appear normatively unreasonable.

Bounded rationality

Herbert Simon introduced the term 'bounded rationality' (Simon 1957b: 198; see also Klaes & Sent 2005) as a shorthand for his brief against neoclassical economics and his call to replace the perfect rationality assumptions of homo economicus with a conception of rationality tailored to cognitively limited agents.

Broadly stated, the task is to replace the global rationality of economic man with the kind of rational behavior that is compatible with the access to information and the computational capacities that are actually possessed by organisms, including man, in the kinds of environments in which such organisms exist. (Simon 1955a: 99)

Bounded rationality

Boundedly rational procedures are in fact fully optimal procedures when one takes account of the cost of computation in addition to the benefits and costs inherent in the problem as originally posed (Arrow 2004).

Models of Bounded rationality

Many exists -few common ones:

- epsilon-optimization - choose our actions so that the payoff is within epsilon of the optimum.
- satisficing - Search available options and choose the first one that exceeds your aspiration level. (Simon 1955a; Hutchinson et al. 2012).

Superrationality

In economics, a participant is considered to have superrationality: perfect rationality (and thus maximize their utility) & assume that all other players are superrational too and that all superrational individuals will come up with the same strategy when facing the same problem.

		Player j C	Player j D
Player i	C	3	0
Player i	D	5	1

Prisoner's Dilemma payoff matrix

Applying this definition, a superrational player playing against a superrational opponent in a prisoner's dilemma will cooperate while a rationally self-interested player would defect.

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